Recursive T-Matrix Algorithm for Multiple Metallic Cylinders *

Adnan Şahin and Eric L. Miller
Center for Electromagnetics Research,
235 Forsyth Building, 360 Huntington Ave.
Northeastern University, Boston, MA 02115
Telephone: (617) 373-8386
Telefax : (617) 373-8627
email: adnan@cdsp.neu.edu

February 14, 1997

Abstract

We present a new application of the recursive T-matrix algorithm to calculate the scattered field from a single or multiple metallic cylinders of arbitrary shapes. Using the equivalence theorem each metallic object is replaced with small metallic cylinders along its perimeter, then scattered fields are calculated using the recursive T-matrix algorithm. Results are verified with those in the literature and analytical calculations.

Keywords: Scattering, recursive T-matrix algorithm

*This work was supported in part DOE contract DE-FC07-95ID13395, NSF Grant MIP-9623721, and by subcontract GC123920NDG from Boston University under the AFOSR MURI Program on Reduced Signature Target Recognition

1 Introduction

The transition matrix (T-matrix) technique is known to be an efficient electromagnetic forward solver for scattering problems involving objects with simple shapes. Finite difference and method of moments (MoM) techniques both discretize the entire region of interest resulting in a large number of unknowns, whereas T-matrix methods replace this discretization with harmonic expansions of the fields thereby reducing the number of unknowns for numerous problems. Waterman [1] developed the T-matrix technique for single metallic or dielectric scatterers. Peterson et.al. [2] introduced an iterative algorithm which finds the scattering due to multiple scatterers. Recently Chew and co-workers have developed a number of fast, recursive T-matrix algorithms for determining the scattered fields in a variety of scenarios [3–9]. Among this work, in [4], problems involving electrically large dielectric objects are considered. By tesselating the objects into many small cylindrical sub-scatterers and using multipole expansions of the fields for each sub-scatterer the authors arrive at a highly efficient, T-matrix based algorithm for computing the scattered fields. In [7], Chew et.al. consider a scattering problem involving a group of metallic strips. Here the method of moments is used to compute the T-matrices for each, individual strip and the same recursion as in [6] is employed to solve the overall, multi-object scattering problem. In [8] the scattered field from an ogive shaped scatterer with metallic and dielectric parts is found using the recursive T-matrix algorithm. In that paper, as in [7], the metallic scatterer is decomposed into a collection of strips arrayed about the boundary and the T-matrices for the individual strips are found using the method of moments.

In this letter we consider an alternative sub-scatterer discretization for metallic objects from that in [7, 8]. Instead of using metallic strips to model the perimeter of scatterers, we use metallic cylinders (similar to the concept in [4]) placed about the perimeter and employ the same recursive

algorithm given in [4] to calculate the scattered field. As a result, we obtained an accurate, efficient forward solver which does not require the use of method of moments to form the single scatterer T-matrices. Rather we obtained these quantities by using closed form, low order harmonic expansions associated with the small metallic cylinders. We apply this method to single electrically large metallic objects and verify the results with those in literature and analytical results. Additionally we demonstrate the usefulness of the method for the multi-object case by verifying against previously published results.

2 Recursive T-matrix Algorithm

The algorithms in [3, 4, 6, 7, 9] are order recursive methods for constructing the T-matrix for a multi-object scattering problem given the T-matrices for each individual object. The algorithm uses the basic principle of the single scatterer T-matrix formulas in that for each object, the scattered fields from others are assumed a part of a total incident field. The recursion starts with the T-matrices of individual scatterers, then one by one scatterers are incorporated into the equation and the T-matrices are updated until the final form of the T-matrix, including all multiple scattering effects, is obtained.

Formally, for L scatterers, the harmonic expansion of scattered field can be written as [4]:

$$\psi^{sca}(\underline{r}) = \sum_{i=1}^{L} \underline{\psi}^T(\underline{r_i})\mathbf{T}_{i(L)}\boldsymbol{\beta}_{i,0}\underline{a} \tag{1}$$

where $\mathbf{T}_{i(L)}$ is the T-matrix for ith object in the presence of L scatterers, $\underline{a}$ is the coefficient vector used in the expansion of the incident plane wave in terms of cylindrical basis functions and $\boldsymbol{\beta}_{i,0}$ is the translation matrix used to translate same type basis functions between reference coordinate systems. (The translation matrices $\boldsymbol{\beta}_{i,0}$ contain Bessel functions and complex exponentials. For details about these matrices see [2, 3].) Expansion of the scattered field in (1) is valid if all observation points

are outside the smallest circle enclosing all scatterers. Following Chew's derivation, the recursive construction of $\mathbf{T}_{i(L)}$ can be written as [4, eq.10-11] :

