Transmission Schemes for the Causal Cognitive Interference Channels

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Abstract

The causal cognitive interference channel (CCIC) has two sender-receiver pairs, in which the second sender obtains information from the first sender causally and assists the transmission of both senders. In this thesis, we study both the full- and half-duplex modes. In each mode, we propose two new coding schemes built successively upon one another to illustrate the impact of different coding techniques. The first scheme, called *partial decode-forward binning* (PDF-binning), combines the ideas of partial decode-forward relaying and Gelfand-Pinsker binning. The second scheme, called *Han-Kobayashi partial decode-forward binning* (HK-PDF-binning), combines PDF-binning with Han-Kobayashi coding by further splitting rates and applying superposition coding, conditional binning and relaxed joint decoding.

In both schemes, the second sender decodes a part of the message from the first sender, then uses the Gelfand-Pinsker binning technique to bin against the decoded codeword, in such a way that allows both state nullifying and forwarding. For Gaussian channels, this PDF-binning essentializes to a correlation between the transmit signal and the binning state, which encompasses the traditional dirty-paper-coding binning as a special case when this correlation factor is zero. We also provide the closed-form optimal binning parameter for each scheme.

The 2-phase half-duplex schemes are adapted from the full-duplex ones by removing the block Markov encoding, sending different message parts in different phases and applying joint decoding across both phases. Analysis shows that the HK-PDF-binning scheme in both modes encompasses the Han-Kobayashi rate region and achieves both the partial decode-forward relaying rate for the first sender and interference-free rate for the second sender. Furthermore, this scheme outperforms several existing schemes.

This thesis further analyzes the maximum rate for the cognitive user while keeping the primal user's rate interference-free in four special channel settings. In each setting, we investigate the optimal time duration for the first phase transmission and the power allocations. We also study the effect of channel gain parameters on this maximum rate for the cognitive user. Simulation results for different channel parameters verify the analysis and show that the cognitive user can achieve significant rates while not affecting the primary user's rate even in the half-duplex causal mode.

Abrégé

Le canal à interférence cognitif causal (CCIC) est constitué de deux paires émetteurs-récepteurs: le premier émetteur peut transmettre de l'information au second de manière causale, de telle sorte que ce dernier puisse améliorer la communication des deux. Dans cette thèse, nous étudions à la fois les liaisons half- et full-duplex. Pour chacune, nous proposons deux nouveaux codages, construits successivement l'un sur l'autre, dans le but d'illustrer l'impact de chaque technique. Le premier codage, que nous nommerons decode-forward partiel avec binning (PDF-binning), associe la stratégie de relayage par decode-forward partiel à la technique de binning de Gelfand-Pinsker. Le second codage, appelé decode-forward partiel avec binning de Han-Kobayashi (HK-PDF-binning), combine la stratégie PDF-binning avec le codage de Han-Kobayashi: il s'agit ici de diviser le débit plus finement et d'utiliser le codage par superposition, le binning conditionnel et un décodage conjoint mais moins contraignant.

Pour ces deux codages, le second émetteur décode seulement une partie du message reçu du premier, puis il applique la technique de binning de Gelfand-Pinsker afin de réduire les interférences générées par les mots de code émis par l'utilisateur primaire, de telle sorte que le message décodé en provenance du premier utilisateur devienne l'état de binning et soit retransmis ainsi sur le canal. Dans le cas des canaux gaussiens, la stratégie de PDF-binning met en lumière la corrélation entre le signal transmis et l'état de binning. La technique classique de binning utilisée dans le "dirty paper coding" est alors incluse en tant que cas particulier, lorsque le facteur de corrélation est nul. Puis, nous présentons une solution analytique au problème d'optimisation du paramètre de binning pour chaque stratégie de codage.

Les codages pour les liaisons half-duplex, constituées de deux phases de transmission, découlent de leurs homologues full-duplex. La stratégie de codage ne repose plus dans ce cas sur l'encodage en bloc de Markov. Les différents messages sont transmis à des instants distincts, et sont décodés conjointement en considérant les deux phases de transmission. L'étude analytique proposée dans cette thèse démontre que la stratégie HK-PDF-binning inclut la région de capacité de Han-Kobayashi à la fois pour les liaisons half-duplex et full-duplex. Elle permet au premier émetteur de transmettre au débit permis par le decode-forward partiel en même temps qu'elle permet au second de transmettre sans interférence. De plus, les performances de cette stratégie surpassent celles des stratégies existantes.

Cette thèse s'intéresse également au débit maximum atteignable par l'utilisateur cognitif lorsque l'utilisateur primaire transmet sans interférence pour quatre situations de canaux. Pour chaque situation, nous recherchons la durée optimale de la première phase, ainsi que les allocations de puissance. Nous analysons également l'impact des différents paramètres de canaux sur ce débit maximum pour l'émetteur cognitif. Les simulations, effectuées pour différents paramètres, permettent de confirmer l'analyse. Elles montrent que l'utilisateur cognitif peut atteindre des débits significatifs sans pour autant affecter le débit de l'émetteur primaire, même dans le cas d'une liaison half-duplex causale.

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Contents

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List of Acronyms

CCIC	Causal Cognitive Interfernce Channel
IC	Interference Channel
RC	Relay Channel
CIC	Cognitive Interference Channel
SC	Source Cooperation
MAC	Multiple Access Channel
DM	Discrete Memoryless
FD	Full Duplex
HD	Half Duplex
DPC	Dirty Paper Coding
PDF-Binning	Partial Decode-Forward Binning
HK-PDF-Binning	Han-Kobayashi Partial Decode-Forward Binning

Chapter 1

Introduction

1.1 Motivation

Wireless communication has undergone tremendous development in the last few decades. With the rapid development of mobile devices and communication applications, radio frequency is becoming one of the most valuable resources in the world. However, according to a report by the Federal Communications Commission [1], nearly 90% of the spectrum is unused and needs to be improved. Cognitive radio technology is one of the most significant and efficient technologies in improving the spectrum efficiency. The prominent characteristic of cognitive radio is the ability for one cognitive device to sense another device and use it to assist its own transmissions.

Devoroye, Mitran and Tarokh first study cognitive radio in an information-theoretic perspective in [2] and put forward the definition of a cognitive radio channel, which consists of two pairs of senders and receivers. The sender which is aware of the message of the other sender is called the cognitive sender, while the other sender is called the primary user. This channel brings new ideas to improving the spectrum efficiency and has received tremendous attention. Many researchers have begun to investigate in this area and proposed new coding schemes for this channel. However, Devoroye, Mitran and Tarokh's paper mainly focuses on the non-causal case, meaning the cognitive user knows the primary user's message beforehand. This is impractical in the implementations. In this thesis, we propose several transmission schemes for the cognitive radio channel in the causal case, in which the cognitive user obtains the primer user's message and information causally through a decoding process.

1.2 Thesis problem statement

In this thesis, we focus on the problem of the Causal Cognitive Interference Channel (CCIC) where the cognitive user serves as a relay by forwarding the primary user's message, while also transmitting its own messages. This channel is an asymmetric channel, which means that only the cognitive user will assist the primary user, not the reverse.

This model has many practical considerations and values. Consider, for example, the cellular system with two users and two base stations. User one and user two want to communicate with base station one and base station two, respectively. If user two is much closer to user one than base station one is, then user two can serve as a relay and help user one transmit information. User two can also send its own message to base station two at the same time.

The CCIC is a special case for a more generalized channel called the Interference Channel with Source Cooperation (IC-SC). IC-SC is also a four-node channel (two sender-receiver pairs). It considers the scenario of cooperations in both directions, i.e. both senders will first decode cooperative messages from the other sender and help each other to transmit. This thesis only investigates CCIC, the uni-directional cooperation case for IC-SC, which has not been much studied yet. Although CCIC is a special case of IC-SC, the study of the CCIC is necessary and valuable. First, some coding schemes for IC-SC requiring bi-directional cooperations between both senders are not applicable to the CCIC channel due to the uni-directional cooperation constraint. For example, IC-SC usually requires senders to both listen and decode. This may be impossible if one of the links between the senders is poor or does not even exist. Second, we propose a new perspective to study this channel. Most of the traditional coding schemes study the cognitive channel and IC-SC in a view of interference channel. This thesis, however, studies this channel more from a relaying perspective.

1.3 Thesis contribution and organization

1.3.1 Thesis contribution

In this thesis, we fully define the causal cognitive interference channel in both the full- and half-duplex modes and propose several coding schemes based on partial decode-forward relaying, Gelfand-Pinsker binning and Han-Kobayashi coding.

Full-duplex case: PDF-binning and HK-PDF-binning

The full-duplex causal cognitive interference channel is a four-node channel with two sender-receiver pairs S_1-D_1 and S_2-D_2, as in Figure 3.1. S_1 and S_2 want to transmit messages to D_1 and D_2, respectively. S_2 also serves as a relay by forwarding S_1's message to D_1 while transmitting its own message to D_2.

We propose two new coding schemes, in which the second scheme is built successively on top of the first one to illustrate the effect of each technique used.

- The first scheme is called *partial decode-forward binning (PDF-binning)*, which utilizes rate splitting, block Markov encoding, partial decode-forward relaying, Gelfand-Pinsker binning and forward joint decoding across two blocks. S_1 divides its message into two parts: one as a private message sent to D_1 directly, the other as a forwarding message, which is sent to D_1 with the help of S_2. S_2 first causally decodes the forwarding message part from S_1, then uses the decoded codeword as the binning state. In this case, however, the binning also allows S_2 to forward a part of the state to D_1, which uses joint decoding across two blocks to decode its messages from both S_1 and S_2. As opposed to the state amplification in [3], we want to decode the state at a different receiver (D_1) from decoding the message (D_2). This scheme achieves the partial decode-forward relaying rate for user 1 and Gelfand-Pinsker rate for user 2.

- The second scheme is called *Han-Kobayashi PDF-binning (HK-PDF-binning)*, which combines PDF-binning with Han-Kobayashi coding by having both users further split their messages. S_1 divides its message into three parts: one as the Han-Kobayashi (HK) private message decoded only at D_1, another as the HK public message decoded at both D_1 and D_2, and the final part as the forwarding message. S_2 divides its message into two parts: one as the HK private message and the other as the HK public message. There are three ideas in addition to the PDF-binning. First, in performing partial decode-forward, S_2 uses conditional binning instead of traditional binning to bin only its private message part. Second, although D_1 uses joint decoding in both schemes, the decoding rule here is relaxed, as D_1 also decodes the public message from S_2 without requiring it to be correct. Third, instead of simple Gelfand-Pinsker decoding, D_2 uses joint decoding of the binning auxiliary random variable and the HK public messages from the two senders, which are encoded independently of the state. HK-PDF-binning achieves both the Han-Kobayashi and the PDF-binning rate regions.

Half-duplex case: HD-PDF-binning and HD-HK-PDF-binning

For the half-duplex CCIC, the transmission is divided into two phases as in Figure 4.1. In the first phase, S_1 transmits to S_2, D_1 and D_2. In the second phase, the two senders transmit messages simultaneously, during which S_2 can both relay and apply cognitive encoding.

We adapt the above two coding schemes to the half-duplex case. The main challenges in adapting full-duplex schemes to the half-duplex mode include deciding which message parts should be sent in which phase and changing the destination decoding rule to joint decoding across both phases.

Specifically, we propose two half-duplex (HD) schemes: HD-PDF-binning and HD-HK-PDF-binning. At the end of the first phase in both schemes, S_2 decodes a message part from S_1 and then applies PDF-binning, but neither D_1 nor D_2 decode here. Both D_1 and D_2 only decode at the end of the second phase. There are several differences from full-duplex coding. First, not all message parts are sent in each phase. Second, there is no need for block Markovity; instead, we use superposition codewords in the two phases of the same block. Third, we use joint decoding at the destinations over two phases of the same block instead of over two consecutive blocks.

Applications to Gaussian channels

When applied to the Gaussian channel, a major difference between PDF-binning and the traditional binning in dirty paper coding (DPC) [4] is that we introduce a correlation between the transmit signal and the state. This correlation allows both binning and forwarding at the same time, thus helping improve the transmission rate for the first user and still allowing the second user to achieve the interference-free rate. We derive the closed-form optimal binning parameter for each coding scheme. This PDF-binning parameter contains the DPC-binning parameter as a special case.

Results show that the HK-PDF-binning scheme outperforms several existing schemes in both the full- and half-duplex modes for the causal cognitive interference channel. Our analysis also shows clearly the impact on rate region for each of the techniques used.

Furthermore, the maximum rate for the primary sender is the rate of partial decode-forward relaying and the maximum rate for the secondary sender is the interference-free rate as in dirty paper coding.

Rate region analysis for the Gaussian HD-HK-PDF-Binning scheme

Since the half-duplex scheme is practical in real systems, we further analyze the rate region for the Gaussian half-duplex Han-Kobayashi Partial Decode-Forward Binning scheme. We study the maximum rate for the second sender while keeping the first sender rate as without interference in four special channel settings. In each setting, we investigate the optimal time duration for the first phase transmission and the power allocations to achieve this rate for the cognitive sender. Furthermore, we study the relationships between the maximum rate for the cognitive sender and three channel gains parameters (two cross channel gains and one channel gain between two transmitters). The simulation results show that when the cross channel gains are strong enough, the cognitive user can achieve the interference-free rate.

Advantages of the proposed scheme

One of the prominent advantages for the proposed scheme is the inclusion of both the Han-Kobayashi rate region and the partial decode-forward rate. Several works have investigated the IC-SC in the full-duplex and half-duplex modes, but none of them achieve both Han-Kobayashi rate region and partial decode-forward rate.

[23], [24], [26] and [27] study the IC-SC in the full-duplex mode. Host-Madsen [23] proposes a coding scheme for IC-SC for transmitter cooperation based on the dirty paper coding and block Markov encoding. This scheme achieves partial decode-forward rate but not Han-Kobayashi region since there is no rate splitting. Probhakaran and Viswanath propose two coding schemes in [24]. The first scheme (see [24] Theorem 4(a)) is based on 3-part message splitting and block Markov encoding. This scheme achieves Han-Kobayashi region but not always the partial decode-forward rate because both destinations need to decode the cooperative public message. The second scheme in [24] is based on 4-part rate splitting and block Markov encoding. But it does not always achieve either the Han-Kobayashi rate region or the partial decode-forward rate. Cao and Chen [26] propose an achievable rate region for the interference channel with source cooperation based on rate

splitting, block Markov encoding, superposition encoding, dirty paper coding and random binning. This scheme achieves the Han-Kobayashi region, but not the decode-forward rate due to no block Markovity between the current and previous cooperative block messages. Yang and Tuninetti [27] propose two schemes for the interference channel with generalized feedback based on block Markov superposition coding, binning and backward decoding. This scheme achieves the Han-Kobayashi region but not the decode-forward rate because both destinations need to decode the cooperative common message.

[2] and [35] study the cognitive channel in the half-duplex mode. However, neither of these two achieves both Han-Kobayashi rate region and partial decode-forward rate. In [2], four protocols are proposed in which the secondary user obtains the message from the primary user causally. Time-sharing these 4 protocols can achieve the Han-Kobayashi rate region but not the decode-forward relaying rate. In [35], the authors proposed a new achievable rate region by a 2-phase scheme based on rate splitting, block Markov encoding, Gelfand-Pinsker binning and backward decoding. This scheme can only achieve the rate of decode-forward relaying, which is less than the partial decode-forward rate in the half-duplex mode. We will discuss these two schemes in more detail in Section 4.3.4.

Another obvious advantage is that we propose a new binning technique called HK-PDF-Binning, which introduces a correlation factor between the codeword and the binning state. This correlation contains both functionalities of binning and message forwarding, thus it enlarges the rate region compared with the traditional Gelfand-Pinsker binning [14]. We also show the analytical expressions for the optimal binning parameter λ in each Gaussian case, which depends on the correlation factor. This correlation factor and the optimal binning parameter are significant in the optimal power allocations.

Last but not the least, we present the compact rate region after Fourier Motzkin Elimination and obtain a region similar to the 7-constraint Han-Kobayashi rate region for the interference channel. This explicit rate region makes it possible to plot the rate region directly, which few of the current coding schemes can do. Many of the current coding schemes only show the rate constraints before Fourier Motzkin Elimination, which is impossible to plot numerically.

1.3.2 Thesis organization

This thesis consists of five major chapters. Chapter 2 reviews the current works related to the causal cognitive interference channel in both full- and half-duplex modes. In each mode, we discuss four channels: relay channel, interference channel, cognitive interference channel and interference channel with source cooperation.

Chapter 3 first defines the discrete memoryless model for the causal cognitive interference channel in the full-duplex case. We then propose two new transmission schemes: PDF-Binning and Han-Kobayashi PDF-Binning. For each scheme, we obtain the rate regions and compare them with existing full-duplex schemes in IC-SC analytically and numerically.

Chapter 4 adapts the two coding schemes to the half-duplex case and follows similar procedures to Chapter 3. We first present the half-duplex channel and two half-duplex coding schemes: HD-PDF-Binning and HD-HK-PDF-Binning, then derive the rate region for both discrete memoryless and Gaussian cases and compare them with the existing half-duplex schemes.

Chapter 5 analyzes the rate region for the Gaussian HD-HK-PDF-Binning and studies the maximum rate for the cognitive user while keeping the primary user in the interference-free rate. We also study four special channel settings to find the optimal time duration for the first phase and the power allocations. Simulation results for different channel parameters provide supports for our analysis in the special channel settings.

Chapter 6 concludes the thesis and states possible future work directions.

1.4 Author's work

The following paper based on the content of this thesis is published in the international conference proceedings:

- Z. Wu and M. Vu, "Partial Decode-Forward Binning for Full-Duplex Causal Cognitive Interference Channels", in *Proc. IEEE Int'l Symp. on Information Theory (ISIT)*, July 2012.

The following paper based on the content of this thesis is submitted to the international journal:

- Z. Wu and M. Vu, "Partial Decode-Forward Binning Schemes for the Causal Cognitive Relay Channels", submitted to *IEEE Trans. on Information Theory*, Dec. 2011, available at http://arxiv.org/abs/1111.3966.

Chapter 2

Literature Review

The Causal Cognitive Interference Channel (CCIC) is a four-node channel with two senders and two receivers, in which the second sender obtains information from the first sender causally, then uses that to assist the transmissions of the first sender and its own message. Different from the assumption in the traditional cognitive channel that the secondary user knows the primary user's message non-causally, we propose several coding schemes in which the secondary user first decodes the primary user's message causally, then transmits the decoded message and its own message cognitively.

In this thesis, we study the causal cognitive interference channel in both full- and half-duplex modes. Analysis for the full-duplex mode gives us insights into the optimal coding schemes, while application to the half-duplex mode is more practical. In the full-duplex mode, there is no time division into sub-phases; both senders transmit all messages during the whole transmission. In the half-duplex mode, however, the transmission is divided into two phases with different message parts sent during each phase. In the first phase, the second user obtains a message from the first sender causally. In the second phase, these two senders transmit the messages concurrently.

This causal cognitive interference channel has not been studied much in the literature. But it has tight relationships with the relay channel (RC), the interference channel (IC) and the cognitive interference channel (CIC). On the one hand, the second sender serves as a relay and helps forward the message from the first sender. On the other hand, these two senders interfere with each other during the transmission, and they can also cooperate cognitively. The closest channel to the CCIC is the interference channel with source cooperation (IC with SC), in which both senders can exchange messages causally.

Next, we review existing work related to the causal cognitive interference channel in both full- and half-duplex modes.

2.1 Full-duplex case

2.1.1 Relay channel

Van der Meulen first proposes the concept of relay channel in [5]. Cover and El Gamal further design several important techniques for relay channels, including decode-forward, compress-forward, and mixed decode-forward and compress-forward in [6]. A variation of the decode-forward scheme is partial decode-forward, in which the relay only decodes a part of the message from the source and forwards it to the destination instead of decoding the whole message. Kramer, Gastpar and Gupta [7] extend these schemes to the multiple-node relay networks and propose several rate regions based on decode-forward, compress-forward and mixed strategies. Lim, Kim, El Gamal and Chung [8] propose a new scheme called noisy network coding (NNC) based on compress-forward relaying. These relay coding techniques have been widely applied in other channels. For example, in [9], Liang and Kramer study the relay broadcast channel using the idea of rate splitting, block Markov encoding and partial decode-forward relaying.

2.1.2 Interference channel

Carleial first introduces the interference channel and proposes inner and outer bounds as well as capacity results for several special cases in [10]. Sato studies the capacity for the Gaussian interference channel with strong interference in [11]. Han and Kobayashi propose the well-known Han-Kobayashi coding technique in [12] using rate splitting at the transmitters and joint decoding at the receivers, which to date achieves the largest rate region for the interference channel. Chong, Motani, Garg and El Gamal [13] propose a variant scheme based on superposition coding, which achieves the same rate region as the original Han-Kobayashi scheme but has fewer auxiliary random variables and hence reduces the encoding and decoding complexities.

2.1.3 Cognitive interference channel

The cognitive interference channel is another closely related channel, which plays a significant role in improving spectrum efficiency. Devroye, Mitran and Tarokh first propose the concept in [2] and provide an achievable rate region based on combining Gelfand-Pinsker coding [14] with Han-Kobayashi scheme. They study both the genie-aided (non-causal) and the non genie-aided (causal) cases. Maric, Yates and Kramer determine the capacity region for the channel with very strong interference in [15]. Wu, Vishwanath and Arapostathis determine the capacity region for the weak interference case in [16]. Other coding schemes for the cognitive interference channel can be seen in [17, 18, 19]. Jovicic and Viswanath [20] analyze the Gaussian cognitive channel and give the largest rate for the cognitive user under the constraint that the primary user experiences no rate degradation and uses single-user decoder. Rini, Tuninetti and Devroye [21] further propose several new inner, outer bounds and capacity results based on rate spitting, superposition coding, a broadcast channel-like binning scheme and Gelfand-Pinsker coding.