$$\mathbf{T}_{n+1(n+1)}\boldsymbol{\beta}_{n+1,0} = \left[\mathbf{I} - \mathbf{T}_{n+1(1)}\sum_{i=1}^{n}\boldsymbol{\alpha}_{n+1,i}\mathbf{T}_{i(n)}\boldsymbol{\beta}_{i,0}\boldsymbol{\alpha}_{0,n+1}\right]^{-1}\mathbf{T}_{n+1(1)}\left[\boldsymbol{\beta}_{n+1,0} + \sum_{i=1}^{n}\boldsymbol{\alpha}_{n+1,i}\mathbf{T}_{i(n)}\boldsymbol{\beta}_{i,0}\right] \tag{2}$$

and

$$\mathbf{T}_{i(n+1)}\boldsymbol{\beta}_{i,0} = \mathbf{T}_{i(n)}\boldsymbol{\beta}_{i,0} + \mathbf{T}_{i(n)}\boldsymbol{\beta}_{i,0}\boldsymbol{\alpha}_{0,n+1}\mathbf{T}_{n+1(n+1)}\boldsymbol{\beta}_{n+1,0} \tag{3}$$

where $n = 1, 2, ..., L$, $i = 1, 2, ..., n$ and $\boldsymbol{\alpha}_{n,i}$ is another translation matrix [3].The recursion starts with the individual T-matrices, $\mathbf{T}_{i(1)}$, of the scatterers, i.e. the T-matrix of the ith scatterer when there are no other scatterers in the medium.

Theoretically the matrices $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$, $\mathbf{T}$ are of infinite dimension. The T-matrix algorithms truncate these matrices with finite values N and M such that the residual error is below the machine precision or acceptable levels. Here N represents the number of harmonics used to expand the fields at the scattering origin and M represents the number of harmonics used to expand the fields in the objects' local coordinate systems. Thus, the T-matrix is of size $M \times M$, $\boldsymbol{\beta}_{i,0}$ is of size $M \times N$ and $\boldsymbol{\alpha}_{i,n+1}$ is of size $M \times M$. It has been shown that computational complexity of (2)-(3) is $O(L^2M^2N)$ for L scatterers [4].

The contribution of this letter is to show that, based on the equivalence theorem, recursive T-matrix algorithms can be used to calculate the scattered fields from metallic objects by placing small metallic cylinders on their perimeter. Traditionally, the recursive T-matrix algorithm has been applied in one of two manners. In the case of dielectric scatterers, the whole object was decomposed into small cylinders, low order harmonic expansions were used to represent the fields from each object, and the recursive algorithm was used to solve the scattering problem. For metallic objects, the equivalence theorem was used to decompose the surface of the scatterer into small

strips, moment methods were then employed to find the individual T-matrices for each strip, and the same T-matrix recursions were used to solve the overall scattering problem. The objective of this letter is to show that one may make use of the cylinder approach (Fig.1) as well for the metallic scattering problem and still obtain highly accurate solutions. In particular, by using cylinders, one may employ the closed-form harmonic expansion method to find the individual scatterers thereby avoiding the moment method computation. In the next section, we will give examples of scattering from circular and rectangular cylinders and the results are verified with those in the literature or analytical calculations.

3 Examples and Discussions

In this section we verify that replacing metallic objects with small metallic cylinders along their perimeters actually produces the results reported in the literature or results obtained analytically. First we define the terms used in this section. The normalized scattering field pattern is defined as:

$$F(\phi) = 10\log_{10}\left\{\lim_{r\to\infty} 2\pi r \frac{|\psi^{sca}(\underline{r})|^2}{\max\{|\psi^{sca}(\underline{r})|^2\}}\right\}. \tag{4}$$

Normalized power density, or the "gain", is defined as:

$$G_E(\phi) = \lim_{r\to\infty} \frac{|\psi^{sca}(\underline{r})|^2}{\frac{1}{2\pi}\int_0^{2\pi} |\psi^{sca}(\underline{r})|^2 d\phi}. \tag{5}$$

In all examples the cylinders are embedded in free space with an E_z polarized plane wave incident on them. The first example is the scattering from a single circular cylinder of radius 0.8λ ($ka = 5$). As seen in Fig. 2(a), the cylinder is approximated by 60 smaller cylinders along its circumference which corresponds to a sampling 12 cylinders per wavelength. The normalized scattering field pattern, $F(\phi)$, obtained from the recursive algorithm is plotted against the analytical solution in Fig. 2(b). The second example is the scattering from two circular cylinders with radii of 0.8λ ($ka = 5$) and separated by a distance 2.55λ ($kd = 16$). As in previous example, each cylinder is

approximated by 60 small metallic cylinders with 12 cylinders per wavelength. Fig. 3(a) shows the scattering geometry and Fig. 3(b) compares the normalized scattering patterns obtained using the recursive T-matrix algorithm with those given in [10]. The last example shows the normalized power densities for a slanted rectangular cylinder for two different sizes. The geometry is shown in Fig. 4(a) and the far field power densities, $G_E(\phi)$, for $ka = 3$ and $ka = 5$ ($a = 0.48\lambda$ and $a = 0.8\lambda$, both with $a = 2b$) are depicted in Fig. 4(b). For both cases, the perimeter is sampled at approximately 13 cylinders per wavelength. In this figure, the scattering patterns are compared with the results reported in [11]. As these plots have shown, the scattered fields from metallic objects can be found by replacing these objects with smaller cylinders along the perimeter and then using the recursive T-matrix algorithm of [4].