An important technique used in all cognitive coding is the binning technique proposed by Gelfand and Pinsker in [14]. In Gelfand-Pinsker binning, the state of the channel is known at the input, but unknown at the output. Marton [22] proposes the double binning scheme and applies it to the broadcast channel. Kim, Sutivong and Cover [3] further analyze Gelfand-Pinsker binning to allow the decoding of a part of state information at the destination at a reduced information rate. Costa [4] applies Gelfand-Pinsker binning to the Gaussian channel and proposes the well-known dirty paper coding (DPC) scheme, which achieves the same rate as if the channel is interference free. A surprising feature of DPC binning is that the transmit signal is independent of the state.

2.1.4 Interference channel with source cooperation

Host-Madsen [23] studies outer and inner bounds for the interference channel with either destination or source cooperation. The achievable rate for source cooperation is based on block Markov encoding and dirty paper coding, which includes the rate for decode-forward relaying but not the Han-Kobayashi region. Prabhakaran and Viswanath [24] investigate the Gaussian interference channel with source cooperation and propose two achievable rate region built on block Markov encoding, superposition coding and Han-Kobayashi scheme, but without binning, as well as several upper bounds on the sum rate.

Wang and Tse [25] study the Gaussian interference channel with conferencing transmitters and propose an achievable rate region within 6.5 bits/s/Hz of the capacity for all channel parameters. The channel is based on conferencing model, in which the common message parts are known though noiseless conference links between the two transmitters before each block transmission begins, hence there is no need for block Markovity. The scheme utilizes Marton's double binning for the cooperative private messages and superposition coding but not dirty paper coding for the non-cooperative private message parts. Cao and Chen [26] propose an achievable rate region for the interference channel with transmitter cooperation using block Markov encoding, rate splitting and superposition coding, dirty paper coding and random binning. This scheme includes the Han-Kobayashi region but not the decode-forward relaying rate. Yang and Tuninetti [27] study the interference channel with generalized feedback (also known as source cooperation) and propose two schemes. The first scheme uses rate splitting and block Markov superposition coding only, in which the two users send common messages cooperatively. The second scheme extends the first one by using both block Markov superposition coding and binning, in which parts of both common and private messages are sent cooperatively. This scheme also achieves the Han-Kobayashi region but not the decode-forward relaying rate. We will discuss the schemes in [26, 27] in more detail in Section 3.2.3. Tandon and Ulukus [28] study an outer bound for the MAC with generalized feedback based on dependence balance [29] and extend this idea to the interference channel with user cooperation. We will apply this outer bound in Section 3.3.6.

2.2 Half-duplex case

For half-duplex communications, results also exist for the above channels, albeit fewer than in the full-duplex case.

2.2.1 Relay channel

Host-Madsen and Zhang study upper and lower bounds for the half-duplex relay channel based on time-division in [30, 31] and give achievable rates for the Gaussian relay channel using partial decode-forward and compress-forward. Zhang, Jiang, Goldsmith and Cui [32] study the half-duplex Gaussian relay channel with arbitrary correlated noises at the relay and destination. They also evaluate the achievable rates using decode-forward, compress-forward and amplify-forward, showing none of these schemes strictly outperforms the others. Liu, Stankovic and Xiong [33] propose a practical compress-forward scheme for the half-duplex Gaussian relay channel based on Wyner-Ziv coding. The practical implementation of this scheme achieves as close as 0.76 dB to the theoretical limit of compress forward if utilizing LDPC codes for error protection at the source and nested scalar quantization and IRA codes for Wyner-Ziv coding at the relay.

2.2.2 Interference channel

Peng and Rajan [34] study the half-duplex Gaussian interference channel and compute several inner and outer bounds for transmitter or receiver cooperation. In both cooperation schemes, the transmission is divided into three phases: two broadcast phases and one MIMO cooperative phase. For transmitter cooperation, each transmitter utilizes the decode-forward scheme and serves as a relay and decodes the message from the other transmitter in the broadcast phase. In the cooperative phase, both transmitters deliver their messages cooperatively. In the receiver cooperation, each receiver acts as a relay and decodes the information from the other destination. The compress-forward scheme is used at the relays. They also compare these two cooperation schemes and show that the transmitter cooperation outperforms the receiver cooperation under the same channel conditions and transmit power constraints.

2.2.3 Cognitive interference channel

For the half-duplex cognitive interference channel, Devroye, Mitran and Tarokh [2] propose four protocols in which the secondary user obtains the message from the primary user causally. Time-sharing these 4 protocols can achieve the Han-Kobayashi rate region but not the decode-forward relaying rate. Chatterjee, Tong and Oyman [35] further propose a new achievable rate region by a 2-phase scheme based on rate splitting, block Markov encoding, Gelfand-Pinsker binning and backward decoding. This scheme can only achieve the rate of decode-forward relaying, which is less than the partial decode-forward rate in the half-duplex mode. We will discuss these two schemes in more detail in Section 4.3.4.

2.2.4 Half-duplex IC-SC

Wu, Prabhakaran and Viswanath study the interference channels with source cooperation under the half-duplex constraint [36]. They create a virtual channel which decomposes the model of both senders transmitting into two separate models that each sender transmits alone. They further propose a scheme generalizing superposition coding and the Han-Kobayashi scheme and study a specific symmetric linear deterministic case and compute its sum rate. Analysis shows that this scheme is optimal for the considered channel by showing the match of upper and lower bounds.

Chapter 3

Full-Duplex Transmission Scheme for the CCIC

3.1 Full-duplex DM-CCIC Models

The full-duplex causal cognitive interference channel consists of two input alphabets $\mathcal{X}_1, \mathcal{X}_2$, and three output alphabets $\mathcal{Y}_1, \mathcal{Y}_2, \mathcal{Y}$. The channel is characterized by a channel transition probability $p(y_1, y_2, y|x_1, x_2)$, where x_1 and x_2 are the transmit signals of S_1 and S_2; y_1, y_2 and y are the received signals of D_1, D_2 and S_2. Figure 3.1 illustrates the channel model, where W_1 and W_2 are the messages of S_1 and S_2. For notation, we use upper case letters to indicate random variables and lower case letters to indicate their realizations. We use x^n and x_k^n to represent the vectors $(x_1, \ldots, x_n)$ and $(x_k, \ldots, x_n)$ respectively.

The causal cognitive interference channel has tight relationships with the interference and the relay channels. For example, this channel model can be converted to the interference channel[10] if S_2 does not forward any information to D_1. Similarly, this channel reduces to the relay channel [5], [6] if S_2 does not have any message for D_2.

A $(2^{nR_1}, 2^{nR_2}, n)$ code, or a communication strategy for n channel uses with rate pair (R_1, R_2), consists of the following:

- Two message sets $\mathcal{W}_1 \times \mathcal{W}_2 = [1, 2^{nR_1}] \times [1, 2^{nR_2}]$ and independent messages W_1, W_2 uniformly distributed over $\mathcal{W}_1$ and $\mathcal{W}_2$, respectively.

- Two encoders: one maps message w_1 into codeword $x_1^n(w_1) \in \mathcal{X}_1^n$, and one maps w_2 and each received sequence y^{k-1} into a symbol $x_{2k}(w_2, y^{k-1}) \in \mathcal{X}_2$.

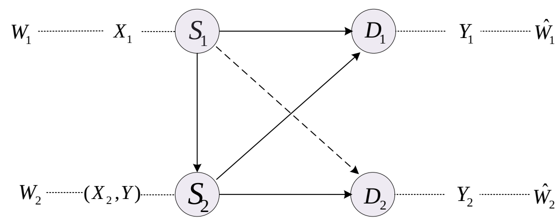

Fig. 3.1 The full-duplex modes for the causal cognitive interference channel.

- Two decoders: one maps y_1^n into $\hat{w}_1 \in \mathcal{W}_1$; one maps y_2^n into $\hat{w}_2 \in \mathcal{W}_2$.

The probability of error when the message pair (W_1, W_2) is sent is defined as $P_e(W_1, W_2) = P\{(\widehat{W}_1, \widehat{W}_2) \neq (W_1, W_2)\}$. A rate pair (R_1, R_2) is said to be *achievable* if, for any $\epsilon > 0$, there exists a code such that the average error probability $P_e \leq \epsilon$ as $n \to \infty$. The capacity region is the convex closure of the set of all achievable rate pairs.

3.2 Full-duplex partial decode-forward binning schemes

3.2.1 PDF-binning scheme

The first scheme uses block Markov superposition encoding at S_1 and partial decode-forward relaying and Gelfand-Pinsker binning at S_2. D_1 uses joint decoding across two blocks while D_2 uses normal Gelfand-Pinsker decoding. The first sender S_1 splits its message w_1 into two parts (w_{10}, w_{11}), which correspond to the common (forwarding) and private parts. We use block Markov encoding at S_1, such that the current-block common message w_{10} is superimposed on the previous-block common message w'_{10}. Then, message w_{11} is superimposed on both w'_{10} and w_{10}. The second sender S_2 decodes the previous common message w'_{10} from the first sender S_1 and then uses binning to bin against the codeword for this message part. Depending on the joint distribution between the binning auxiliary random variable and the state, S_2 can also forward a part of the state (i.e. message w'_{10}) to D_1. The encoding and decoding structure can be seen in Figure 3.2, in which w'_{10} corresponds to $w_{10[i-1]}$.

Theorem 1. *The convex hull of the following rate region is achievable for the full-duplex*

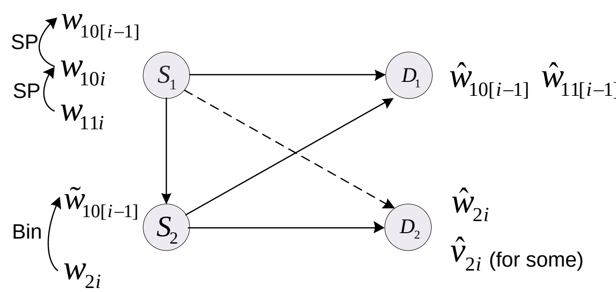

Fig. 3.2 Coding structure for the full-duplex PDF-binning scheme at block i. (SP stands for superposition, Bin stands for binning.)

causal cognitive interference channel using PDF-binning:

$$\bigcup_{P_1} \begin{cases} R_1 & \leq I(U_{10};Y|T_{10}) + I(X_1;Y_1|U_{10},T_{10}) \\ R_1 & \leq I(T_{10},U_{10},X_1;Y_1) \\ R_2 & \leq I(U_2;Y_2) - I(U_2;T_{10}), \end{cases} \tag{3.1}$$

where

$$P_1 = p(t_{10})p(u_{10}|t_{10})p(x_1|t_{10},u_{10})p(u_2|t_{10})p(x_2|t_{10},u_2)p(y_1,y_2,y|x_1,x_2).$$

Remark 1. *The maximum rate for each user.*

- *The first user S_1 achieves the maximum rate of partial decode-forward relaying if we set $U_2 = \emptyset$, $X_2 = T_{10}$.*

$$R_1^{\max} = \max_{p(u_{10},x_2)p(x_1|u_{10},x_2)} \min\{I(U_{10};Y|X_2) + I(X_1;Y_1|U_{10},X_2), I(X_1,X_2;Y_1)\}. \tag{3.2}$$

In this case, there is no binning but only forwarding at S_2.

- *The second user S_2 achieves the maximum rate of Gelfand-Pinsker's binning if we set $T_{10} = U_{10} = X_1$.*

$$R_2^{\max} = \max_{p(x_1,u_2)p(x_2|x_1,u_2)} \{I(U_2;Y_2) - I(U_2;X_1)\}. \tag{3.3}$$

In this case, there is no forwarding of the state at S_2.

Proof. The transmission is done in B blocks, each of which consists of n channel uses. S_1

splits each message w_1 into two independent parts (w_{10}, w_{11}). During the first $B-1$ blocks, S_1 encodes and sends a message tuple $(w_{10[i-1]}, w_{10i}, w_{11i}) \in [1, 2^{nR_{10}}] \times [1, 2^{nR_{10}}] \times [1, 2^{nR_{11}}]$; S_2 encodes and sends message $(w_{10[i-1]}, w_{2i}) \in [1, 2^{nR_{10}}] \times [1, 2^{nR_2}]$, where $i = 1, 2, \ldots, B-1$ denotes the block index. When $B \to \infty$, the average rate triple $\left(R_{10}\frac{B-1}{B}, R_{11}\frac{B-1}{B}, R_2\frac{B-1}{B}\right)$ approaches to (R_{10}, R_{11}, R_2).

We use random codes and fix a joint probability distribution

$$p(t_{10})p(u_{10}|t_{10})p(x_1|t_{10}, u_{10})p(u_2|t_{10})p(x_2|t_{10}, u_2).$$

Codebook generation

For each block i (we can also just generate two independent codebooks for the odd and even blocks to make the error events of two consecutive blocks independent [9]):

- Independently generate $2^{nR_{10}}$ sequences $t_{10}^n \sim \prod_{k=1}^n p(t_{10k})$. Index these codewords as $t_{10}^n(w'_{10})$, $w'_{10} \in [1, 2^{nR_{10}}]$.
- For each $t_{10}^n(w'_{10})$, independently generate $2^{nR_{10}}$ sequences $u_{10}^n \sim \prod_{k=1}^n p(u_{10k}|t_{10k})$. Index these codewords as $u_{10}^n(w_{10}|w'_{10})$, $w_{10} \in [1, 2^{nR_{10}}]$. w_{10} contains the common message of the current block, while w'_{10} contains the common message of the previous block.
- For each $t_{10}^n(w'_{10})$ and $u_{10}^n(w_{10}|w'_{10})$, independently generate $2^{nR_{11}}$ sequences $x_1^n \sim \prod_{k=1}^n p(x_{1k}|t_{10k}, u_{10k})$. Index these codewords as $x_1^n(w_{11}, w_{10}|w'_{10})$, $w_{11} \in [1, 2^{nR_{11}}]$, $w_{10} \in [1, 2^{nR_{10}}]$.
- Independently generate $2^{n(R_2+R'_2)}$ sequences $u_2^n \sim \prod_{k=1}^n p(u_{2k})$. Index these codewords as $u_2^n(w_2, v_2)$, $w_2 \in [1, 2^{nR_2}]$ and $v_2 \in [1, 2^{nR'_2}]$.
- For each $t_{10}(w'_{10})$ and $u_2^n(w_2, v_2)$, generate one $x_2^n \sim \prod_{k=1}^n p(x_{2k}|t_{10k}, u_{2k})$. Denote x_2^n by $x_2^n(w'_{10}, w_2, v_2)$.

Encoding

At the beginning of block i, let $(w_{10i}, w_{11i}, w_{2i})$ be the new messages to be sent in block i, and $(w_{10[i-1]}, w_{11[i-1]}, w_{2[i-1]})$ be the messages sent in block $i-1$.

- S_1 knows $w_{10[i-1]}$, in order to send (w_{10i}, w_{11i}), S_1 transmits $x_1^n(w_{11i}, w_{10i}|w_{10[i-1]})$.
- S_2 searches for a v_{2i} such that

$$(t_{10}^n(w_{10[i-1]}), u_2^n(w_{2i}, v_{2i})) \in A_\epsilon^{(n)}(P_{T_{10}U_2}).$$

  Such a v_{2i} exists with high probability if

$$R_2' \geq I(U_2; T_{10}). \tag{3.4}$$

  S_2 then transmits $x_2^n(w_{10[i-1]}, w_{2i}, v_{2i})$.

Decoding

At the end of block i:

- S_2 knows $w_{10[i-1]}$ and declares message $\hat{w}_{10i}$ was sent if it is the unique message such that

$$(t_{10}^n(w_{10[i-1]}), u_{10}^n(\hat{w}_{10i}|w_{10[i-1]}), y^n(i)) \in A_\epsilon^{(n)}(P_{T_{10}U_{10}Y}),$$

  where $y^n(i)$ indicates the received signal at S_2 in block i. We can show that the decoding error probability goes to 0 as $n \to \infty$ if

$$R_{10} \leq I(U_{10}; Y|T_{10}). \tag{3.5}$$

- D_1 knows $w_{10[i-2]}$ and decodes $(w_{10[i-1]}, w_{11[i-1]})$ based on the signals received at block $i-1$ and block i. It declares that message pair $(\hat{w}_{10[i-1]}, \hat{w}_{11[i-1]})$ was sent if it is the unique pair such that

$$(t_{10}^n(w_{10[i-2]}), u_{10}^n(\hat{w}_{10[i-1]}|w_{10[i-2]}), x_1^n(\hat{w}_{11[i-1]}, \hat{w}_{10[i-1]}|w_{10[i-2]}), y_1^n(i-1)) \in A_\epsilon^{(n)}(P_{T_{10}U_{10}X_1Y_1})$$
$$\text{and} \quad (t_{10}^n(\hat{w}_{10[i-1]}), y_1^n(i)) \in A_\epsilon^{(n)}(P_{T_{10}Y_1}).$$

The decoding error probability goes to 0 as $n \to \infty$ if

$$\begin{aligned} R_{11} &\leq I(X_1; Y_1 | U_{10}, T_{10}) \\ R_{10} + R_{11} &\leq I(T_{10}, U_{10}, X_1; Y_1). \end{aligned} \tag{3.6}$$

- D_2 treats T_{10}, a part of the signal from S_1, as the state and decodes w_{2i} based on the signal received at block i. Specifically, D_2 decodes w_{2i} directly using joint typicality between u_2 and y_2. It declares that message $\hat{w}_{2i}$ was sent if it is unique such that

$$(u_2^n(\hat{w}_{2i}, \hat{v}_{2i}), y_2^n(i)) \in A_\epsilon^{(n)}(P_{U_2 Y_2})$$

for some $\hat{v}_{2i}$. The decoding error probability goes to 0 as $n \to \infty$ if

$$R_2 + R_2' \leq I(U_2; Y_2). \tag{3.7}$$

Let $R_1 = R_{10} + R_{11}$, apply Fourier-Motzkin Elimination [37] on constraints (3.4)-(3.7), and we get the rate region in (3.1). □

Remark 2. *While the idea of the basic PDF-binning scheme is straightforward, this scheme allows the understanding of binning to achieve the maximum rates of partial decode-forward relaying at user 1 as in* (3.2) *and Gelfand-Pinsker coding at user 2 as in* (3.3). *The importance magnifies in the Gaussian application in Section 3.3. This scheme helps build the base for more complicated schemes later.*

3.2.2 Han-Kobayashi PDF-binning scheme

Figure 3.3 illustrates the idea of the full-duplex Han-Kobayashi PDF-binning scheme. Built upon PDF-binning, each user further splits its message to incorporate Han-Kobayashi coding. Message w_1 is split into three parts: w_{10}, w_{11}, w_{12}, corresponding to the common (forwarding), public and private parts, and message w_2 is split into two parts: w_{21}, w_{22}, corresponding to the public and private parts. Take the transmission in block i as an example. At S_1, the current common message w_{10i} is superimposed on the previous commons message $w_{10[i-1]}$; message w_{11i} is encoded independently of both $w_{10[i-1]}$ and w_{10i}; message w_{12i} is then superimposed on all three messages $w_{10[i-1]}$, w_{10i} and w_{10i}. S_2 decodes $\tilde{w}_{10[i-1]}$

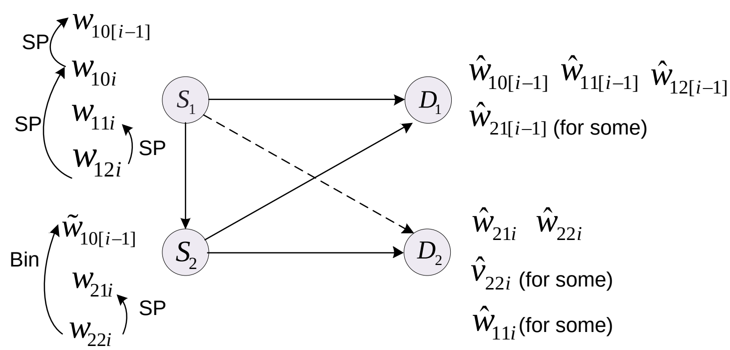

Fig. 3.3 Coding structure for the full-duplex Han-Kobayashi PDF-binning scheme at block i.

of the previous block and uses conditional binning to bin its private part $w_{22[i]}$ against $\tilde{w}_{10[i-1]}$, conditionally on knowing the public part $w_{21[i]}$. At the end of block i, D_1 uses joint decoding over two blocks to decode a unique tuple $(\hat{w}_{10[i-1]}, \hat{w}_{11[i-1]}, \hat{w}_{12[i-1]})$ for some $\hat{w}_{21[i-1]}$ without requiring this message part to be correct. D_2 treats the codeword for $w_{10[i-1]}$ as the state and searches for a unique pair (w_{21i}, w_{22i}) for some w_{11i}. The detailed coding and decoding procedures are shown in the proof of Theorem 2 below.

Theorem 2. *The convex hull of the following rate region is achievable for the causal cognitive interference channel using HK-PDF-binning:*

$$\bigcup_{P_2} \begin{cases} R_1 & \leq \min\{I_2 + I_5, I_6\} \\ R_2 & \leq I_{12} - I_1 \\ R_1 + R_2 & \leq \min\{I_2 + I_7, I_8\} + I_{13} - I_1 \\ R_1 + R_2 & \leq \min\{I_2 + I_3, I_4\} + I_{14} - I_1 \\ R_1 + R_2 & \leq \min\{I_2 + I_9, I_{10}\} + I_{11} - I_1 \\ 2R_1 + R_2 & \leq \min\{I_2 + I_3, I_4\} + \min\{I_2 + I_9, I_{10}\} + I_{13} - I_1 \\ R_1 + 2R_2 & \leq \min\{I_2 + I_7, I_8\} + I_{11} - I_1 + I_{14} - I_1, \end{cases} \tag{3.8}$$

where

$$P_2 = p(t_{10})p(u_{10}|t_{10})p(u_{11})p(x_1|t_{10}, u_{10}, u_{11})p(u_{21})p(u_{22}|u_{21}, t_{10})p(x_2|t_{10}, u_{21}, u_{22})p(y_1, y_2, y|x_1, x_2), \tag{3.9}$$

and I_1 — I_{14} are defined as

$$
\begin{aligned}
I_1 &= I(U_{22};T_{10}|U_{21}) \\
I_2 &= I(U_{10};Y|T_{10}) \\
I_3 &= I(X_1;Y_1|T_{10},U_{10},U_{11},U_{21}) \\
I_4 &= I(U_{10},X_1;Y_1|T_{10},U_{11},U_{21}) + I(T_{10};Y_1) \\
I_5 &= I(U_{11},X_1;Y_1|T_{10},U_{10},U_{21}) \\
I_6 &= I(U_{10},U_{11},X_1;Y_1|T_{10},U_{21}) + I(T_{10};Y_1) \\
I_7 &= I(X_1,U_{21};Y_1|T_{10},U_{10},U_{11}) \\
I_8 &= I(U_{10},X_1,U_{21};Y_1|T_{10},U_{11}) + I(T_{10};Y_1) \\
I_9 &= I(U_{11},X_1,U_{21};Y_1|T_{10},U_{10}) \\
I_{10} &= I(T_{10},U_{10},U_{11},X_1,U_{21};Y_1) \\
I_{11} &= I(U_{22};Y_2|U_{21},U_{11}) \\
I_{12} &= I(U_{21},U_{22};Y_2|U_{11}) \\
I_{13} &= I(U_{11},U_{22};Y_2|U_{21}) \\
I_{14} &= I(U_{11},U_{21},U_{22};Y_2).
\end{aligned}
\tag{3.10}
$$

Remark 3. *Inclusion of PDF-binning and Han-Kobayashi schemes.*

- *The HK-PDF-binning scheme becomes PDF-binning if $U_{11} = U_{21} = \emptyset$.*
- *The HK-PDF-binning scheme becomes the Han-Kobayashi scheme if $T_{10} = U_{10} = \emptyset$ and $X_2 = U_{22}$.*
- *The maximum rates for S_1 and S_2 are the same as in the PDF-binning scheme in* (3.2) *and* (3.3)*.*

Proof. We use random codes and fix a joint probability distribution

$$p(t_{10})p(u_{10}|t_{10})p(u_{11})p(x_1|t_{10},u_{10},u_{11})p(u_{21})p(u_{22}|u_{21},t_{10})p(x_2|t_{10},u_{21},u_{22}).$$

Codebook generation

For each block i (or for odd and even blocks):

- Independently generate $2^{nR_{10}}$ sequences $t_{10}^n \sim \prod_{k=1}^n p(t_{10k})$. Index these codewords as $t_{10}^n(w_{10}')$, $w_{10}' \in [1, 2^{nR_{10}}]$.
- For each $t_{10}^n(w_{10}')$, independently generate $2^{nR_{10}}$ sequences $u_{10}^n \sim \prod_{k=1}^n p(u_{10k}|t_{10k})$. Index these codewords as $u_{10}^n(w_{10}|w_{10}')$, $w_{10} \in [1, 2^{nR_{10}}]$. w_{10} is the common message of the current block, while w_{10}' is the common message of the previous block.
- Independently generate $2^{nR_{11}}$ sequences $u_{11}^n \sim \prod_{k=1}^n p(u_{11k})$. Index these codewords as $u_{11}^n(w_{11})$, $w_{11} \in [1, 2^{nR_{11}}]$.
- For each $t_{10}^n(w_{10}')$, $u_{10}^n(w_{10}|w_{10}')$ and $u_{11}^n(w_{11})$, independently generate $2^{nR_{12}}$ sequences $x_1^n \sim \prod_{k=1}^n p(x_{1k}|t_{10k}, u_{10k}, u_{11k})$. Index these codewords as $x_1^n(w_{12}|w_{11}, w_{10}, w_{10}')$, $w_{12} \in [1, 2^{nR_{12}}]$.
- Independently generate $2^{nR_{21}}$ sequences $u_{21}^n \sim \prod_{k=1}^n p(u_{21k})$. Index these codewords as $u_{21}^n(w_{21})$, $w_{21} \in [1, 2^{nR_{21}}]$.
- For each $u_{21}^n(w_{21})$, independently generate $2^{n(R_{22}+R_{22}')}$ sequences $u_{22}^n \sim \prod_{k=1}^n p(u_{22k}|u_{21k})$. Index these codewords as $u_{22}^n(w_{22}, v_{22}|w_{21})$, $w_{22} \in [1, 2^{nR_{22}}]$ and $v_{22} \in [1, 2^{nR_{22}'}]$.
- For each $t_{10}(w_{10}')$, $u_{21}^n(w_{21})$ and $u_{22}^n(w_{22}, v_{22}|u_{21})$, generate one $x_2^n \sim \prod_{k=1}^n p(x_{2k}|t_{10k}, u_{21i}, u_{22i})$. Denote x_2^n by $x_2^n(w_{10}', w_{21}, w_{22}, v_{22})$.

Encoding

At the beginning of block i, let $(w_{10i}, w_{11i}, w_{12i}, w_{21i}, w_{22i})$ be the new messages to be sent in block i, and $(w_{10[i-1]}, w_{11[i-1]}, w_{12[i-1]}, w_{21[i-1]}, w_{22[i-1]})$ be the messages sent in block $i-1$.

- S_1 knows $w_{10[i-1]}$; in order to send $(w_{10i}, w_{11i}, w_{12i})$, it transmits $x_1^n(w_{12}|w_{11i}, w_{10i}, w_{10[i-1]})$.
- S_2 searches for a v_{22i} such that

$$(t_{10}^n(w_{10[i-1]}), u_{21}^n(w_{21i}), u_{22}^n(w_{22i}, v_{22i}|w_{21i})) \in A_\epsilon^{(n)}(P_{T_{10}U_{22}|U_{21}}). \tag{3.11}$$

Such a v_{22i} exists with high probability if

$$R_{22}' \geq I(U_{22}; T_{10}|U_{21}). \tag{3.12}$$

S_2 then transmits $x_2^n(w_{10[i-1]}, w_{21i}, w_{22i}, v_{22i})$.

Decoding

At the end of block i:

- S_2 knows $w_{10[i-1]}$ and declares message $\hat{w}_{10i}$ was sent if it is the unique message such that

$$(t_{10}^n(w_{10[i-1]}), u_{10}^n(\hat{w}_{10i}|w_{10[i-1]}), y^n(i)) \in A_\epsilon^{(n)}(P_{T_{10}U_{10}Y}),$$

where $y^n(i)$ indicates the received signal at S_2 in block i. We can show that the decoding error probability goes to 0 when $n \to \infty$ if

$$R_{10} \leq I(U_{10}; Y|T_{10}). \tag{3.13}$$

- D_1 knows $w_{10[i-2]}$ and searches for a unique tuple $(\hat{w}_{10[i-1]}, \hat{w}_{11[i-1]}, \hat{w}_{12[i-1]})$ for some $\hat{w}_{21[i-1]}$ such that

$$\begin{aligned}(t_{10}^n(w_{10[i-2]}), u_{10}^n(\hat{w}_{10[i-1]}|w_{10[i-2]}), u_{11}^n(w_{11[i-1]}), &x_1^n(\hat{w}_{12[i-1]}|\hat{w}_{11[i-1]}, \hat{w}_{10[i-1]}, w_{10[i-2]}),\\ u_{21}^n(\hat{w}_{21[i-1]}), y_1^n(i-1)) &\in A_\epsilon^{(n)}(P_{T_{10}U_{10}U_{11}X_1U_{21}Y_1})\\ \text{and} \quad (t_{10}^n(\hat{w}_{10[i-1]}), y_1^n(i)) &\in A_\epsilon^{(n)}(P_{T_{10}Y_1}).\end{aligned} \tag{3.14}$$

The decoding error probability goes to 0 as $n \to \infty$ if

$$\begin{aligned}R_{12} &\leq I(X_1; Y_1|T_{10}, U_{10}, U_{11}, U_{21})\\ R_{10} + R_{12} &\leq I(U_{10}, X_1; Y_1|T_{10}, U_{11}, U_{21}) + I(T_{10}; Y_1)\\ R_{11} + R_{12} &\leq I(U_{11}, X_1; Y_1|T_{10}, U_{10}, U_{21})\\ R_{10} + R_{11} + R_{12} &\leq I(U_{10}, U_{11}, X_1; Y_1|T_{10}, U_{21}) + I(T_{10}; Y_1)\\ R_{12} + R_{21} &\leq I(X_1, U_{21}; Y_1|T_{10}, U_{10}, U_{11})\\ R_{10} + R_{12} + R_{21} &\leq I(U_{10}, X_1, U_{21}; Y_1|T_{10}, U_{11}) + I(T_{10}; Y_1)\\ R_{11} + R_{12} + R_{21} &\leq I(U_{11}, X_1, U_{21}; Y_1|T_{10}, U_{10})\\ R_{10} + R_{11} + R_{12} + R_{21} &\leq I(T_{10}, U_{10}, U_{11}, X_1, U_{21}; Y_1).\end{aligned} \tag{3.15}$$

- D_2 treats $T_{10}^n(w'_{10[i-1]})$ as the state and decodes $(w_{21i}, w_{22i}, v_{22i})$ based on the signal received in block i. Specifically, D_2 searches for a unique $(\hat{w}_{21i}, \hat{w}_{22i})$ for some $(\hat{w}_{11i}, \hat{v}_{22i})$ such that

$$(u_{11}^n(\hat{w}_{11i}), u_{21}^n(\hat{w}_{21i}), u_{22}^n(\hat{w}_{22i}, \hat{v}_{22i}|\hat{w}_{21i}), y_2^n(i)) \in A_\epsilon^{(n)}(P_{U_{11}U_{21}U_{22}Y_2}). \tag{3.16}$$

The decoding error probability goes to 0 as $n \to \infty$ if

$$\begin{aligned} R_{22} + R'_{22} &\leq I(U_{22}; Y_2|U_{21}, U_{11}) \\ R_{21} + R_{22} + R'_{22} &\leq I(U_{21}, U_{22}; Y_2|U_{11}) \\ R_{11} + R_{22} + R'_{22} &\leq I(U_{11}, U_{22}; Y_2|U_{21}) \\ R_{11} + R_{21} + R_{22} + R'_{22} &\leq I(U_{11}, U_{21}, U_{22}; Y_2). \end{aligned} \tag{3.17}$$

Applying Fourier-Motzkin Elimination to (3.12)-(3.17), we get rate region (3.8). See Appendix A.1 for more details. □

Remark 4. *Several features of the HK-PDF-binning scheme are worth noting:*

- *In encoding, w_{10} and w_{11} are encoded independently, then w_{12} is superpositioned on both. This independent coding between the forwarding part (w_{10}) and Han-Kobayashi public part (w_{11}), rather than superposition, is important to ensure the rate region includes both PDF-binning and Han-Kobayashi regions.*

- *In the binning step* (3.11) *at S_2, we use conditional binning instead of the usual (unconditional) binning. The binning is only between the Han-Kobayashi private message part (w_{22}) and the state (w'_{10}), conditionally on knowing the Han-Kobayashi public messsage part w_{21}. This conditional binning is possible since w_{21} is decoded at both destinations.*

- *In the decoding step* (3.16) *at D_2, we use joint decoding of both the Gelfand-Pinsker auxiliary random variable (u_{22}) and the Han-Kobayashi public message parts (w_{11} and w_{21}), instead of decoding Gelfand-Pinsker and Han-Kobayashi codewords separately. This joint decoding is possible since the codewords for w_{11} and w_{21} (i.e. u_{11}^n and u_{21}^n) are independent of the state in Gelfand-Pinsker coding (i.e. t_{10}^n). Joint decoding at both D_1* (3.14) *and D_2* (3.16) *helps achieve the largest rate region for this coding structure.*

3.2.3 Comparison with existing schemes for the interference channel with source cooperation

In this section, we analyze in detail four existing schemes, [23, 24, 26, 27], for the interference channel with source cooperation that are most closely related to the proposed schemes. The interference channel with source cooperation is a 4-node channel in which both S_1 and S_2 can receive signals from each other and use those signals cooperatively in sending messages to D_1 and D_2. This channel therefore includes the CCIC as a special case (when S_2 sends no information to S_1).

Host-Madsen scheme

Host-Madsen [23] proposes a transmitter cooperation scheme in the synchronous case (see the section VI.C) for IC-SC based on the dirty paper coding and block Markov encoding. In this scheme, both senders need to decode each other's message first, then use the decoded message as the binning state and bin against it. There is no rate splitting but block Markov encoding. Sender two generates codewords U_2^0, which encode the message in the previous block. Sender one decodes this message from U_2^0 and generates codewords U_1^0 for the previous block message to bin against U_2^0. Then, it generates codewords U_1 for its current block message, which is superimposed on its previous message codewords U_1^0 and U_2^0. Sender two generates codewords U_2 to encode its current block message and bins it with the previous message codeword U_1^0 from sender one. The final codeword for sender one is X_1, which is superimposed on U_1, U_1^0 and U_2^0. Similarly, the final codeword for sender two is X_2, which is superimposed on U_2, U_1^0 and U_2^0. For decoding, destination one uses backward decoding to decode the message from sender one. Destination two uses joint decoding over two blocks to decode the message from sender two in the previous block. The Host-Madsen scheme differs from our scheme in the following respects:

- Both senders need to decode the whole message from the other sender. For example, sender two decodes the current block message from sender one since it knows the previous block message of sender one by forward decoding assumption and also U_1^0 and U_2^0. Similarly for sender one. These decodings at both senders are mandatory for the Host-Madsen scheme because the decoded messages are used as binning states in the next block. In our scheme, sender one does not need to decode any message from the second sender, since there is no binning at the first sender.

- Both senders use binning techniques; but in our scheme, we only use binning in the second sender, which eliminates the rate constraint at the first sender.

- As a result of the previous two points, the Host-Madsen scheme cannot be applied to the CCIC. This is because, in his scheme, both sources first decode the messages from the other source and then use binning techniques. Without the link from sender two to sender one, sender one can not decode any message from the second sender, thus the binning process at sender one is impossible, and the whole coding scheme fails for the CCIC.

- There is no rate splitting in the Host-Madsen scheme, thus it cannot achieve the Han-Kobayashi rate.

Prabhakaran-Viswanath coding schemes

Probhakaran and Viswanath propose two coding schemes in [24]. In this section, we will analyze these two coding schemes and compare them with the Han-Kobayashi rate region and partial decode-forward rate.

Prabhakaran-Viswanath scheme one

The first scheme (see [24] Theorem 4(a)) is based on 3-part message splitting and block Markov encoding. Take the encoding at source one at block i as an example. V_1 is the codeword for the current cooperative public message. W encodes the cooperative messages in the previous time slot at both sources. V_1 is superimposed on W. U_1 corresponds to the Han-Kobayashi public message, and X_1 encodes the Han-Kobayashi private message. The codeword X_1 is superimposed on U_1, which is superimposed on V_1. Similarly for source two. For decoding, source one decodes the cooperative message (m_{V_2}) from source two. Destination three (destination for source one) uses backward decoding to decode the cooperative message m_W, the Han-Kobayashi public message m_{U_1} and Han-Kobayashi public message m_{X_1}. Destination three also decodes the Han-Kobayashi public message m_{U_2} from source two, but it does not care if this decoding is correct. Similarly for source one and destination four.

This scheme achieves the Han-Kobayashi rate region, which can be verified if we set the cooperative message to 0 (in both the current block and also the previous block). However, this scheme does not always include the partial decode-forward region since it has one more extra rate constraint than the partial decode-forward rate. This is because destination four also decodes the cooperative public message V_1 from sender one, which adds one more extra constraint than the partial decode-forward rate. See Appendix A.2 for more details on the the comparison with both the Han-Kobayashi rate and the partial decode-forward rate.

Prabhakaran-Viswanath scheme two

The second scheme in [24] is based on 4-part rate splitting and block Markov encoding. These four parts acts as cooperative public, Han-Kobayashi public, Han-Kobayashi private and cooperative private messages. We only illustrate the codebook generation at sender one as an example, since the codebook generation is similar in both senders. First, the codeword W encodes the cooperative public messages in both senders in the previous block. Then, the codeword V_1 encodes the current cooperative public message, which is superimposed on W. The codeword U_1 encodes the Han-Kobayashi public message and is superimposed on W and V_1. The codeword S_1 encodes the previous cooperative private message, which is superimposed on W. (Also generated is the codeword S_2 for the previous cooperative private message of the second sender.) The codeword Z_1 encodes the current Han-Kobayashi private message and is superimposed on W, V_1, U_1 and S_1. Finally, one can generate the codeword X_1 for the current cooperative private message, which is superimposed on W, V_1, U_1, S_1, Z_1 and S_2. Sender two has similar codebook generation procedures. For encoding, senders one and two transmit X_1 and X_2 as their codewords. For decoding, sender one decodes the messages m_{V_1}, m_{U_1}, m_{Z_1} and m_{S_1}. Destination three uses backward decoding to decode the unique message tuples $(m_W, m_{U_1}, m_{S_1}, m_{Z_1})$ for some m_{U_2}. But it does not care if m_{U_2} is decoded correctly. Similarly for source two and destination four. See Figure 3.4 for the encoding procedures at source one, and similarly for source two.

This second scheme in [24] does not always include the Han-Kobayashi region or the partial decode-forward rate depending on channel parameters. To compare it with the Han-Kobayashi rate region, we need to set cooperative public and private messages of both sources to 0 ($V_2 = S_2 = \emptyset$ and $W = V_1 = S_1 = \emptyset$). After these settings, we get a rate region of 10 constraints (see (A.36)), which has two extra rate constraints compared with the traditional Han-Kobayashi rate region. These two extra constraints are introduced

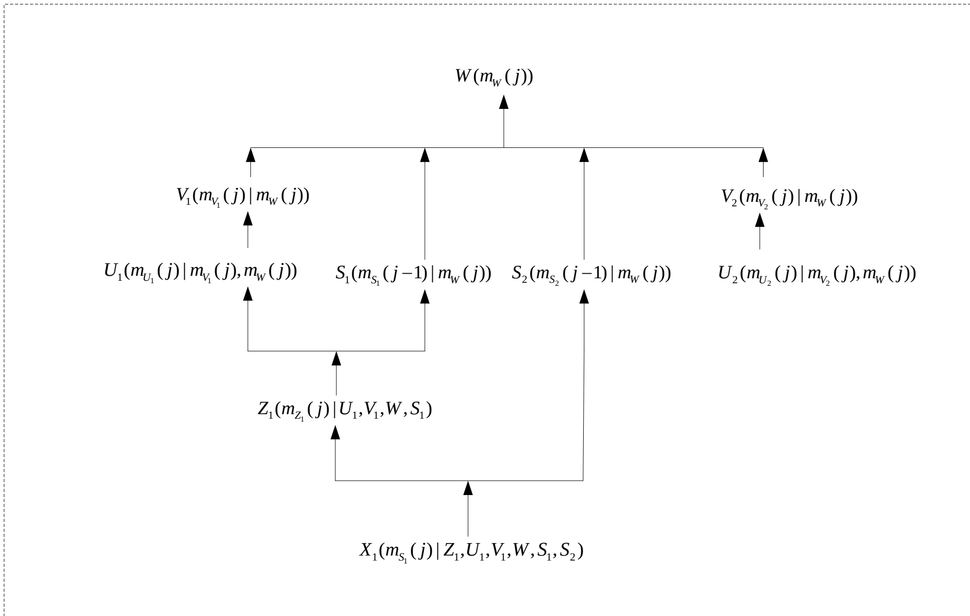

Fig. 3.4 Encoding procedures for Prabhakaran and Viswanath scheme two at source one, $m_W(j) = (m_{V_1}(j-1), m_{V_2}(j-1))$.

since the source two needs to decode the non-cooperative public message (U_1) and the non-cooperative private message (Z_1) from the source one. See Appendix A.3.1 for more details for the comparisons with the Han-Kobayashi region.

To compare with the partial decode-forward rate, we need to set all the messages belonging to source two to 0 ($V_2 = U_2 = S_2 = Z_2 = \emptyset$), also set the non-cooperative public (U_1) to 0. It reduces to a 7-constraint rate region (see (A.38)). Since in the PDF rate, we only have one forwarding part, we can set either the cooperative public (V_1) or the cooperative private message (S_1) to 0. When setting $V_1 = \emptyset$, it reduces to a 4-rate region (see (A.39)). The last constraint in (A.39) is an extra constraint compared with the PDF rate. This is because, in Prabhakaran and Viswanath scheme two, the current cooperative private message (X_1) is superimposed on the non-cooperative private message (Z_1) and the previous cooperative message (S_1), which differs with the partial decode-forward scheme in that the non-cooperative private message is superimposed on both the current and the previous cooperative messages. Because of this difference in the order of superposition, source two needs to decode both the current cooperative and the non-cooperative private messages instead of only the current cooperative message. Since source two needs to decode two message parts, this incurs an extra rate constraint and can reduce the rate to below the PDF rate. When setting $S_1 = \emptyset$, it reduces to a 5-rate region (see (A.40)). The last constraint is an extra constraint compared with the PDF rate because destination four needs to decode the non-cooperative public message m_{V_1}. Thus, in both cases, Prabhakaran-Viswanath scheme two does not always include the PDF rate. See Appendix A.3.2 for the comparison between Prabhakaran and Viswanath scheme two and the partial decode-forward rate.