4 Conclusion

In this letter an alternative discretization along the perimeter of metallic scatterers is used with recursive T-matrix algorithm to calculate the scattered field. The results are verified with previous work.

References

[1] P.C. Waterman. New formulation of acoustic scattering. *J. Acoust. Soc. Am.*, 45(6):1417–1429, 1969.

[2] B. Peterson S. Ström. Matrix formulation of acoustic scattering from an arbitrary number of scatterers. *J. Acoust. Soc. Am.*, 56(3):771–780, September 1974.

[3] W.C. Chew. *Waves and Fields in Inhomogeneous Media.* Van Nostrand Reinhold, 1990.

[4] Y.M. Wang W.C. Chew. An efficient algorithm for solution of a scattering problem. *Microwave and Optical Technology Letters*, 3(3):102–106, March 1990.

[5] W.C. Chew J.A. Friedrich R. Geiger. A multiple scattering solution for effective permittivity of a sphere mixture. *IEEE Trans. Geoscience and Remote Sensing*, 28(2):207–214, March 1990.

[6] W.C. Chew Y.M. Wang. A fast algorithm for solution of a scattering problem using a recursive aggregate τ matrix method. *Microwave and Optical Technology Letters*, 3(5):164–169, May 1990.

[7] W.C. Chew Y.M. Wang L. Gürel. Recursive algorithm for wave-scattering solutions using windowed addition theorem. *Journal of Electromagnetic Waves and Applications*, 6(11):1537–1560, 1992.

[8] J.-H. Lin W.C. Chew. Scattering solution of oblong, dielectric coated, metallic scatterers by recursive algorithm. *Electronics Letters*, 28(2):185–187, January 1992.

[9] W.C. Chew L. Gürel Y.M. Wang G. Otto R.L. Wagner Q.H. Liu. A generalized recursive algorithm for wave-scattering solutions in two dimensions. *IEEE Trans. on Microwave Theory and Techniques*, 40(4):716–722, April 1992.

[10] A.Z. Elsherbeni M. Hamid. Scattering by parallel conducting circular cylinders. *IEEE Trans. on Antennas and Propagation*, 35(3):355–358, March 1987.

[11] K.K. Mei J.G. Van Bladel. Scattering by perfectly conducting rectangular cylinders. *IEEE Trans. on Antennas and Propagation*, 11:185–192, 1963.

Figure Captions

Figure 1: Tessellation of metallic cylinders along their perimeters

Figure 2: Comparison of normalized scattering field pattern calculated using the recursive T-matrix algorithm with the analytically calculated one.

Figure 3: Comparison of normalized scattering field pattern calculated using the recursive T-matrix algorithm with [10].

Figure 4: Comparison of normalized power density calculated using the recursive T-matrix algorithm with [11].

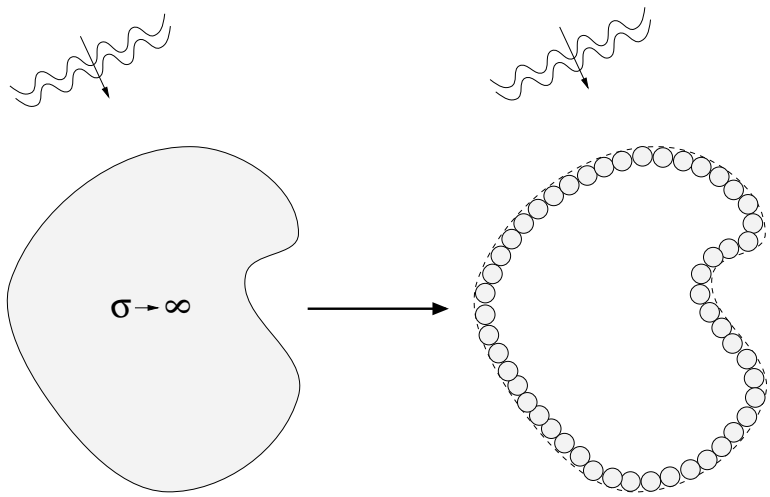
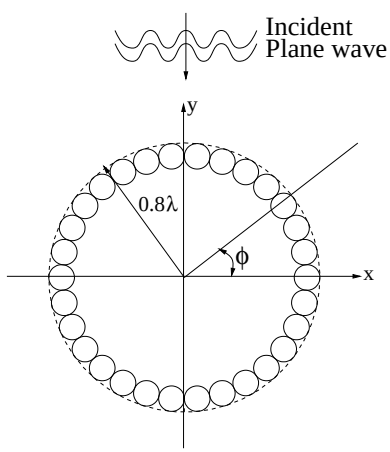
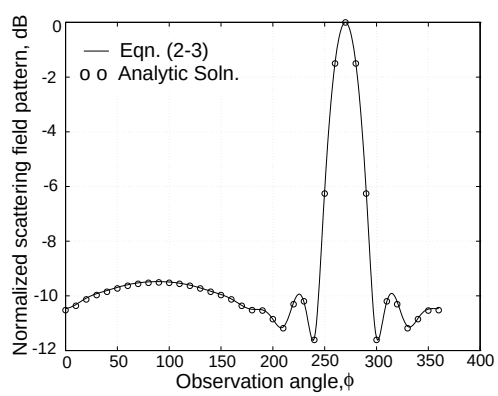