We also apply Prabhakaran and Viswanath scheme two in our CCIC (by setting the cooperative public and private message V_2 and S_2 at source two to 0, $V_2 = S_2 = \emptyset$) and compare it numerically with our scheme in the Section 3.3.6.

Cao-Chen scheme

Cao and Chen [26] propose an achievable rate region for the interference channel with source cooperation based on rate splitting, block Markov encoding, superposition encoding, dirty paper coding and random binning. Each user splits its message into three parts: common, private and cooperative messages and divides the cooperative message into cells. The second user generates independent codewords for the current common message (u_2^n), previous cooperative cell index (s_2^n) and current cooperative message (w_2^n). The codeword for the current private message is then superimposed on the current common message and previous cooperative cell index ($v_2^n|u_2^n, s_2^n$). Then, the first user treats the previous cooperative-cell-index codeword (s_2^n) as the state and jointly bins its codewords for the current common message (n_1^n), previous cooperative cell index (h_1^n) and current cooperative message (g_1^n). Finally, the codeword for the first user's private message (m_1^n) is conditionally binned with s_2^n given n_1^n and h_1^n. A two-step decoding with list decoding is then used at each destination.

The common, private and cooperative message parts in [26] correspond roughly to our HK public, HK private and forwarding (common) part, respectively. As such, when applied to the CCIC, their scheme differs from the proposed HK-PDF-binning scheme in the following aspects:

- Block Markovity is applied only on the HK private part, whereas in our scheme, block Markovity is applied on all message parts.
- Block Markovity is based on cell division of the previous cooperative message, while in our scheme, block Markovity is on the whole previous common message. This,

however, is a minor difference, since if each cell contains only one message, then cell index reduces to message index.

- The first user bins both its HK public and private parts (the user labels are switched in [26]), whereas we only bin the HK private part (see Remark 4).

- The scheme in [26] cannot achieve the decode-forward relaying rate because of no block Markovity between the current cooperative-message codeword (w_2^n) and the previous cooperative-cell codeword (s_2^n). In other words, there is no coherent transmission between the source and relay, which can be readily verified from the code distribution. Consider setting $V_1 = V_2 = U_1 = U_2 = 0$, $M_1 = M_2 = N_1 = N_2 = 0$ and $W_2 = S_2 = G_2 = H_2 = 0$ in equation (8) of [26], then the code distribution reduces to

$$p(q)p(g_1|q)p(h_1|q)p(x_1|g_1, q)p(x_2|h_1, q) = p(q, g_1, x_1)p(q, h_1, x_2) \neq p(q, x_1, x_2),$$

  where q is the time sharing variable. This distribution implies that the first user splits its message into two parts and independently encodes each of them (by g_1 and h_1). The second user then decodes one part in g_1 and forwards this part to the destination. But because of the independence between g_1 and h_1, the achievable rate is less than in coherent decode-forward relaying.

  Thus, the claim in Remark 2 of [26] that this scheme achieves the capacity region of the degraded relay channel is in fact unfounded.

Yang-Tuninetti scheme

Yang and Tuninetti [27] propose two schemes for the interference channel with generalized feedback based on block Markov superposition coding, binning and backward decoding. Since the first scheme is a special case of the second, we analyze only their second scheme. Each user splits its message into four parts: cooperative common (w_{10c}), cooperative private (w_{11c}), non-cooperative common (w_{10n}) and non-cooperative private (w_{11n}). Consider the transmission in block b. First, generate independent codewords for the previous cooperative-common messages of both users ($Q^n(w_{10c,b-1}, w_{20c,b-1})$). Then the cooperative-common ($w_{10c,b}$), non-cooperative common ($w_{10n,b}$) and non-cooperative private ($w_{11n,b}$)

messages are superimposed on each other successively as V_1, D_1, U_1, respectively (according to $p(v_1, D_1, u_1|q)$). There are three binning steps after the above codebook generation. First, the codewords S_1, S_2 for the previous cooperative-private messages of both users are binned with each other given Q. Second, V_1, U_1 and D_1 are binned with S_1 and S_2 given Q. Third, the codeword Z_1 for the cooperative-private message ($w_{11c,b}$) is conditionally binned with S_2, U_1 and D_1 given V_1, S_1 and Q. Backward decoding is used, in which each destination applies relaxed joint decoding of all interested messages.

The non-cooperative messages in [27] correspond to our HK public and private parts. Their scheme has two cooperative message parts (the common is decoded at both destinations while the private is not), whereas the proposed HK-PDF-binning has only one common part. To compare these two schemes, we consider the following two special settings to make the message parts equivalent:

i) Set the cooperative-common message (w_{10c}) to $\emptyset$: their cooperative private message then corresponds to our forwarding (common) message. Their scheme differs markedly from HK-PDF-binning as follows.

- User one uses binning among the three message parts instead of superposition coding as in HK-PDF-binning. Block Markov superposition is also replaced by binning with the codeword for the previous cooperative message.
- User two applies joint binning of both the non-cooperative common and private parts instead of conditional binning of only the non-cooperative private part, given the non-cooperative common part (see Remark 4).

ii) Set the cooperative-private message (w_{11c}) to $\emptyset$: Their cooperative common message then corresponds to our forwarding (common) message. Their scheme is more similar to HK-PDF-binning, but there are several important differences as follows.

- User one now uses superposition coding, but superimposes all three message parts successively, whereas we generate codewords for the forwarding part and the HK public part independently (see Remark 4).
- User two also applies joint binning of both non-cooperative message parts instead of conditional binning, similar to case i).

- Destination two decodes the cooperative-common part of user one, thus limiting the rate of user one to below the decode-forward relaying rate because of the extra rate constraint at destination two (this applies even with relaxed decoding). In our proposed scheme, the forwarding part of user one is not decoded at destination two.

As a result, both schemes in [26] and [27], when applied to the CCIC, achieve the Han-Kobayashi region but not the decode-forward relaying rate for the first user. Thus, the maximum rates for user one in both schemes are smaller than in (3.2).

Another point is that, in both [26] and [27], joint decoding of both the state and the binning auxiliary random variables is used at the destinations. But this joint decoding is invalid and results in a rate region larger than is possible. In our proposed scheme, all message parts that are jointly decoded with the binning auxiliary variable at the second destination are encoded independently of the state.

Remark 5. *Based on our analysis, we conjecture that splitting the common (forwarding) message further into two parts is not necessary for the CCIC. In [27, 25], the common message is split into two parts: one for decoding at the other destination and the other for binning. Our analysis shows that both these operations can be included in one-step binning by varying the joint distribution between the state and the auxiliary random variable. This joint distribution becomes apparent when applied to the Gaussian channel as in Section 3.3 next.*

3.3 Full-duplex Gaussian CCIC rate regions

3.3.1 Full-duplex Gaussian CCIC model

In this section, we analyze the standard full-duplex Gaussian causal cognitive interference channel model as follows.

$$\begin{aligned} Y_1 &= X_1 + bX_2 + Z_1 \\ Y_2 &= aX_1 + X_2 + Z_2 \\ Y &= cX_1 + Z, \end{aligned} \tag{3.18}$$

where $Z_1,\ Z_2,\ Z \sim \mathcal{N}(0,1)$ are independent Gaussian noises. Assume that the transmit signals X_1 and X_2 are subject to power constraints P_1 and P_2, respectively.

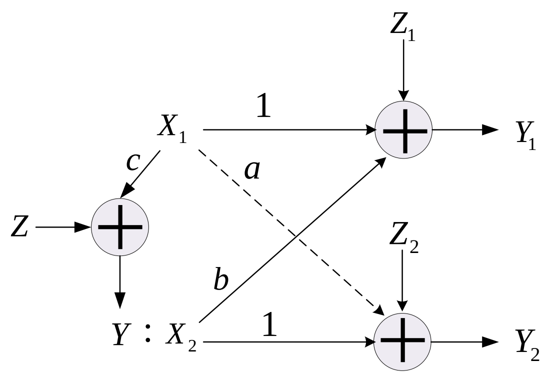

Fig. 3.5 The standard full-duplex Gaussian causal cognitive interference channel.

The standard Gaussian CCIC is shown in Figure 3.5. If the original channel is not in this standard form, we can always transform it into the standard form using a procedure similar to the interference channel [10].

3.3.2 Signaling and rates for full-duplex PDF-binning

In the Gaussian channel, the signals T_{10}, U_{10}, U_2, X_1 and X_2 of the PDF-binning scheme in Section 3.2.1 can be represented as follows.

$$\begin{aligned} T_{10} &= \alpha S'_{10}(w'_{10}), \\ U_{10} &= \alpha S'_{10}(w'_{10}) + \beta S_{10}(w_{10}), \\ X_1 &= \alpha S'_{10}(w'_{10}) + \beta S_{10}(w_{10}) + \gamma S_{11}(w_{11}), \\ X_2 &= \mu \left(\rho S'_{10}(w'_{10}) + \sqrt{1-\rho^2} S_{22} \right), \\ U_2 &= X_2 + \lambda S'_{10} = (\mu\rho + \lambda) S'_{10} + \mu \sqrt{1-\rho^2} S_{22}, \end{aligned} \tag{3.19}$$

where S'_{10}, S_{10}, S_{11} and S_{22} are independent $\mathcal{N}(0,1)$ random variables to encode w'_{10}, w_{10}, w_{11} and w_2 respectively. U_2 is the auxiliary random variable for binning that encodes w_2. X_1 and X_2 are the transmit signals of S_1 and S_2. The parameters α, β, γ, μ are power allocation factors satisfying the power constraints

$$\begin{aligned} \alpha^2 + \beta^2 + \gamma^2 &\le P_1, \\ \mu^2 &\le P_2, \end{aligned} \tag{3.20}$$

where P_1 and P_2 are transmit power constraints of S_1 and S_2.

An important feature of the signaling design in (3.19) is ρ ($-1 \leq \rho \leq 1$), the correlation factor between the transmit signal (X_2) and the state (S'_{10}) at S_2. In traditional dirty paper coding, the transmit signal and the state are independent. Here we introduce correlation between them, which includes dirty paper coding as a special case when $\rho = 0$. This correlation allows both signal forwarding and traditional binning at the same time. λ is the partial decode-forward binning parameter which will be optimized later.

Substituting X_1, X_2 into Y_1, Y_2 and Y in (3.18), we get

$$\begin{aligned} Y_1 &= (\alpha + b\mu\rho)S'_{10} + \beta S_{10} + \gamma S_{11} + b\mu\sqrt{1-\rho^2}S_{22} + Z_1, \\ Y_2 &= (a\alpha + \mu\rho)S'_{10} + a\beta S_{10} + a\gamma S_{11} + \mu\sqrt{1-\rho^2}S_{22} + Z_2, \\ Y &= c\alpha S'_{10} + c\beta S_{10} + c\gamma S_{11} + Z. \end{aligned} \tag{3.21}$$

Corollary 1. *The achievable rate region for the full-duplex Gaussian-CCIC using the PDF-binning scheme is the convex hull of all rate pairs (R_1, R_2) satisfying*

$$\begin{aligned} R_1 &\leq C\left(\frac{c^2\beta^2}{c^2\gamma^2+1}\right) + C\left(\frac{\gamma^2}{b^2\mu^2(1-\rho^2)+1}\right) \\ R_1 &\leq C\left(\frac{(\alpha+b\mu\rho)^2+\beta^2+\gamma^2}{b^2\mu^2(1-\rho^2)+1}\right) \\ R_2 &\leq C\left(\frac{\mu^2(1-\rho^2)}{a^2\beta^2+a^2\gamma^2+1}\right), \end{aligned} \tag{3.22}$$

where $-1 \leq \rho \leq 1$, $C(x) = \frac{1}{2}\log(1+x)$, and the power allocation factors α, β, γ and μ satisfy the power constraints (3.20).

Proof. Applying Theorem 1 with the signaling in (3.19), we get the rate region in Corollary 1. □

Remark 6. *Maximum rates for each sender*

- *Setting $\rho = \pm 1$, $\mu = \rho\sqrt{P_2}$, we obtain the maximum rate for R_1 as in partial decode-*

forward relaying:

$$R_1^{\max} = \max_{\alpha^2+\beta^2+\gamma^2 \leq P_1} \min \left\{ C\left(\frac{c^2\beta^2}{c^2\gamma^2+1}\right) + C(\gamma^2), C\left(\left(\alpha + b\sqrt{P_2}\right)^2 + \beta^2 + \gamma^2\right) \right\}. \tag{3.23}$$

- *Setting* $\rho = 0$, $\beta = \gamma = 0$ *and* $\mu = \sqrt{P_2}$, *we obtain the maximum rate for* R_2 *as in dirty paper coding:*

$$R_2^{\max} = C(P_2). \tag{3.24}$$

3.3.3 Optimal binning parameter for full-duplex PDF-binning

In this section, we derive in closed form the optimal binning parameter λ for (3.19) to achieve rate region (3.22). This optimal binning parameter is different from the optimal binning parameter in dirty paper coding, as we introduce the correlation factor ρ between the transmit signal and the state. This correlation contains the function of message forwarding. For example, if we set $\rho = \pm 1$, X_2 will only encode w'_{10} without any actual binning, and hence realize the function of message forwarding. If we set $\rho = 0$, PDF-binning becomes dirty paper coding without any message forwarding. For $0 < |\rho| < 1$, PDF-binning has both the functions of binning and message forwarding. Thus, PDF-binning generalizes dirty paper coding.

Theorem 3. *The optimal* λ *for the full-duplex PDF-binning scheme is*

$$\lambda^* = \frac{a\alpha\mu^2(1-\rho^2) - \mu\rho(a^2\beta^2 + a^2\gamma^2 + 1)}{a^2\beta^2 + a^2\gamma^2 + \mu^2(1-\rho^2) + 1}. \tag{3.25}$$

Proof. The optimal λ^* is obtained by maximizing both rates R_1 and R_2. In rate region (3.1), through the Fourier Motzkin Elimination process, we can see that if we maximize the term $I(U_2;Y_2) - I(U_2;T_{10})$, both R_1 and R_2 are maximized simultaneously. We have

$$\begin{aligned}
&I(U_2;Y_2) - I(U_2;T_{10}) \\
&= H(Y_2) - H(Y_2|U_2) - H(U_2) + H(U_2|T_{10}) \\
&= H(Y_2) + H(U_2|T_{10}) - H(U_2,Y_2).
\end{aligned}$$

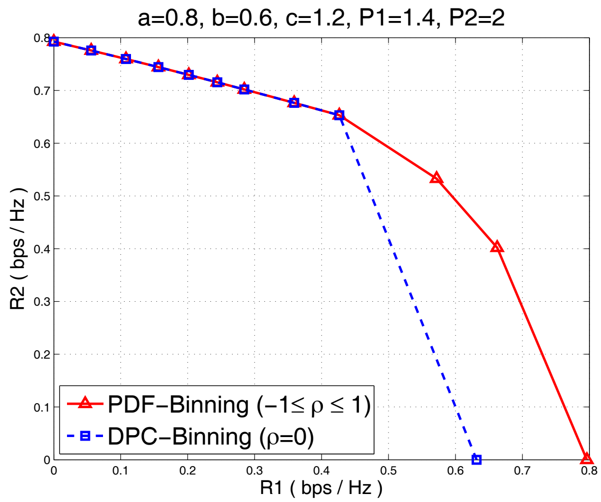

Fig. 3.6 Effect of the binning correlation factor ρ.

Here λ only affects the last term $H(U_2, Y_2)$. The covariance matrix between U_2 and Y_2 is

$$\text{cov}(U_2, Y_2) = \begin{bmatrix} \text{var}(U_2) & \text{E}(U_2, Y_2) \\ \text{E}(U_2, Y_2) & \text{var}(Y_2) \end{bmatrix}, \tag{3.26}$$

where

$$\begin{aligned} \text{var}(U_2) &= \mu^2 + \lambda^2 + 2\mu\rho\lambda, \\ \text{E}(U_2, Y_2) &= (\mu\rho + \lambda)(a\alpha + \mu\rho) + \mu^2(1 - \rho^2), \\ \text{var}(Y_2) &= (a\alpha + \mu\rho)^2 + a^2\beta^2 + a^2\gamma^2 + \mu^2(1 - \rho^2) + 1. \end{aligned}$$

Minimizing the determinant of the covariance matrix in (3.26), we obtain the optimal λ^* in (3.25). (See Appendix A.4 for details.) □

Remark 7. *Effect of* ρ*:*

- *If* $\rho = 0$, λ^* *becomes the optimal* λ *for traditional dirty paper coding [4], which achieves the maximum rate for* R_2 *as in* (3.24)*.*
- *If* $\rho = \pm 1$, λ^* *differs from the* λ *in traditional dirty paper coding and achieves the maximum rate for* R_1 *as in* (3.23)*.*

- *The effect of ρ can be seen in Figure 3.6. The dashed line represents the resulting rate region using only DPC-binning ($\rho = 0$), while the solid line represents the region for PDF-binning when we adapt $\rho \in [-1, 1]$. Figure 3.6 illustrates that the correlation factor ρ can enlarge the rate region.*

3.3.4 Signaling and rates for full-duplex Han-Kobayashi PDF-binning

In the Gaussian channel, input signals for the HK-PDF-binning scheme in Section 3.2.2 can be represented as

$$\begin{aligned}
T_{10} &= \alpha S'_{10}(w'_{10}), \\
U_{10} &= \alpha S'_{10}(w'_{10}) + \beta S_{10}(w_{10}), \\
U_{11} &= \gamma S_{11}(w_{11}), \\
X_1 &= \alpha S'_{10}(w'_{10}) + \beta S_{10}(w_{10}) + \gamma S_{11}(w_{11}) + \delta S_{12}(w_{12}), \\
U_{21} &= \theta S_{21}(w_{21}), \\
X_2 &= \theta S_{21}(w_{21}) + \mu \left(\rho S'_{10}(w'_{10}) + \sqrt{1-\rho^2} S_{22} \right), \\
U_{22} &= X_2 + \lambda S'_{10} = (\mu\rho + \lambda) S'_{10} + \theta S_{21}(w_{21}) + \mu\sqrt{1-\rho^2} S_{22},
\end{aligned} \tag{3.27}$$

where S'_{10}, S_{10}, S_{11}, S_{12}, S_{21}, S_{22} are independent $\mathcal{N}(0, 1)$ random variables to encode w'_{10}, w_{10}, w_{11}, w_{12}, w_{21}, w_{22}, respectively. U_{22} is the auxiliary random variable for binning that encodes w_{22}. X_1 and X_2 are the transmit signals of S_1 and S_2. ρ is the correlation coefficient between the transmit signal and the binning state at S_2 ($-1 \le \rho \le 1$). λ is the PDF-binning parameter. The parameters α, β, γ, δ, θ and μ are power allocation factors satisfying the power constraints

$$\begin{aligned}
\alpha^2 + \beta^2 + \gamma^2 + \delta^2 &\le P_1, \\
\theta^2 + \mu^2 &\le P_2,
\end{aligned} \tag{3.28}$$

where P_1 and P_2 are transmit power constraints of S_1 and S_2.

Substituting these variables into the Gaussian channel in (3.18), we get

$$\begin{aligned}
Y &= c\alpha S'_{10} + c\beta S_{10} + c\gamma S_{11} + c\delta S_{12} + Z,\\
Y_1 &= (\alpha + b\mu\rho)S'_{10} + \beta S_{10} + \gamma S_{11} + \delta S_{12} + b\theta S_{21} + b\mu\sqrt{1-\rho^2}S_{22} + Z_1,\\
Y_2 &= (a\alpha + \mu\rho)S'_{10} + a\beta S_{10} + a\gamma S_{11} + a\delta S_{12} + \theta S_{21} + \mu\sqrt{1-\rho^2}S_{22} + Z_2.
\end{aligned} \tag{3.29}$$

Corollary 2. *The achievable rate region for the full-duplex Gaussian-CCIC using the Han-Kobayashi PDF-binning scheme is the convex hull of all rate pairs (R_1, R_2) satisfying*

$$\begin{aligned}
R_1 &\le \min\{I_2 + I_5, I_6\}\\
R_2 &\le I_{12} - I_1\\
R_1 + R_2 &\le \min\{I_2 + I_7, I_8\} + I_{13} - I_1\\
R_1 + R_2 &\le \min\{I_2 + I_3, I_4\} + I_{14} - I_1\\
R_1 + R_2 &\le \min\{I_2 + I_9, I_{10}\} + I_{11} - I_1\\
2R_1 + R_2 &\le \min\{I_2 + I_3, I_4\} + \min\{I_2 + I_9, I_{10}\} + I_{13} - I_1\\
R_1 + 2R_2 &\le \min\{I_2 + I_7, I_8\} + I_{11} - I_1 + I_{14} - I_1,
\end{aligned} \tag{3.30}$$

where

$$\begin{aligned}
I_2 &= C\left(\frac{c^2\beta^2}{c^2\gamma^2 + c^2\delta^2 + 1}\right)\\
I_3 &= C\left(\frac{\delta^2}{b^2\mu^2(1-\rho^2)+1}\right)\\
I_4 &= C\left(\frac{\beta^2+\delta^2}{b^2\mu^2(1-\rho^2)+1}\right) + C\left(\frac{(\alpha+b\mu\rho)^2}{\beta^2+\gamma^2+\delta^2+b^2\theta^2+b^2\mu^2(1-\rho^2)+1}\right)\\
I_5 &= C\left(\frac{\gamma^2+\delta^2}{b^2\mu^2(1-\rho^2)+1}\right)\\
I_6 &= C\left(\frac{\beta^2+\gamma^2+\delta^2}{b^2\mu^2(1-\rho^2)+1}\right) + C\left(\frac{(\alpha+b\mu\rho)^2}{\beta^2+\gamma^2+\delta^2+b^2\theta^2+b^2\mu^2(1-\rho^2)+1}\right)\\
I_7 &= C\left(\frac{\delta^2+b^2\theta^2}{b^2\mu^2(1-\rho^2)+1}\right)
\end{aligned}$$

$$
\begin{aligned}
I_8 &= C\left(\frac{\beta^2+\delta^2+b^2\theta^2}{b^2\mu^2(1-\rho^2)+1}\right)+C\left(\frac{(\alpha+b\mu\rho)^2}{\beta^2+\gamma^2+\delta^2+b^2\theta^2+b^2\mu^2(1-\rho^2)+1}\right)\\
I_9 &= C\left(\frac{\gamma^2+\delta^2+b^2\theta^2}{b^2\mu^2(1-\rho^2)+1}\right)\\
I_{10} &= C\left(\frac{(\alpha+b\mu\rho)^2+\beta^2+\gamma^2+\delta^2+b^2\theta^2}{b^2\mu^2(1-\rho^2)+1}\right)\\
I_{11}-I_1 &= C\left(\frac{\mu^2(1-\rho^2)}{a^2\beta^2+a^2\delta^2+1}\right)\\
I_{12}-I_1 &= C\left(\frac{\mu^2(1-\rho^2)}{a^2\beta^2+a^2\delta^2+1}\right)+C\left(\frac{\theta^2}{(a\alpha+\mu\rho)^2+a^2\beta^2+a^2\delta^2+\mu^2(1-\rho^2)+1}\right)\\
I_{13}-I_1 &= C\left(\frac{\mu^2(1-\rho^2)}{a^2\beta^2+a^2\delta^2+1}\right)+C\left(\frac{a^2\gamma^2}{(a\alpha+\mu\rho)^2+a^2\beta^2+a^2\delta^2+\mu^2(1-\rho^2)+1}\right)\\
I_{14}-I_1 &= C\left(\frac{\mu^2(1-\rho^2)}{a^2\beta^2+a^2\delta^2+1}\right)+C\left(\frac{a^2\gamma^2+\theta^2}{(a\alpha+\mu\rho)^2+a^2\beta^2+a^2\delta^2+\mu^2(1-\rho^2)+1}\right),
\end{aligned}
$$

and α, β, γ, δ, θ *and* μ *are power allocation factors satisfying the power constraints* (3.28) *and* $-1 \le \rho \le 1$.

Proof. Applying Theorem 2 with the signaling in (3.27), we obtain the rate region in Corollary 2. □

Note that rate region (3.30) includes both the Han-Kobayashi rate region and the PDF-binning region in (3.22). Furthermore, the maximum rates for user 1 and user 2 are the same as in (3.23) and (3.24).

3.3.5 Optimal binning parameter for full-duplex Han-Kobayashi PDF-binning

Corollary 3. *The optimal* λ^* *for the full-duplex HK-PDF-binning scheme is*

$$
\lambda^* = \frac{a\alpha\mu^2(1-\rho^2)-\mu\rho(a^2\beta^2+a^2\delta^2+1)}{a^2\beta^2+a^2\delta^2+\mu^2(1-\rho^2)+1}. \tag{3.31}
$$

Proof. To simultaneously maximize R_1 and R_2 in region (3.8), we can simply maximize the term $I_{11} - I_1$ as follows.

$$\begin{aligned}
&I(U_{22}; Y_2|U_{21}, U_{11}) - I(U_{22}; T_{10}|U_{21})\\
&= H(Y_2|U_{21}, U_{11}) - H(Y_2|U_{21}, U_{22}, U_{11}) - H(U_{22}|U_{21}) + H(U_{22}|T_{10}, U_{21})\\
&= H(Y_2') - H(Y_2'|U_{22}') - H(U_{22}') + H(U_{22}|T_{10}, U_{21})\\
&= H(Y_2') + H(U_{22}|T_{10}, U_{21}) - H(U_{22}', Y_2'),
\end{aligned}$$

where

$$\begin{aligned}
Y_2' &= Y_2|U_{21}, U_{11} = (a\alpha + \mu\rho)S_{10}' + a\beta S_{10} + a\delta S_{12} + \mu\sqrt{1-\rho^2}S_{22} + Z_2\\
U_{22}' &= U_{22}|U_{21}, U_{11} = (\mu\rho + \lambda)S_{10}' + \mu\sqrt{1-\rho^2}S_{22}.
\end{aligned}$$

Note that λ only affects the last term $H(U_{22}', Y_2')$. The covariance matrix between U_{22}' and Y_2' is

$$\text{cov}(U_{22}', Y_2') = \begin{bmatrix} \text{var}(U_{22}') & \text{E}(U_{22}', Y_2') \\ \text{E}(U_{22}', Y_2') & \text{var}(Y_2') \end{bmatrix}, \tag{3.32}$$

where

$$\begin{aligned}
\text{var}(U_{22}') &= \mu^2 + \lambda^2 + 2\mu\rho\lambda,\\
\text{E}(U_{22}', Y_2') &= (\mu\rho + \lambda)(a\alpha + \mu\rho) + \mu^2(1-\rho^2),\\
\text{var}(Y_2') &= (a\alpha + \mu\rho)^2 + a^2\beta^2 + a^2\delta^2 + \mu^2(1-\rho^2) + 1.
\end{aligned}$$

Minimizing the determinant of the matrix in (3.32) leads to the optimal λ as in (3.31).

Note that the optimal λ^* in (3.31) contains both the optimal λ^* for PDF-binning in (3.25) and the optimal λ for DPC binning [4] as special cases. □

3.3.6 Numerical examples

In this section, we provide numerical comparison among the proposed PDF-binning and HK-PDF-binning schemes, the original Han-Kobayashi scheme, and an outer bound as discussed below.

Outer bounds for the CCIC capacity

We obtain a simple outer bound for the CCIC capacity by combining the capacity for the (non-causal) CIC and the outer bound for interference channel with user cooperation (IC-UC) [28]. Where the CIC capacity result is not available, we use the MISO broadcast capacity.

$$\begin{aligned}\text{CCIC capacity} &\subset \text{CIC capacity} \bigcap \text{IC-UC outer bound} \\ &\subset \text{MISO BC capacity} \bigcap \text{IC-UC outer bound.}\end{aligned}$$

a) Capacity of the CIC as an outer bound: The capacity of the ideal CIC (with non-causal knowledge of S_1's message at S_2) is an outer bound to the CCIC rate region. The CIC capacity is known in the cases of (i) weak interference [16, 20]; (ii) very strong interference [15]; (iii) the primary-decode-cognitive region [38]. For strong interference, we can also use the outer bound to the CIC capacity in [19] as an outer bound to the CCIC.

b) IC-UC outer bound: Tandon and Ulukus [28] obtain an outer bound for the MAC with generalized feedback based on dependence balance, which was first proposed by Hekstra and Willems [29] to study outer bounds for the single-output two-way channels. The basic idea of dependence balance is that no more information can be consumed than produced. Tandon and Ulukus apply this idea to obtain a new outer bound for IC-UC. It is shown that this dependence-balance-based outer bound is strictly tighter than the cutset bound (see Section V of [28]). Thus, this bound can be used instead of the relay channel (RC) cutset bound for R_1.

c) Gaussian Vector Broadcast Outer Bound: Consider a 2×1 MISO broadcast system as

$$\begin{aligned} Y_1 &= [1 \quad b]X + Z_1, \\ Y_2 &= [a \quad 1]X + Z_2, \end{aligned} \tag{3.33}$$

where a, b are the channel gains, Z_1 and Z_2 are white Gaussian noises with identity covariance. The vector codeword X consists of two independent parts:

$$X = U + V,$$

where $X = \begin{pmatrix} X_1 \\ X_2 \end{pmatrix}$, $U = \begin{pmatrix} U_1 \\ V_1 \end{pmatrix}$, $V = \begin{pmatrix} U_2 \\ V_2 \end{pmatrix}$, and U_1, V_1, U_2, V_2 are zero-mean Gaussian codewords with covariances:

$$K_U = \begin{bmatrix} \alpha^2 & \rho_1\alpha\beta \\ \rho_1\alpha\beta & \beta^2 \end{bmatrix}; \quad K_V = \begin{bmatrix} \gamma^2 & \rho_2\gamma\delta \\ \rho_2\gamma\delta & \delta^2 \end{bmatrix},$$

in which the power allocation factors satisfy

$$\alpha^2 + \beta^2 \leq P_1, \quad \gamma^2 + \delta^2 \leq P_2, \tag{3.34}$$

and the input correlation factors $\rho_1, \rho_2 \in [-1, 1]$.

The Gaussian vector broadcast capacity region is the convex closure of $R_{o1} \bigcup R_{o2}$ [39], where R_{o1} is the region

$$\begin{aligned} R_1 &\leq C\left(\frac{\alpha^2 + 2b\rho_1\alpha\beta + b^2\beta^2}{\gamma^2 + 2b\rho_2\gamma\delta + b^2\delta^2 + 1}\right) \\ R_2 &\leq C\left(a^2\gamma^2 + 2a\rho_2\gamma\delta + \delta^2\right). \end{aligned} \tag{3.35}$$

And R_{o2} is the region

$$\begin{aligned} R_1 &\leq C\left(\alpha^2 + 2b\rho_1\alpha\beta + b^2\beta^2\right) \\ R_2 &\leq C\left(\frac{a^2\gamma^2 + 2a\rho_2\gamma\delta + \delta^2}{a^2\alpha^2 + 2a\rho_1\alpha\beta + \beta^2 + 1}\right). \end{aligned} \tag{3.36}$$

Numerical comparison

Figure 3.7 shows the comparison in the full-duplex mode among the Han-Kobayashi scheme, PDF-binning, HK-PDF-binning, and the outer bound. We can see that the proposed HD-PDF-binning scheme contains both the Han-Kobayashi and the PDF-binning rate regions, as analyzed in Remark 3. Note that the outer bound is the intersection of the two bounds drawn and is loose as this bound is not achievable. However, we observe that as b decreases, the HK-PDF-binning rate region becomes closer to the outer bound.

We further apply Prabhakaran and Viswanath scheme two in our channel (set the cooperative message V_2 and S_2 at source two to 0, $V_2 = S_2 = \emptyset$). After these settings and

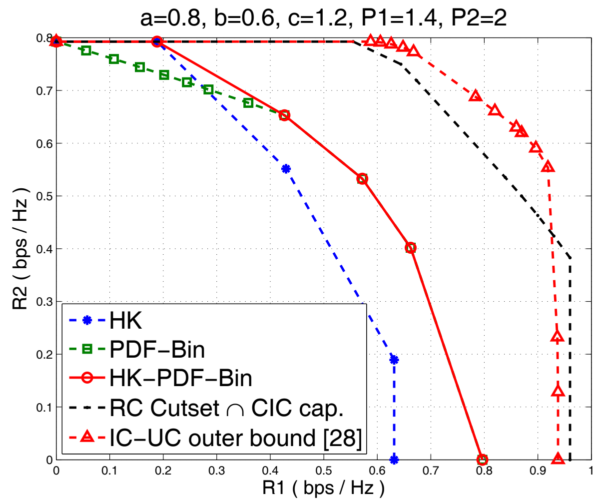

Fig. 3.7 Rate regions for full-duplex schemes in the Gaussian causal cognitive interference channel.

Fourier-Motzkin Elimination (see Appendix A.3.3), the rate region for the second scheme when applied in the CCIC (uni-cooperation) is shown as

$$\begin{aligned} R_1 &\leq I_4 \\ R_2 &\leq I_{15} \\ R_1 + R_2 &\leq I_{13} + I_{14} \\ R_1 + R_2 &\leq \min\{I_1 + I_5, I_2, I_7\} + I_{18} \\ R_1 + 2R_2 &\leq \min\{I_1 + I_9, I_{11}\} + I_{14} + I_{18}, \end{aligned} \tag{3.37}$$

where the term I_1-I_{18} is defined in (A.18) - (A.35).

Appendix A.3.4 shows the rate region for Prabhakaran and Viswanath scheme two in the Gaussian case. The simulation results for both weak and strong interference in Gaussian cases are shown in Figure 3.8 and Figure 3.9. From the weak interference result (Figure 3.8), we can see that Prabhakaran and Viswanath scheme two achieves the partial decode-forward rate but not the HK region. Moreover, the maximum rate for R_2 is less than the maximum rate ($C(P_2)$) that our scheme achieves. From the strong interference result (see Figure 3.9), their scheme achieves HK region and the maximum rate for the cognitive sender, but the maximum rate for R_1 is less than the partial decode-forward rate.

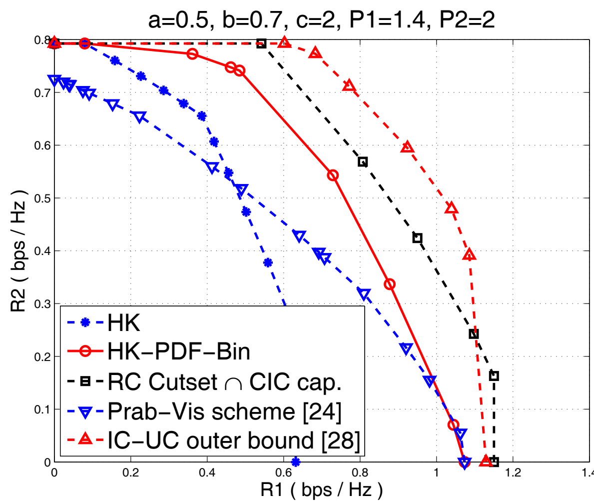

Fig. 3.8 Rate regions comparison for full-duplex schemes in weak interference.

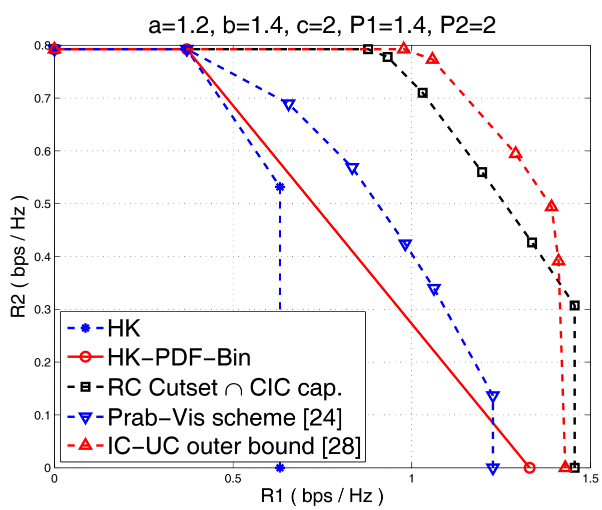

Fig. 3.9 Rate regions comparison for full-duplex schemes in strong interference.

Chapter 4

Half-Duplex Coding Schemes for the CCIC

4.1 Half-duplex DM-CCIC model

The half-duplex causal cognitive interference channel also consists of four nodes: two senders S_1, S_2 and two receivers D_1, D_2 as in Figure 4.1. S_1 wants to send a message to D_1. S_2 serves as a causal relay node and helps forward messages from S_1 to D_1, while also sending its own message to D_2. The transmission in the half-duplex mode is divided into two phases. In the first phase, S_1 transmits its message and S_2, D_1 and D_2 listen. In the second phase, both S_1 and S_2 transmit and D_1 and D_2 listen. This 2-phase transmission allows, for example, S_2 to decode a part of the message from S_1 in the first phase and then forward this part with its own message in the second phase.

Formally, the half-duplex causal cognitive interference channel consists of three input alphabets $\mathcal{X}_{11}$, $\mathcal{X}_{12}$, $\mathcal{X}_{22}$, and five output alphabets $\mathcal{Y}_{11}$, $\mathcal{Y}_{21}$, $\mathcal{Y}$, $\mathcal{Y}_{12}$, $\mathcal{Y}_{22}$. The channel is characterized by a channel transition probability
$p_c(y_{11}, y_{21}, y, y_{12}, y_{22}, |x_{11}, x_{12}, x_{22})$ defined as

$$p_c(y_{11}, y_{21}, y, y_{12}, y_{22}, |x_{11}, x_{12}, x_{22}) = \begin{cases} p(y_{11}, y_{21}, y|x_{11}) & \text{if } 0 \leq t \leq \tau, \\ p(y_{12}, y_{22}|x_{12}, x_{22}) & \text{if } \tau \leq t \leq 1, \end{cases} \tag{4.1}$$

where t is the normalized transmission time within 1 block, x_{11} and x_{21} refer to the transmit signals of S_1 in the first and second phases, respectively; x_{22} refers to the transmit signal

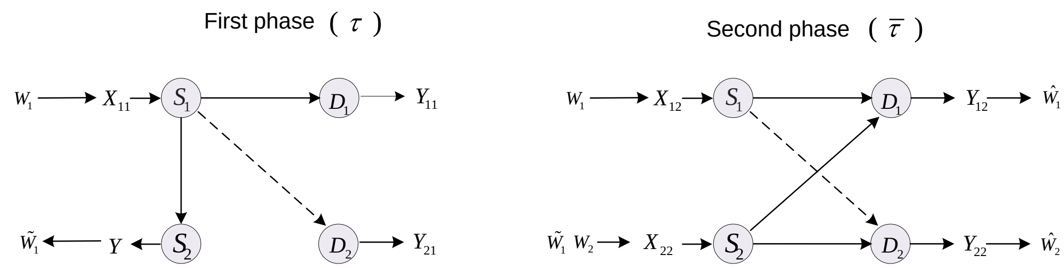

Fig. 4.1 The half-duplex discrete memoryless causal cognitive interference channel.

of S_2 in the second phase (S_2 does not send any signal in the first phase); y_{11} and y_{12} are the received signals of D_1 in the first and second phases; y_{21} and y_{22} are the received signals of D_2 in the two phases; and y is the received signal of S_2 in the first phase. We assume the channel is memoryless. Figure 4.1 illustrates the channel model, where W_1 and W_2 are the messages of S_1 and S_2. We use the notations $x^{\tau n} = (x_1, x_2, \cdots, x_{\tau n})$ and $x^{\bar{\tau} n} = (x_{\tau n+1}, \cdots, x_n)$, which correspond to the codewords sent during the first and second phases.

A (2^{nR_1}, 2^{nR_2}, n) code, or a communication strategy for n channel uses with rate pair (R_1, R_2), consists of the following:

- Two message sets $\mathcal{W}_1 \times \mathcal{W}_2 = [1, 2^{nR_1}] \times [1, 2^{nR_2}]$ and independent messages W_1, W_2 that are uniformly distributed over $\mathcal{W}_1$ and $\mathcal{W}_2$.
- Three encoders: two that map message w_1 into codewords $x_{11}^n(w_1) \in \mathcal{X}_{11}^n$ and $x_{12}^n(w_1) \in \mathcal{X}_{12}^n$, and one that maps w_2 and $y^{\tau n}$ into a codeword $x_{22}^n(w_2, y^{\tau n}) \in \mathcal{X}_{22}^n$.
- Two decoders: One maps y_1^n into $\hat{w}_1 \in \mathcal{W}_1$, and one maps y_2^n into $\hat{w}_2 \in \mathcal{W}_2$.

The probabilities of error, achievable rate and capacity region are defined in a similar way to the full-duplex case.

4.2 Half-Duplex coding schemes

In this section, we adapt the two full-duplex schemes to the half-duplex mode. The half-duplex schemes are also based on rate splitting, superposition encoding, partial decode-forward binning and Han-Kobayashi coding. There are several differences between the half-

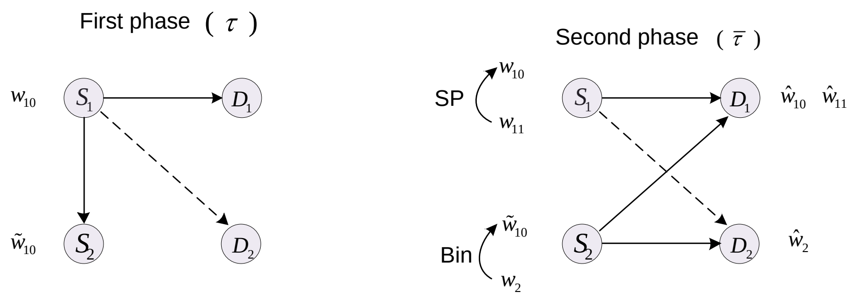

Fig. 4.2 Coding structure for the half-duplex CCIC based on partial decode-forward binning.

and full-duplex cases. First, under the half-duplex constraint, no node can both transmit and receive at the same time, thus leading us to divide each transmission block into two phases. In the first phase, S_1 sends a message to S_2, D_1 and D_2, while S_2 only receives but sends no messages. In the second phase, both S_1 and S_2 send messages concurrently. Second, S_1 sends different message parts in different phases. Specifically, S_1 only sends one part of its message to other nodes in the first phase, but will send all message parts in the second phase. Third, there is no block Markovity in the encoding since the superposition coding can be done between 2 phases of the same block instead of between 2 consecutive blocks. Finally, both D_1 and D_2 apply joint decoding only at the end of the second phase to make use of the received signals in both phases.

4.2.1 Half-duplex partial decode-forward binning scheme

The coding structure for the half-duplex PDF-binning scheme is shown in Figure 4.2. This scheme uses superposition encoding at the first sender, and partial decode-forward relaying and binning at the second sender. The first sender S_1 splits its message into two parts (w_{10}, w_{11}), corresponding to the forwarding and private parts. In the first phase, S_1 sends a codeword $X_{11}^{\tau n}$ containing the message part w_{10}; S_2 sends no information but only listens. At the end of the first phase, S_2 decodes w_{10} from S_1 . Note that neither D_1 nor D_2 decodes during this phase. In the second phase, S_1 sends a codeword $X_{12}^{\bar{\tau} n}$ containing both parts (w_{10}, w_{11}), in which w_{11} is superimposed on w_{10}. S_2 now sends both w_2 and w_{10} and uses Gelfand-Pinsker binning technique to bin against the codeword $X_{11}^{n}(w_{10})$ decoded from S_1 in the first phase. At the destinations, D_1 uses joint decoding to decode (w_{10}, w_{11}) from the signals received in both phases; D_2 decodes w_2 using the received signal in the second phase.

Specifically, at the end of the first phase, S_2 searches for a unique $\hat{w}_{10}$ such that

$$(x_{11}^{\tau n}(\hat{w}_{10}), \mathbf{y}) \in A_{\epsilon}^{(\tau n)}(P_{X_{11}Y}),$$

where $\mathbf{y}$ is the received signal vector at S_2 in the first phase. It then performs binning by looking for a v_2 such that

$$(x_{11}^{\bar{\tau}n}(\hat{w}_{10}), u_2^{\bar{\tau}n}(w_2, v_2)) \in A_{\epsilon}^{(\bar{\tau}n)}(P_{X_1U_2}),$$

and sends $x_{22}^{\bar{\tau}n}(\mathbf{x}_{11}, \mathbf{u}_2)$ as a function of $x_{11}^{\bar{\tau}n}$ and $u_2^{\bar{\tau}n}$ in the second phase.

At the end of the second phase, D_1 searches for a unique $(\hat{w}_{10}, \hat{w}_{11})$ such that

$$\begin{aligned}(x_{11}^{\bar{\tau}n}(\hat{w}_{10}), x_{12}^{\bar{\tau}n}(\hat{w}_{11}|\hat{w}_{10}), \mathbf{y_{12}}) &\in A_{\epsilon}^{(\bar{\tau}n)}(P_{X_{11}X_{12}Y_{12}}) \\ \text{and} \quad (x_{11}^{\tau n}(\hat{w}_{10}), \mathbf{y_{11}}) &\in A_{\epsilon}^{(\tau n)}(P_{X_{11}Y_{11}}),\end{aligned}$$

where $\mathbf{y_{11}}$ and $\mathbf{y_{12}}$ indicate the received vectors at D_1 during the first and second phases, respectively. D_2 treats the codeword X_{11}^n as the state and decodes w_2. It searches for a unique $\hat{w}_2$ for some $\hat{v}_2$ such that

$$(u_2^{\bar{\tau}n}(\hat{w}_2, \hat{v}_2), \mathbf{y_{22}}) \in A_{\epsilon}^{(\bar{\tau}n)}(P_{U_2Y_{22}}),$$

where $\mathbf{y_{22}}$ is the received vector at D_2 in the second phase.

Theorem 4. *The convex hull of the following rate region is achievable for the half-duplex causal cognitive interference channel using PDF-binning:*

$$\bigcup_{P_3} \begin{cases} R_1 & \leq \tau I(X_{11}; Y) + \bar{\tau} I(X_{12}; Y_{12}|X_{11}) \\ R_1 & \leq \tau I(X_{11}; Y_{11}) + \bar{\tau} I(X_{11}, X_{12}; Y_{12}) \\ R_2 & \leq \bar{\tau} I(U_2; Y_{22}) - \bar{\tau} I(U_2; X_{11}), \end{cases} \tag{4.2}$$

where

$$P_3 = p(x_{11})p(x_{12}|x_{11})p(u_2|x_{11})p(x_{22}|x_{11}, u_2)p_c(y_{11}, y_{21}, y, y_{12}, y_{22}|x_{11}, x_{12}, x_{22}),$$

and p_c is given in (4.1), $\bar{\tau} = 1 - \tau, 0 \leq \tau \leq 1$.

Proof. We use random codes and fix a joint probability distribution

$$p(x_{11})p(x_{12}|x_{11})p(u_2|x_{11})p(x_{22}|x_{11},u_2).$$

Codebook generation

- Independently generate $2^{nR_{10}}$ sequences $x_{11}^n \sim \prod_{k=1}^n p(x_{11k})$. Index these codewords as $x_{11}^n(w_{10})$, $w_{10} \in [1, 2^{nR_{10}}]$.
- For each $x_{11}^n(w_{10})$, independently generate $2^{nR_{11}}$ sequences $x_{12}^n \sim \prod_{k=1}^n p(x_{12k}|x_{11k})$. Index these codewords as $x_{12}^n(w_{11}|w_{10})$, $w_{11} \in [1, 2^{nR_{11}}]$, $w_{10} \in [1, 2^{nR_{10}}]$.
- Independently generate $2^{n(R_2+R_2')}$ sequences $u_2^n \sim \prod_{k=1}^n p(u_{2k})$. Index these codewords as $u_2^n(w_2, v_2)$, $w_2 \in [1, 2^{nR_2}]$ and $v_2 \in [1, 2^{nR_2'}]$.
- For each $x_{11}(w_{10})$ and $u_2^n(w_2, v_2)$, generate one $x_{22}^n \sim \prod_{k=1}^n p(x_{22i}|x_{11k}, u_{2k})$. Index these codewords as $x_{22}^n(w_{10}, w_2, v_2)$, $w_2 \in [1, 2^{nR_2}]$, $v_2 \in [1, 2^{nR_2'}]$.

Encoding

- In the first phase, S_1 sends the codewords $x_{11}^{\tau n}(w_{10})$. S_2 does not send anything.
- In the second phase, S_1 sends $x_{12}^{\bar{\tau} n}(w_{11}|w_{10})$.

  For S_2, it searches for a v_2 such that

$$(x_{11}^{\bar{\tau} n}(w_{10}), u_2^{\bar{\tau} n}(w_2, v_2)) \in A_\epsilon^{(\bar{\tau} n)}(P_{X_{11}U_2}).$$

  Such v_2 exists with high probability if

$$R_2' \geq \bar{\tau} I(U_2; X_{11}). \tag{4.3}$$

  S_2 then transmits $x_{22}^{\bar{\tau} n}(w_{10}, w_2, v_2)$.

Decoding

- At the end of the first phase, S_2 searches for a unique $\hat{w}_{10}$ such that

$$(x_{11}^{\tau n}(\hat{w}_{10}), \mathbf{y}) \in A_\epsilon^{(\tau n)}(P_{X_{11}Y}).$$

We can show that the decoding error probability goes to 0 as $n \to \infty$ if

$$R_{10} \le \tau I(X_{11}; Y). \tag{4.4}$$

- At the end of the second phase, D_1 searches for a unique $(\hat{w}_{10}, \hat{w}_{11})$ such that

$$(x_{11}^{\bar{\tau}n}(\hat{w}_{10}), x_{12}^{\bar{\tau}n}(\hat{w}_{11}|\hat{w}_{10}), \mathbf{y_{12}}) \in A_\epsilon^{(\bar{\tau}n)}(P_{X_{11}X_{12}Y_{12}})$$
$$\text{and} \quad (x_{11}^{\tau n}(\hat{w}_{10}), \mathbf{y_{11}}) \in A_\epsilon^{(\tau n)}(P_{X_{11}Y_{11}}).$$

Here $\mathbf{y_{11}}$ and $\mathbf{y_{11}}$ indicate the received vectors at D_1 during the first and second phases. The decoding error probability goes to 0 as $n \to \infty$ if

$$R_{11} \le \bar{\tau} I(X_{12}; Y_{12}|X_{11})$$
$$R_{10} + R_{11} \le \bar{\tau} I(X_{11}, X_{12}; Y_{12}) + \tau I(X_{11}; Y_{11}). \tag{4.5}$$

- D_2 treats the codeword $X_{11}^{\bar{\tau}n}$ from S_1 as the state and decodes w_2. It searches for a unique $\hat{w}_2$ for some $\hat{v}_2$ such that

$$(u_2^{\bar{\tau}n}(\hat{w}_2, \hat{v}_2), \mathbf{y_{22}}) \in A_\epsilon^{(\bar{\tau}n)}(P_{U_2Y_{22}}).$$

The decoding error probability goes to 0 as $n \to \infty$ if

$$R_2 + R_2' \le \bar{\tau} I(U_2; Y_{22}). \tag{4.6}$$

Combining all the above rate constraints, we get

$$R_2' \ge \bar{\tau} I(U_2; X_{11})$$
$$R_{10} \le \tau I(X_{11}; Y)$$
$$R_{11} \le \bar{\tau} I(X_{12}; Y_{12}|X_{11})$$
$$R_{10} + R_{11} \le \bar{\tau} I(X_{11}, X_{12}; Y_{12}) + \tau I(X_{11}; Y_{11})$$
$$R_2 + R_2' \le \bar{\tau} I(U_2; Y_{22}). \tag{4.7}$$

Let $R_1 = R_{10} + R_{11}$, apply Fourier-Motzkin Elimination, and we get region (4.2). □

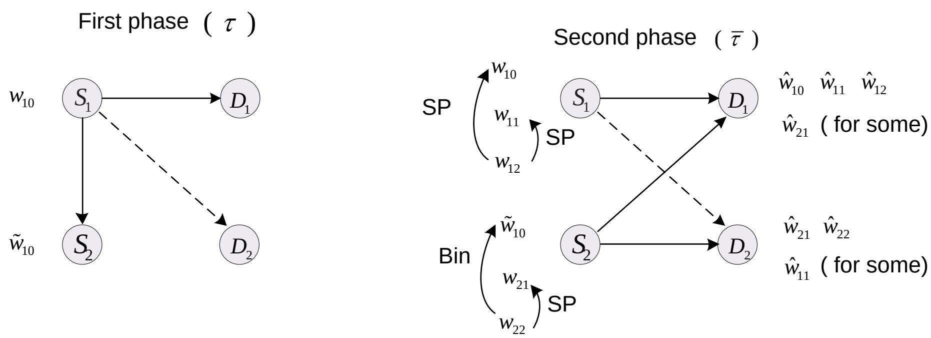

Fig. 4.3 Coding structure for the half-duplex CCIC based on Han-Kobayashi partial decode-forward binning.

Remark 8. *The maximum rate for each user.*

- *The first user S_1 achieves the maximum rate of half-duplex partial decode-forward relaying if we set $U_2 = \emptyset$.*

$$R_1^{\max} = \max_{\substack{0\leq\tau\leq 1\\ p(x_{11},x_{12})}} \min\{\tau I(X_{11};Y) + \bar{\tau} I(X_{12};Y_{12}|X_{11}), \tau I(X_{11};Y_{11}) + \bar{\tau} I(X_{11},X_{12};Y_{12})\}. \tag{4.8}$$

This half-duplex $R_1^{\max}$ is slightly smaller than in the full-duplex case of (3.2)*.*

- *The second user S_2 achieves the maximum rate of the Gelfand-Pinsker binning if we set $\tau = 0$, $X_{12} = X_{11}$.*

$$R_2^{\max} = \max_{p(x_{11},u_2)p(x_{22}|x_{11},u_2)} \{I(U_2;Y_{22}) - I(U_2;X_{11})\}. \tag{4.9}$$

This half-duplex $R_2^{\max}$ is the same as in the full-duplex case of (3.3)*. Even though this equality seems somewhat surprising, it is indeed the case in the limit of $\tau \to 0$, given that user 1 sends just enough information for S_2 to be able to decode completely in the first phase and then bin against it in the second phase. At $\tau = 0$ and $X_{12} = X_{11} = \emptyset$, S_2 can achieve the interference-free rate.*

4.2.2 Half-duplex Han-Kobayashi PDF-binning scheme

The first half-duplex coding scheme utilizes PDF-binning at the second sender and achieves the maximum possible rates for both user 1 and user 2. But it does not include the Han-Kobayashi scheme for the interference channel. In this section, we extend this scheme to combine with the Han-Kobayashi scheme by further splitting the messages in the second phase.

The coding structure for half-duplex HK-PDF-binning is shown in Figure 4.3. The encoding and decoding procedure in the first phase is the same as that of half-duplex PDF-binning. The major difference is in the second phase. Message w_1 of the first sender S_1 is split into three parts (w_{10}, w_{11}, w_{12}), corresponding to the forwarding, public and private parts. Message w_2 is split into 2 parts (w_{21}, w_{22}), corresponding to the public and private parts. We generate independent codewords for messages w_{10} and w_{11} and superimpose w_{12} on both of them. In the first phase, S_1 sends a codeword containing w_{10}, while S_2 does not send any message. At the end of the first phase, S_2 decodes $\tilde{w}_{10}$ using the received signal vector $\mathbf{y}$ and then bins its private part w_{22} against the decoded message $\tilde{w}_{10}$, conditionally on knowing the public part w_{21}. In the second phase, S_1 sends a codeword containing (w_{10}, w_{11}, w_{12}) while S_2 sends the binned signal containing (w_{10}, w_{21}, w_{22}). At the end of the second phase, D_1 uses joint decoding across both phases and searches for a unique triple $(\hat{w}_{10}, \hat{w}_{11}, \hat{w}_{12})$ for some w_{21}. D_2 also uses joint decoding based on the received signal in the second phase and searches for a unique pair $(\hat{w}_{21}, \hat{w}_{22})$ for some $\hat{w}_{11}$.

Specifically, in the first phase, S_1 sends $x_{11}^{\tau n}(w_{10})$; S_2 does not transmit. In the second phase, S_1 sends $x_{12}^{\bar{\tau} n}(w_{12}|w_{10}, w_{11})$; S_2 searches for some v_{22} such that

$$(x_{11}^{\bar{\tau} n}(w_{10}), u_{21}^{\bar{\tau} n}(w_{21}), u_{22}^{\bar{\tau} n}(w_{22}, v_{22}|w_{21})) \in A_{\epsilon}^{(\bar{\tau} n)}(P_{X_{11}U_{22}|U_{21}}), \tag{4.10}$$

and then sends $x_2^{\bar{\tau} n}(w_{10}, w_{21}, w_{22}, v_{22})$.

For decoding, at the end of the first phase, S_2 searches for a unique $\hat{w}_{10}$ such that

$$(x_{11}^{\tau n}(\hat{w}_{10}), \mathbf{y}) \in A_{\epsilon}^{(\tau n)}(P_{X_{11}Y}). \tag{4.11}$$

At the end of the second phase, D_1 searches for a unique $(\hat{w}_{10}, \hat{w}_{11}, \hat{w}_{12})$ for some $\hat{w}_{21}$ such that

$$(x_{11}^{\bar{\tau}n}(\hat{w}_{10}), u_{11}^{\bar{\tau}n}(\hat{w}_{11}), x_{12}^{\bar{\tau}n}(\hat{w}_{12}|\hat{w}_{10}, \hat{w}_{11}), u_{21}^{\bar{\tau}n}(\hat{w}_{21}), \mathbf{y_{12}}) \in A_{\epsilon}^{(\bar{\tau}n)}(P_{X_{11}U_{11}X_{12}U_{21}Y_{12}})$$
$$\text{and} \quad x_{11}^{\tau n}(\hat{w}_{10}), \mathbf{y_{11}}) \in A_{\epsilon}^{(\tau n)}(P_{X_{11}Y_{11}}). \tag{4.12}$$

D_2 searches for a unique $(\hat{w}_{21}, \hat{w}_{22})$ for some $(\hat{w}_{11}, \hat{v}_{22})$ such that

$$(u_{11}^{\bar{\tau}n}(\hat{w}_{11}), u_{21}^{\bar{\tau}n}(\hat{w}_{21}), u_{22}^{\bar{\tau}n}(\hat{w}_{22}, \hat{v}_{22}|\hat{w}_{21}), \mathbf{y_{22}}) \in A_{\epsilon}^{(\bar{\tau}n)}(P_{U_{11}U_{21}U_{22}Y_{22}}). \tag{4.13}$$

Note that similar to the full-duplex scheme in Section 3.2.2, we use conditional binning in step (4.10) and joint decoding at both destinations in steps (4.12) and (4.13) (see Remark 4).

Theorem 5. *The convex hull of the following rate region is achievable for the half-duplex causal cognitive interference channel using the HK-PDF-binning scheme:*

$$\bigcup_{P_4} \begin{cases} R_1 & \leq \min\{I_2 + I_5, I_6\} \\ R_2 & \leq I_{12} - I_1 \\ R_1 + R_2 & \leq \min\{I_2 + I_7, I_8\} + I_{13} - I_1 \\ R_1 + R_2 & \leq \min\{I_2 + I_3, I_4\} + I_{14} - I_1 \\ R_1 + R_2 & \leq \min\{I_2 + I_9, I_{10}\} + I_{11} - I_1 \\ 2R_1 + R_2 & \leq \min\{I_2 + I_3, I_4\} + \min\{I_2 + I_9, I_{10}\} + I_{13} - I_1 \\ R_1 + 2R_2 & \leq \min\{I_2 + I_7, I_8\} + I_{11} - I_1 + I_{14} - I_1, \end{cases} \tag{4.14}$$

where

$$\begin{aligned} P_4 = & p(x_{11})p(u_{11})p(x_{12}|u_{11}, x_{11})p(u_{21})p(u_{22}|u_{21}, x_{11}) \\ & p(x_{22}|x_{11}, u_{21}, u_{22})p_c(y_{11}, y_{21}, y, y_{12}, y_{22}|x_{11}, x_{12}, x_{22}), \end{aligned} \tag{4.15}$$

with p_c as given in (4.1) *and*

$$I_1 = \bar{\tau} I(U_{22}; X_{11}|U_{21})$$
$$I_2 = \tau I(X_{11}; Y)$$

$$
\begin{aligned}
I_3 &= \bar{\tau} I(U_{12}; Y_{12}|X_{11}, U_{11}, U_{21}) \\
I_4 &= \tau I(X_{11}; Y_{11}) + \bar{\tau} I(X_{11}, X_{12}; Y_{12}|U_{11}, U_{21}) \\
I_5 &= \bar{\tau} I(U_{11}, X_{12}; Y_{12}|X_{11}, U_{21}) \\
I_6 &= \tau I(X_{11}; Y_{11}) + \bar{\tau} I(X_{11}, U_{11}, X_{12}; Y_{12}|U_{21}) \\
I_7 &= \bar{\tau} I(X_{12}, U_{21}; Y_{12}|X_{11}, U_{11}) \\
I_8 &= \tau I(X_{11}; Y_{11}) + \bar{\tau} I(X_{11}, X_{12}, U_{21}; Y_{12}|U_{11}) \\
I_9 &= \bar{\tau} I(U_{11}, X_{12}, U_{21}; Y_{12}|X_{11}) \\
I_{10} &= \tau I(X_{11}; Y_{11}) + \bar{\tau} I(X_{11}, U_{11}, X_{12}, U_{21}; Y_{12}) \\
I_{11} &= \bar{\tau} I(U_{22}; Y_{22}|U_{21}, U_{11}) \\
I_{12} &= \bar{\tau} I(U_{21}, U_{22}; Y_{22}|U_{11}) \\
I_{13} &= \bar{\tau} I(U_{11}, U_{22}; Y_{22}|U_{21}) \\
I_{14} &= \bar{\tau} I(U_{11}, U_{21}, U_{22}; Y_{22}),
\end{aligned}
\tag{4.16}
$$

where $\bar{\tau} = 1 - \tau, 0 \leq \tau \leq 1$.

Proof. We use random codes and fix a joint probability distribution

$$p(x_{11})p(u_{11})p(x_{12}|x_{11}, u_{11})p(u_{21})p(u_{22}|u_{21}, x_{11})p(x_{22}|x_{11}, u_{21}, u_{22}).$$

Codebook generation

- Independently generate $2^{nR_{10}}$ sequences $x_{11}^n \sim \prod_{k=1}^n p(x_{11k})$. Index these codewords as $x_{11}^n(w_{10})$, $w_{10} \in [1, 2^{nR_{10}}]$.
- Independently generate $2^{nR_{11}}$ sequences $u_{11}^n \sim \prod_{k=1}^n p(u_{11k})$. Index these codewords as $u_{11}^n(w_{11})$, $w_{11} \in [1, 2^{nR_{11}}]$.
- For each $x_{11}^n(w_{10})$ and $u_{11}^n(w_{11})$, independently generate $2^{nR_{12}}$ sequences $x_{12}^n \sim \prod_{k=1}^n p(x_{12k}|x_{11k}, u_{11k})$. Index these codewords as $x_{12}^n(w_{12}|w_{10}, w_{11})$, $w_{12} \in [1, 2^{nR_{12}}]$.
- Independently generate $2^{n(R_{21})}$ sequences $u_{21}^n \sim \prod_{k=1}^n p(u_{21k})$. Index these codewords as $u_{21}^n(w_{21})$, $w_{21} \in [1, 2^{nR_{21}}]$.

- For each $u_{21}^n(w_{21})$, independently generate $2^{n(R_{22}+R'_{22})}$ sequences $u_{22}^n \sim \prod_{k=1}^n p(u_{22k}|u_{21k})$. Index these codewords as $u_{22}^n(w_{22}, v_{22}|w_{21})$, $w_{22} \in [1, 2^{nR_{22}}]$, $v_{22} \in [1, 2^{nR_{22}}]$.

- For each $x_{11}(w_{10})$, $u_{21}^n(w_{21})$ and $u_{22}^n(w_{22}, v_{22}|w_{21})$, generate one $x_{22}^n \sim \prod_{k=1}^n p(x_{22k}|u_{22k}, u_{21k}, x_{11k})$. Index these codewords as $x_{22}^n(w_{10}, w_{21}, w_{22}, v_{22})$.

Encoding

- In the first phase, S_1 sends the codewords $X_{11}^{\tau n}(w_{10})$. S_2 does not send anything.

- In the second phase, S_1 sends $x_{12}^{\bar{\tau} n}(w_{12}|w_{10}, w_{11})$.

  S_2 searches for some v_{22} such that

$$(x_{11}^{\bar{\tau} n}(w_{10}), u_{21}^{\bar{\tau} n}(w_{21}), u_{22}^{\bar{\tau} n}(w_{22}, v_{22}|w_{21})) \in A_\epsilon^{(\bar{\tau} n)}(P_{X_{11}U_{22}|U_{21}}).$$

  Such v_{22} exists with high probability if

$$R'_{22} \geq \bar{\tau} I(U_{22}; X_{11}|U_{21}). \tag{4.17}$$

  S_2 then transmits $x_{22}^{\bar{\tau} n}(w_{10}, w_{21}, w_{22}, v_{22})$.

Decoding

- At the end of the first phase, S_2 searches for a unique $\hat{w}_{10}$ such that

$$(x_{11}^{\tau n}(\hat{w}_{10}), \mathbf{y}) \in A_\epsilon^{(\tau n)}(P_{X_{11}Y}).$$

  We can show that the decoding error probability goes to 0 as $n \to \infty$ if

$$R_{10} \leq \tau I(X_{11}; Y). \tag{4.18}$$

- At the end of the second phase, D_1 searches for a unique $(\hat{w}_{10}, \hat{w}_{11}, \hat{w}_{12})$ for some $\hat{w}_{21}$ such that

$$(x_{11}^{\bar{\tau} n}(\hat{w}_{10}), u_{11}^{\bar{\tau} n}(\hat{w}_{11}), x_{12}^{\bar{\tau} n}(\hat{w}_{12}|\hat{w}_{10}, \hat{w}_{11}), u_{21}^{\bar{\tau} n}(\hat{w}_{21}), \mathbf{y_{12}}) \in A_\epsilon^{(\bar{\tau} n)}(P_{X_{11}U_{11}X_{12}U_{21}Y_{12}})$$
$$\text{and} \quad x_{11}^{\tau n}(\hat{w}_{10}), \mathbf{y_{11}}) \in A_\epsilon^{(\tau n)}(P_{X_{11}Y_{11}}).$$

The decoding error probability goes to 0 as $n \to \infty$ if

$$\begin{aligned}
R_{12} &\leq \bar{\tau} I(X_{12}; Y_{12} | X_{11}, U_{11}, U_{21}) \\
R_{10} + R_{12} &\leq \tau I(X_{11}; Y_{11}) + \bar{\tau} I(X_{11}, X_{12}; Y_{12} | U_{11}, U_{21}) \\
R_{11} + R_{12} &\leq \bar{\tau} I(U_{11}, X_{12}; Y_{12} | X_{11}, U_{21}) \\
R_{10} + R_{11} + R_{12} &\leq \tau I(X_{11}; Y_{11}) + \bar{\tau} I(X_{11}, U_{11}, X_{12}; Y_{12} | U_{21}) \\
R_{12} + R_{21} &\leq \bar{\tau} I(X_{12}, U_{21}; Y_{12} | X_{11}, U_{11}) \\
R_{10} + R_{12} + R_{21} &\leq \tau I(X_{11}; Y_{11}) + \bar{\tau} I(X_{11}, X_{12}, U_{21}; Y_{12} | U_{11}) \\
R_{11} + R_{12} + R_{21} &\leq \bar{\tau} I(U_{11}, X_{12}, U_{21}; Y_{12} | X_{11}) \\
R_{10} + R_{11} + R_{12} + R_{21} &\leq \tau I(X_{11}; Y_{11}) + \bar{\tau} I(X_{11}, U_{11}, X_{12}, U_{21}; Y_{12}). \qquad (4.19)
\end{aligned}$$

- D_2 uses joint decoding to decode (w_{11}, w_{21}, w_{22}). It searches for a unique $(\hat{w}_{21}, \hat{w}_{22})$ for some $(\hat{w}_{11}, \hat{v}_{22})$ such that

$$(u_{11}^{\bar{\tau}n}(\hat{w}_{11}), u_{21}^{\bar{\tau}n}(\hat{w}_{21}), u_{22}^{\bar{\tau}n}(\hat{w}_{22}, \hat{v}_{22} | \hat{w}_{21}), \mathbf{y_{22}}) \in A_{\epsilon}^{(\bar{\tau}n)}(P_{U_{11}U_{21}U_{22}Y_{22}}).$$

The decoding error probability goes to 0 as $n \to \infty$ if

$$\begin{aligned}
R_{22} + R'_{22} &\leq \bar{\tau} I(U_{22}; Y_{22} | U_{21}, U_{11}) \\
R_{21} + R_{22} + R'_{22} &\leq \bar{\tau} I(U_{21}, U_{22}; Y_{22} | U_{11}) \\
R_{11} + R_{22} + R'_{22} &\leq \bar{\tau} I(U_{11}, U_{22}; Y_{22} | U_{21}) \\
R_{11} + R_{21} + R_{22} + R'_{22} &\leq \bar{\tau} I(U_{11}, U_{21}, U_{22}; Y_{22}). \qquad (4.20)
\end{aligned}$$

Let $R_1 = R_{10} + R_{11} + R_{12}$ and $R_2 = R_{21} + R_{22}$ and apply Fourier-Motzkin Elimination on the above constraints, we get region (4.14). □

Remark 9. *Inclusion of half-duplex PDF-binning and Han-Kobayashi schemes.*

- *The half-duplex HK-PDF-binning scheme becomes half-duplex PDF-binning if* $U_{11} = U_{21} = \emptyset$.
- *The half-duplex HK-PDF-binning scheme becomes the Han-Kobayashi scheme if* $\tau = 0$, $X_{11} = \emptyset$ *and* $X_{22} = U_{22}$.

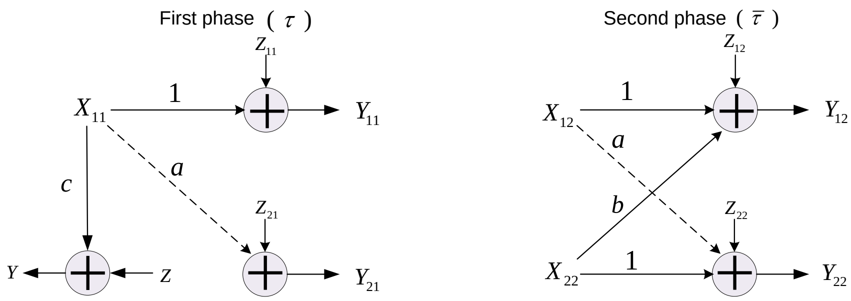

Fig. 4.4 The half-duplex Gaussian causal cognitive interference channel model.

- *The maximum rates for* S_1 *and* S_2 *are the same as in* (4.8) *and* (4.9)*.*

4.3 Half-duplex Gaussian CCIC rate regions

4.3.1 Half-duplex Gaussian CCIC model

The Gaussian model for the half-duplex causal cognitive interference channel is shown in Figure 4.4. The input-output signals can be represented as

$$\begin{aligned} \text{First phase}: \quad Y &= cX_{11} + Z, \\ Y_{11} &= X_{11} + Z_{11}, \\ Y_{21} &= aX_{11} + Z_{21}; \qquad (4.21) \\ \text{Second phase}: \quad Y_{12} &= X_{12} + bX_{22} + Z_{12}, \\ Y_{22} &= aX_{12} + X_{22} + Z_{22}, \qquad (4.22) \end{aligned}$$

where X_{11} is the transmit signal of S_1 in the first phase and X_{12} and X_{22} are the transmit signals of S_1 and S_2 in the second phase, respectively. Y, Y_{11} and Y_{21} are the received signals at S_2, D_1 and D_2 in the first phase. Y_{21} and Y_{22} are the received signals at D_1 and D_2 in the second phase. a, b, and c are the channel gains where the direct links are normalized to 1 as in the standard interference channel[10]. Z, Z_{11}, Z_{21}, Z_{12}, and Z_{22} are independent white Gaussian noises with unit variance.

In the following section, we provide analysises for both the half-duplex PDF-binning and half-duplex HK-PDF-binning schemes in the Gaussian case.

4.3.2 Signaling and rates for the half-duplex HK-PDF-binning

In a Gaussian channel, input signals for the HK-PDF-binning scheme as in Section 4.2.2 can be represented as

$$\begin{aligned} X_{11} &= \alpha_1 S_{10}(w_{10}), \\ X_{12} &= \alpha_2 S_{10}(w_{10}) + \beta_2 S_{11}(w_{11}) + \gamma_2 S_{12}(w_{12}), \\ X_{22} &= \theta S_{21}(w_{21}) + \mu \left(\rho S_{10}(w_{10}) + \sqrt{1-\rho^2} S_{22}\right), \\ U_{22} &= X_{22} + \lambda S_{10} = (\mu\rho + \lambda) S_{10} + \theta S_{21} + \mu\sqrt{1-\rho^2} S_{22}, \end{aligned} \tag{4.23}$$

where S_{10}, S_{11}, S_{12}, S_{21} and S_{22} are independent $\mathcal{N}(0,1)$ random variables that encode w_{10}, w_{11}, w_{12}, w_{21} and w_{22}, respectively; U_{22} is the Gelfand-Pinsker binning variable that encodes w_{22}. The parameter ρ is the correlation factor between the transmit signal X_{22} and the state X_{11}, similar to that in Section 3.3.2. λ is a parameter for binning. Parameters α_1, α_2, β_2, γ_2, θ and μ are the corresponding power allocations that satisfy the power constraints

$$\begin{aligned} \tau\alpha_1^2 + \bar{\tau}(\alpha_2^2 + \beta_2^2 + \gamma_2^2) &\leq P_1, \\ \bar{\tau}(\mu^2 + \theta^2) &\leq P_2, \end{aligned} \tag{4.24}$$

where τ and $\bar{\tau} = 1 - \tau$ are the time duration for the two phases.

Substituting X_{11}, X_{12}, X_{22} into Y, Y_{11}, Y_{21}, Y_{12}, Y_{22} in (4.21) and (4.22), we get

$$\begin{aligned} Y &= c\alpha_1 S_{10} + Z, \\ Y_{11} &= \alpha_1 S_{10} + Z_{11}, \\ Y_{21} &= a\alpha_1 S_{10} + Z_{21}, \\ Y_{12} &= (\alpha_2 + b\mu\rho) S_{10} + \beta_2 S_{11} + \gamma_2 S_{12} + b\theta S_{21} + b\mu\sqrt{1-\rho^2} S_{22} + Z_{12}, \\ Y_{22} &= (a\alpha_2 + \mu\rho) S_{10} + a\beta_2 S_{11} + a\gamma_2 S_{12} + \theta S_{21} + \mu\sqrt{1-\rho^2} S_{22} + Z_{22}. \end{aligned} \tag{4.25}$$

Corollary 4. *The achievable rate region for the half-duplex causal cognitive interference channel using Han-Kobayashi PDF-binning is the convex hull of all rate pairs* $(R_1,\ R_2)$

satisfying

$$
\begin{aligned}
R_1 &\leq \min\{I_2 + I_5, I_6\}, \\
R_2 &\leq I_{12} - I_1, \\
R_1 + R_2 &\leq \min\{I_2 + I_7, I_8\} + I_{13} - I_1, \\
R_1 + R_2 &\leq \min\{I_2 + I_3, I_4\} + I_{14} - I_1, \\
R_1 + R_2 &\leq \min\{I_2 + I_9, I_{10}\} + I_{11} - I_1, \\
2R_1 + R_2 &\leq \min\{I_2 + I_3, I_4\} + \min\{I_2 + I_9, I_{10}\} + I_{13} - I_1, \\
R_1 + 2R_2 &\leq \min\{I_2 + I_7, I_8\} + I_{11} - I_1 + I_{14} - I_1,
\end{aligned} \tag{4.26}
$$

where

$$
\begin{aligned}
I_2 &= \tau C\left(c^2\alpha_1^2\right), \\
I_3 &= \bar{\tau} C\left(\frac{\gamma_2^2}{b^2\mu^2(1-\rho^2)+1}\right), \\
I_4 &= \tau C\left(\alpha_1^2\right) + \bar{\tau} C\left(\frac{(\alpha_2 + b\mu\rho)^2 + \gamma_2^2}{b^2\mu^2(1-\rho^2)+1}\right), \\
I_5 &= \bar{\tau} C\left(\frac{\beta_2^2 + \gamma_2^2}{b^2\mu^2(1-\rho^2)+1}\right), \\
I_6 &= \tau C\left(\alpha_1^2\right) + \bar{\tau} C\left(\frac{(\alpha_2 + b\mu\rho)^2 + \beta_2^2 + \gamma_2^2}{b^2\mu^2(1-\rho^2)+1}\right), \\
I_7 &= \bar{\tau} C\left(\frac{\gamma_2^2 + b^2\theta^2}{b^2\mu^2(1-\rho^2)+1}\right), \\
I_8 &= \tau C\left(\alpha_1^2\right) + \bar{\tau} C\left(\frac{(\alpha_2 + b\mu\rho)^2 + \gamma_2^2 + b^2\theta^2}{b^2\mu^2(1-\rho^2)+1}\right), \\
I_9 &= \bar{\tau} C\left(\frac{\beta_2^2 + \gamma_2^2 + b^2\theta^2}{b^2\mu^2(1-\rho^2)+1}\right), \\
I_{10} &= \tau C\left(\alpha_1^2\right) + \bar{\tau} C\left(\frac{(\alpha_2 + b\mu\rho)^2 + \beta_2^2 + \gamma_2^2 + b^2\theta^2}{b^2\mu^2(1-\rho^2)+1}\right), \\
I_{11} - I_1 &= \bar{\tau} C\left(\frac{\mu^2(1-\rho^2)}{a^2\gamma_2^2+1}\right), \\
I_{12} - I_1 &= \bar{\tau} C\left(\frac{\mu^2(1-\rho^2)}{a^2\gamma_2^2+1}\right) + \bar{\tau} C\left(\frac{\theta^2}{(a\alpha_2 + \mu\rho)^2 + a^2\gamma_2^2 + \mu^2(1-\rho^2)+1}\right),
\end{aligned}
$$

$$I_{13} - I_1 = \bar{\tau} C\left(\frac{\mu^2(1-\rho^2)}{a^2\gamma_2^2+1}\right) + \bar{\tau} C\left(\frac{a^2\beta_2^2}{(a\alpha_2+\mu\rho)^2 + a^2\gamma_2^2 + \mu^2(1-\rho^2)+1}\right),$$
$$I_{14} - I_1 = \bar{\tau} C\left(\frac{\mu^2(1-\rho^2)}{a^2\gamma_2^2+1}\right) + \bar{\tau} C\left(\frac{a^2\beta_2^2+\theta^2}{(a\alpha_2+\mu\rho)^2 + a^2\gamma_2^2 + \mu^2(1-\rho^2)+1}\right),$$

and $C(x) = 0.5\log_2(1+x)$*;* $\tau \in [0,1]$ *and* $\tau + \bar{\tau} = 1$*;* $\rho \in [-1,1]$ *is the correlation factor between* S_2*'s transmit signal* X_{22} *and the state* X_{11}*; and the power allocations* α_1*,* α_2*,* β_2*,* γ_2*,* θ *and* μ *satisfy the power constraints* (4.24)*.*

Proof. Applying the signaling in (4.23) to Theorem 5, we obtain the rate region in the Corollary 4. □

Remark 10. *The optimal binning parameter can be found in a way similar to the full-duplex case as follows.*

Corollary 5. *The optimal parameter* λ *for the half-duplex Han-Kobayashi partial decode-forward binning scheme is*

$$\lambda^* = \frac{a\alpha_2\mu^2(1-\rho^2) - \mu\rho(a^2\gamma_2^2+1)}{a^2\gamma_2^2 + \mu^2(1-\rho^2)+1}. \tag{4.27}$$

Proof. Similar approach to the proof of Corollary 3. □

4.3.3 Inclusion of HD-PDF-Binning and Han-Kobayashi schemes

Remark 11. *Inclusion of half-duplex PDF-binning and Han-Kobayashi schemes.*

- *If we set* $\tau = 0$*,* $\alpha_1 = \alpha_2 = 0$*,* $\rho = 0$*, rate region* (4.26) *becomes the Han-Kobayashi region [12].*
- *If we set* $\beta_2 = \theta = 0$*, rate region* (4.26) *becomes the half-duplex PDF-binning region.*
- *The half-duplex PDF-binning region is the convex hull of all rate pairs* $(R_1,\ R_2)$

satisfying

$$R_1 \leq \tau C\left(c^2\alpha_1^2\right) + \bar{\tau} C\left(\frac{\gamma_2^2}{b^2\mu^2(1-\rho^2)+1}\right),$$
$$R_1 \leq \tau C\left(\alpha_1^2\right) + \bar{\tau} C\left(\frac{(\alpha_2 + b\mu\rho)^2 + \gamma_2^2}{b^2\mu^2(1-\rho^2)+1}\right),$$
$$R_2 \leq \bar{\tau} C\left(\frac{\mu^2(1-\rho^2)}{a^2\gamma_2^2+1}\right), \tag{4.28}$$

where the power allocations α_1, α_2, γ_2 *and* μ *satisfy the power constraints*

$$\tau\alpha_1^2 + \bar{\tau}(\alpha_2^2 + \gamma_2^2) \leq P_1,$$
$$\bar{\tau}\mu^2 \leq P_2. \tag{4.29}$$

- *The maximum rate for* S_1 *is achieved by setting* $\beta_2 = \theta = 0$, $\rho = \pm 1$ *and* $\mu = \rho\sqrt{P_2}$ *as*

$$R_1^{\max} = \max_{\tau\alpha_1^2+\bar{\tau}(\alpha_2^2+\gamma_2^2)\leq P_1} \min\left\{\tau C(c^2\alpha_1^2) + \bar{\tau} C(\gamma_2^2), \tau C(\alpha_1^2) + \bar{\tau} C\left(\left(\alpha_2 + b\sqrt{P_2}\right)^2 + \gamma_2^2\right)\right\}. \tag{4.30}$$

A solution for this optimization problem is available in [31]. Note that in the half-duplex mode, partial decode-forward achieves a strictly higher rate than pure decode-forward for the Gaussian channel.

- *The maximum rate for* S_2 *is achieved by setting* $\tau = 0$, $\rho = 0$, $\alpha_1 = \alpha_2 = \beta_2 = \gamma_2 = \theta = 0$, *and* $\mu = \sqrt{P_2}$ *as*

$$R_2^{\max} = C(P_2). \tag{4.31}$$

4.3.4 Performance comparison

Existing results

Very few results currently exist for the CCIC. We can find only two results for the half-duplex mode. These two coding schemes are different from our scheme and are not directly comparable with us. But one of the obvious limits from these two schemes is that neither of them achieves both the Han-Kobayashi region and the partial decode-forward rate. Next we provide comments for these two existing schemes.

Devroye, Mitran and Tarokh [2] propose four half-duplex protocols with rate region as the convex hull of the four regions. One protocol is the Han-Kobayashi scheme for the interference channel, and the other three are 2-phase protocols in which S_2 obtains S_1's message causally in the first phase as in a broadcast channel, then transmits cognitively in the second phase. All these 3 protocols have D_1 decode at the end of both phases instead of only at the end of the second phase, hence they are suboptimal. Protocol 2 has the idea of decode-forward by keeping the same input distribution at S_1 in both phases; but because it reduces the rate at S_1 in the second phase, S_1 does not achieve the rate of decode-forward relaying. Thus, even though the rate region includes the Han-Kobayashi region (in protocol 3), it does not include partial decode-forward relaying.

Chatterjee, Tong and Oyman [35] propose an achievable rate region for the half-duplex CCIC based on rate-splitting, block Markov encoding, Gelfand-Pinsker binning and backward decoding. The transmission is performed in B blocks, each is divided into two phases. In each phase, each user splits its message into two parts, one common and one private. The primary user (S_1) superimposes its messages in both phases of the current block on the messages in the first phase of the previous block. The cognitive user (S_2) only transmits in the second phase and bins both its message parts against the private message of S_1 in the first phase of the previous block. Backward decoding is then used to decode the messages after B blocks. We have several comments on this scheme:

- Block Markovity is not necessary in half-duplex mode. We can superimpose the second-phase signal on the first-phase signal of the same block, instead of superimposing both phase signals on the first-phase signal of the previous block and using backward decoding as in [35]. Such a half-duplex block Markovity incurs unnecessarily long decoding delay and also wastes power to transmit the first-phase information of the current block, which is decoded backwards.

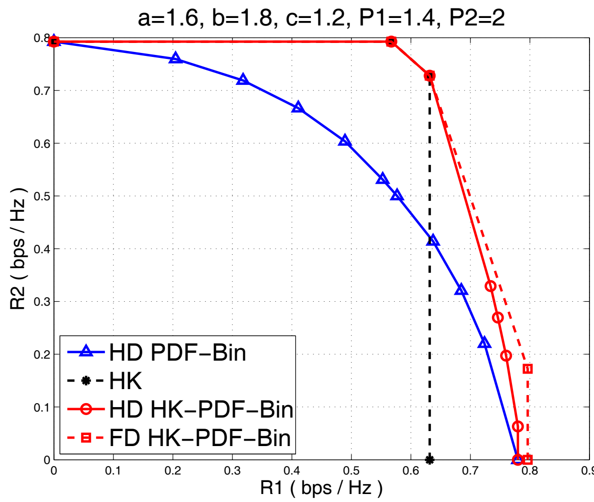

Fig. 4.5 Comparison of four coding schemes (HD = half-duplex, FD = full-duplex).

- Joint decoding of both the state and the binning auxiliary random variable at D_1 is not valid (similar to [26, 27]). The rate region is thus larger than possible, but can be corrected in this step.

- This scheme only covers half-duplex decode-forward relaying (when there is no binning) instead of partial decode-forward relaying and hence achieves a maximum rate for R_1 smaller than (4.30).

Numerical Examples

In this section, we provide numerical results to compare the two proposed schemes with the Han-Kobayashi and other known coding schemes [2, 35] for the half-duplex CCIC.

Figure 4.5 shows the comparison between half-duplex PDF-binning, HK-PDF-binning and the Han-Kobayashi scheme. It can be seen that although PDF-binning has a larger maximum rate for R_1 than the Han-Kobayashi scheme, it is not always better. But the half-duplex HK-PDF-binning rate region encompasses both the Han-Kobayashi and the PDF-binning regions.

In Figure 4.6, we compare the HK-PDF-binning schemes with existing half-duplex schemes for the CCIC in [2, 35]. We can see that HK-PDF-binning is strictly better than all existing schemes. Furthermore, the proposed scheme is more comprehensive than

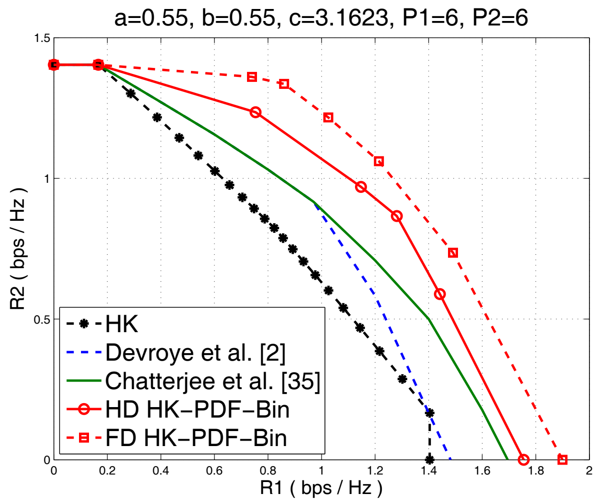

Fig. 4.6 Comparison of the HK-PDF-binning schemes with existing schemes.

the protocols in [2] and simpler than the scheme in [35].

These figures also show that the gap in achievable rates by the HK-PDF-binning scheme in the half- and full-duplex modes is quite small. Thus, the rate loss caused by the half-duplex constraint appears to be insignificant.

Chapter 5

Rate Region Analysis for the Gaussian HD-HK-PDF-Binning Scheme

This chapter focuses on the rate region for the Gaussian HD-HK-PDF-Binning scheme (see (4.26)). It studies the problem of finding the maximum R_2, under the constraint $R_1 = C(P_1)$, and finding the corresponding optimal parameter τ (time duration for the first phase transmission) and power allocations. We set the constraint $R_1 = C(P_1)$ since $C(P_1)$ is the interference-free rate for the primary user. This problem has several practical considerations. First, although S_1 can achieve a maximum rate larger than $C(P_1)$, we are interested, from the practical point of view, in the rate that the cognitive user can transmit while the primary user still transmits at an interference-free rate as if undisturbed by the cognitive user. Second, the optimal parameter τ and power allocations provide useful guidance for practical design and implementations. The optimal, τ, indicates the time ratio for the first phase transmission so as to balance the listening and the transmitting phases of the cognitive user optimally.

5.1 Corner point analysis and optimal power allocations

As an initial analysis, we focus on four special cases and derive the conditions under which the maximum rate for the cognitive sender is achieved. Although these cases do not solve the problem completely, they are important bases for the more general case and can provide intuitions and guidance. These four special cases are: $a = 0$ and b is strong; $b = 0$ and a is weak; $b = 0$ and a is strong; a and b are both strong. The results in these corner cases agree with the existing capacity results in the interference channel without cooperation for strong interference [11].

5.1.1 $a = 0$ and b is strong

Corollary 6. *When $a = 0$ and b is strong, R_2 can achieve the maximum rate of $C(P_2)$ while $R_1 = C(P_1)$, if*

$$b^2 \geq (1 + P_1). \tag{5.1}$$

Proof. Because $a = 0$, D_2 does not receive anything from S_1 and it does not need to decode anything from S_1. Thus we can immediately set $\beta_2 = 0$ in (4.26). When $\beta_2 = 0$, then $I_3 = I_5$, $I_4 = I_6$, $I_7 = I_9$, $I_8 = I_{10}$, $I_{11} = I_{13}$, $I_{12} = I_{14}$. Since b is strong, D_1 can decode everything from S_2, which means that it does not suffer from interference from S_2. Thus the primary user can always achieve its interference-free rate $C(P_1)$ and there is no need for S_2 to decode and forward anything. Thus we can set

$$\tau = 0, \alpha_1 = \alpha_2 = 0, \rho = 0, \theta = \sqrt{P_2}, \mu = 0.$$

We set $\mu = 0$ so that D_1 can decode all the messages from S_2. Therefore the only parameter left is $\gamma_2 = \sqrt{P_1}$. It is easy to verify that, with the above setting and $a = 0$, we can simplify the rate region as

$$\begin{aligned} R_1 &\leq C(P_1) \\ R_2 &\leq C(P_2) \\ R_1 + R_2 &\leq C(P_1 + b^2 P_2). \end{aligned} \tag{5.2}$$

If $b^2 \geq (1+P_1)$, the third constraint will be redundant, leading to the conclusion in Corollary 6. □

5.1.2 $b = 0$ and a is weak

Corollary 7. *R_2 can achieve the maximum rate of $C\left(\frac{P_2}{a^2P_1+1}\right)$ while $R_1 = C(P_1)$, if $b = 0$ and a is weak ($0 < a \leq 1$).*

Proof. Since $b = 0$, there is no interference from the cognitive to the primary user, and no need for the cognitive user to decode and forward anything or to do dirty paper coding, thus we can set $\tau = 0$, $\alpha_1 = \alpha_2 = 0$, $\theta = 0, \mu = \sqrt{P_2}$, $\rho = 0$. But we need to optimize for β_2 and γ_2.

After simplification, we can see that only the first three rate constraints matter, the other four constraints are redundant. The rate region can be simplified to

$$\begin{aligned} R_1 &\leq C(P_1) \\ R_2 &\leq C\left(\frac{P_2}{a^2\gamma_2^2+1}\right) \\ R_1 + R_2 &\leq C(\gamma_2^2) + C\left(\frac{P_2}{a^2P_1+1}\right) + C\left(\frac{a^2\beta_2^2}{a^2\gamma_2^2+P_2+1}\right). \end{aligned} \tag{5.3}$$

The optimal value occurs when the sum of the RHS of the first two constraints equals that of the third. Together with the power constraint, we have the following 2 equations:

$$\beta_2^2 + \gamma_2^2 = P_1 \tag{5.4}$$

$$C(P_1) + C\left(\frac{P_2}{a^2\gamma_2^2+1}\right) = C(\gamma_2^2) + C\left(\frac{P_2}{a^2P_1+1}\right) + C\left(\frac{a^2\beta_2^2}{a^2\gamma_2^2+P_2+1}\right). \tag{5.5}$$

From (5.5), after simplification, we get

$$(a^2P_1 + P_2 + 1)^2(a^2\gamma_2^2 + 1)(\gamma_2^2 + 1) - (a^2P_1 + 1)(P_1 + 1)(a^2\gamma_2^2 + P_2 + 1)^2 = 0.$$

We do a substitution $x = \gamma_2^2$, and let

$$\begin{aligned} f(x) &= (a^2P_1 + P_2 + 1)^2(a^2x + 1)(x + 1) - (a^2P_1 + 1)(P_1 + 1)(a^2x + P_2 + 1)^2 \\ &= c_2x^2 + c_1x + c_0 = 0, \end{aligned} \tag{5.6}$$

where

$$
\begin{aligned}
c_2 &= a^2\left(a^2P_1(2P_2+1-a^2)+(P_2+1)^2-a^2\right),\\
c_1 &= (a^2P_1+P_2+1)^2(a^2+1)-2(a^2P_1+1)(P_1+1)(P_2+1)a^2,\\
c_0 &= -(a^2P_1^2)((P_2+1)^2-a^2)-(a^2P_1)(P_2+1)P_2-P_1(P_2+1)((P_2+1)-a^2).
\end{aligned}
$$

Since $0 < a \leq 1$, we can see $1-a^2 \geq 0$ and $(P_2+1)^2-a^2 \geq 0$. Thus, the parameter c_2 is always greater than 0. For c_0, since $0 < a \leq 1$, all three terms in c_0 are negative. Hence, $c_0 < 0$.

For a quadratic function $f(x) = c_2x^2 + c_1x + c_0$, since $c_2 > 0$, $c_0 < 0$, $f(x)$ will have exactly one positive and one negative root. Note that $x = P_1$ is a root for $f(x) = 0$, which can be verified easily from (5.5) if we substitute γ_2^2 with P_1. Thus $x = P_1$ is the only positive root in $[0, P_1]$. Equivalently, $\gamma_2 = \sqrt{P_1}$ is the only root in $[0, \sqrt{P_1}]$.

Thus, we prove analytically that when $b = 0$ and $0 < a \leq 1$ we should put all the power in the private part and no power in the public part in the second phase ($\gamma_2 = \sqrt{P_1}$ and $\beta_2 = 0$). Therefore, the third rate constraint in (5.3) will be redundant and the rate region will become

$$
\begin{aligned}
R_1 &\leq C(P_1)\\
R_2 &\leq C\left(\frac{P_2}{a^2P_1+1}\right).
\end{aligned}
\tag{5.7}
$$

Thus, when $b = 0$, a is weak, the optimal power allocations are: $\gamma_2^2 = P_1$, $\mu^2 = P_2$, τ, α_1^2, α_2^2, β_2^2, θ^2, μ^2, and ρ are all 0. □

5.1.3 $b = 0$ and a is strong

Corollary 8. *When $b = 0$ and a is strong, R_2 can achieve the maximum rate of $C(P_2)$ while $R_1 = C(P_1)$, if*

$$
a^2 \geq (1+P_2). \tag{5.8}
$$

Proof. Again, since $b = 0$, there is no need for the cognitive user to decode and forward anything or to do dirty paper coding, thus we can set $\tau = 0$, $\alpha_1 = \alpha_2 = 0$, $\theta = 0, \mu = \sqrt{P_2}$,

$\rho = 0$. Since a is strong, D_2 can decode all the message of S_1. Thus $\beta_2 = \sqrt{P_1}, \gamma_2 = 0$.

The rate region can be simplified as

$$\begin{aligned} R_1 &\leq C(P_1) \\ R_2 &\leq C(P_2) \\ R_1 + R_2 &\leq C(a^2 P_1 + P_2). \end{aligned} \tag{5.9}$$

If $a^2 \geq (1 + P_2)$, the sum rate constraint will be redundant, leading to the conclusions in Corollary 8. □

5.1.4 Both a and b are strong

Corollary 9. *R_2 can achieve the maximum rate of $C(P_2)$ while $R_1 = C(P_1)$, if*

$$a^2 \geq (1 + P_2), \quad b^2 \geq (1 + P_1). \tag{5.10}$$

Proof. When both a and b are strong, then both destinations can decode all the messages, so the capacity is achieved for strong interference and there is no need to decode-forward. Thus we can set $\tau = 0, \alpha_1 = \alpha_2 = 0, \rho = 0$, and all the private parts equal 0, or $\gamma_2 = \mu = 0$. The only thing left are the public parts: $\beta_2 = \sqrt{P_1}, \theta = \sqrt{P_2}$.

After simplification, we get

$$\begin{aligned} R_1 &\leq C(P_1) \\ R_2 &\leq C(P_2) \\ R_1 + R_2 &\leq C(a^2 P_1 + P_2) \\ R_1 + R_2 &\leq C(P_1 + b^2 P_2). \end{aligned} \tag{5.11}$$

If $a^2 \geq (1 + P_2)$, $b^2 \geq (1 + P_1)$, the last two sum rate constraints will be redundant. □

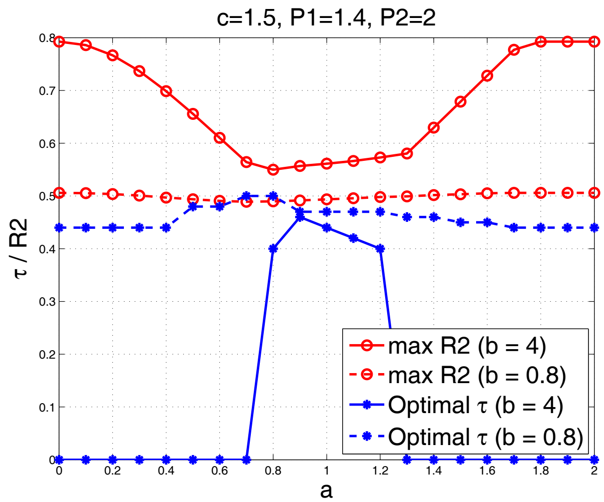

Fig. 5.1 Effect of a.

5.2 Effect of channel parameters

We now investigate the effect of the parameters a, b and c separately, and plot the relationships between the maximum R_2 and optimal τ given that $R_1 = C(P_1)$.

5.2.1 Effect of a

In Figure 5.1, we show the relationship among the maximum R_2, optimal τ and a while fixing other variables. Some comments are of interest. First, when a is very large, the maximum R_2 is the same as that of $a = 0$. This means when the interference is strong enough, it is equivalent to having no interferences, no matter whether b is strong or weak. This is valid since, when a is large, D_2 receives more useful information than noise from S_1, and D_2 can decode all the information from S_1. Second, when b is very strong, the maximum R_2 can achieve $C(P_2)$, which is the maximum possible value for R_2, either if a is 0 or a is very large. This also verifies the conclusion in Corollary 6. For example, the channel setting $a = 0, b = 4$ satisfies the conditions in Corollary 6, and the maximum rate for R_2 is 0.7925 bps/Hz, which is exactly the same result as $C(P_2)$ in Corollary 6. Third, when b is strong, the optimal τ to achieve $C(P_2)$ is 0, meaning we only have the second phase. However, when b is weak, the optimal τ is nonzero. This is possible since, if b is strong enough, what S_2 decodes from S_1 in the second phase is enough for forwarding.

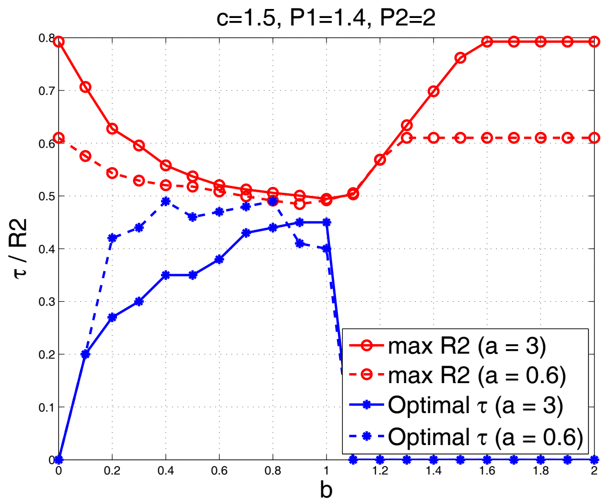

Fig. 5.2 Effect of b.

However, if b is small, the lack of forwarding messages in S_2 will create a bottleneck. Thus, S_2 needs an extra phase to obtain the forwarding messages.

5.2.2 Effect of b

In Figure 5.2, we show the relationships between the maximum R_2, optimal τ and b under the same constraint as before. Here, we also give two different cases when a is either weak or strong. Figure 5.2 provides supports for Corollary 7 and Corollary 8. For example, when $b = 0$ and a is weak (see max $R_2(a = 0.6)$), and the maximum rate for R_2 is 0.6101 bps/Hz, which is exactly $C\left(\frac{P_2}{a^2P_1+1}\right)$. Similarly for Corollary 8 (see max $R_2(a = 3)$). Furthermore, the maximum rate for R_2 is the same either $a = 0$ or a is large.

5.2.3 Effect of c

In Figure 5.3, we show the relationship among the maximum R_2, the optimal τ and c. Here we only give the case when a and b are both weak, since when they are both strong, the cooperation via link c is not necessary (see capacity for strong interference channels [11]). The value for c starts from 1.0 because the cooperative link between transmitters should be no worse than the direct links; otherwise, there is no incentive for the cooperations between transmitters. The simulation result for the parameter c is different from that of a or b. First, when c increases, the maximum R_2 always increases, and thus the maximum

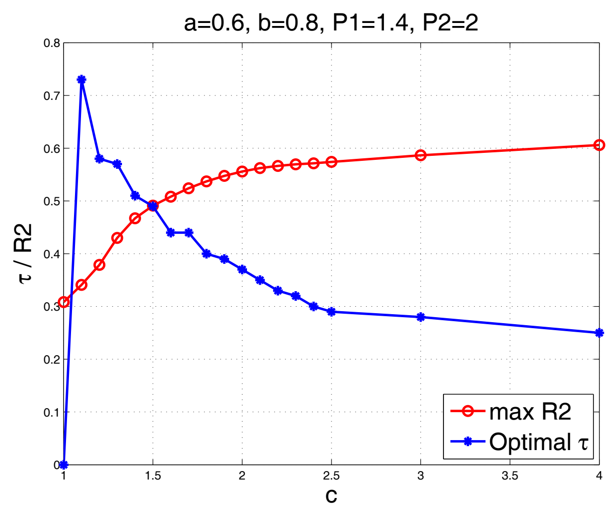

Fig. 5.3 Effect of c.

R_2 differs when $c = 0$ or c is strong. This makes sense because c does not introduce any interference to the transmission, thus for the value of c, the larger the better. Second, there is a jump for τ when c increases from 1.0 to 1.1. This jump is valid since when $c = 1.0$, the channel between the two transmitters is no better than the direct link. Since the cross link $b < 1$, there is no need for S_2 to forward the messages, thus $\tau = 0$. When c is above 1.0, the channel between the two senders gets better and S_2 can assist S_1 to transmit. Furthermore, as c increases, the optimal τ decreases since a stronger cooperation link requires less time for user two to decode the message from user one, leaving more time for the forwarding this message.

Chapter 6

Conclusion

6.1 Summary

In this thesis, we have proposed two new coding schemes for both the full- and half-duplex causal cognitive interference channels. These two schemes are based on partial decode-forward relaying, Gelfand-Pinsker binning and Han-Kobayashi coding. The half-duplex schemes are adapted from the full-duplex schemes by sending different message parts in different phases, removing the block Markov encoding and applying joint decoding across both phases.

When applied to Gaussian channels, different from the traditional binning in dirty paper coding, in which the transmit signal is independent of the state, we introduce a correlation between the transmit signal and the state, which enlarges the rate region by allowing both binning and forwarding. We also derive the optimal binning parameters for each coding scheme. Results show that the proposed binning schemes achieve a higher rate than all existing schemes for user one by allowing user two to also forward a part of the message of user 1. Furthermore, the Han-Kobayashi PDF-binning scheme contains both the Han-Kobayashi scheme and partial decode-forward relaying and outperforms all existing schemes by achieving a larger rate region for both users. Numerical results also suggest that the difference in achievable rates between the half- and full-duplex modes for the CCIC is small.

This thesis further analyzes the Gaussian rate region for the half-duplex HK-PDF-Binning scheme. It studies the problem of finding the maximum rate for the cognitive sender while keeping the primary sender in interference-free rate, and of finding the optimal time duration for the first phase and the optimal power allocations. As an initial analysis, this thesis focuses on four special channel settings. In each setting, the conditions for the channel gain parameters to achieve the maximum rate for the cognitive sender are derived. This thesis also studies the effect of channel gain parameters on the maximum rate for the cognitive sender. The simulation results for different channel parameters verify the analysis for these four special cases and show that the cognitive user can achieve significant rates while keeping the primary user's rate interference-free.

6.2 Future works

This thesis studies four special channel settings in Chapter 5. The future works will focus on more general cases. For example, we will remove the constraints for either a or b being zero, or both being strong and study the cases when a and b are both non zero and not both strong. In each case, we will also derive the optimal time duration for the first phase as well as the optimal power allocations.

Appendix A

Appendices

A.1 Fourier-Motzkin Elimination for the Full-duplex HK-PDF-binning scheme

Combining all the rate constraints (3.12)-(3.17), we get

$$\begin{aligned}
R'_{22} &\geq I(U_{22};T_{10}|U_{21}) = I_1\\
R_{10} &\leq I(U_{10};Y|T_{10}) = I_2\\
R_{12} &\leq I(X_1;Y_1|T_{10},U_{10},U_{11},U_{21}) = I_3\\
R_{10}+R_{12} &\leq I(U_{10},X_1;Y_1|T_{10},U_{11},U_{21}) + I(T_{10};Y_1) = I_4\\
R_{11}+R_{12} &\leq I(U_{11},X_1;Y_1|T_{10},U_{10},U_{21}) = I_5\\
R_{10}+R_{11}+R_{12} &\leq I(U_{10},U_{11},X_1;Y_1|T_{10},U_{21}) + I(T_{10};Y_1) = I_6\\
R_{12}+R_{21} &\leq I(X_1,U_{21};Y_1|T_{10},U_{10},U_{11}) = I_7\\
R_{10}+R_{12}+R_{21} &\leq I(U_{10},X_1,U_{21};Y_1|T_{10},U_{11}) + I(T_{10};Y_1) = I_8\\
R_{11}+R_{12}+R_{21} &\leq I(U_{11},X_1,U_{21};Y_1|T_{10},U_{10}) = I_9\\
R_{10}+R_{11}+R_{12}+R_{21} &\leq I(T_{10},U_{10},U_{11},X_1,U_{21};Y_1) = I_{10}\\
R_{22}+R'_{22} &\leq I(U_{22};Y_2|U_{21},U_{11}) = I_{11}\\
R_{21}+R_{22}+R'_{22} &\leq I(U_{21},U_{22};Y_2|U_{11}) = I_{12}\\
R_{11}+R_{22}+R'_{22} &\leq I(U_{11},U_{22};Y_2|U_{21}) = I_{13}\\
R_{11}+R_{21}+R_{22}+R'_{22} &\leq I(U_{11},U_{21},U_{22};Y_2) = I_{14}.
\end{aligned} \tag{A.1}$$

Eliminating R'_{22}, we get

$$\begin{aligned}
R_{10} &\le I_2\\
R_{12} &\le I_3\\
R_{10}+R_{12} &