Figure 1:

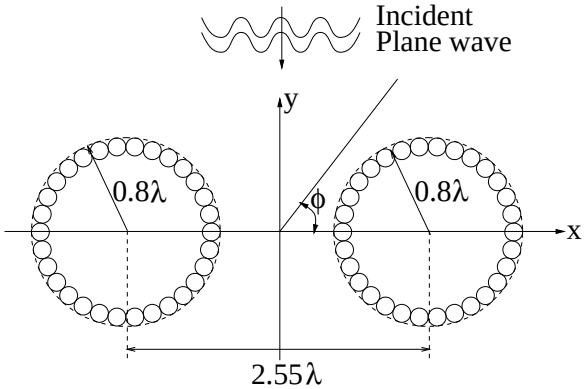


(a) Tessellated geometry showing the scattering for single metallic cylinder

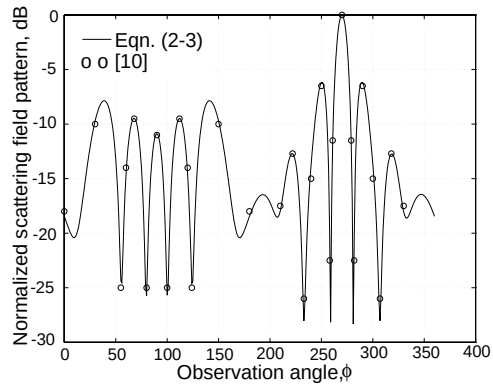


(b) Normalized scattering field pattern for geometry of (a)

Figure 2:

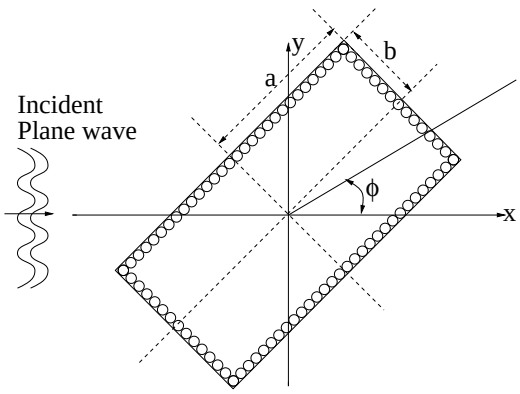


(a) Tessellated geometry showing the scattering for two metallic cylinders

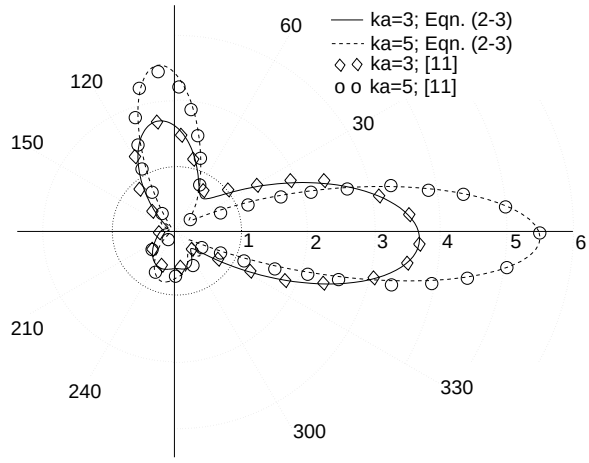


(b) Normalized scattering field pattern for geometry of (a)

Figure 3:



(a) Tessellated geometry showing the scattering for 45° slanted metallic rectangle



(b) Polar plot showing the normalized power density vs. ϕ for geometry of (a)

Figure 4: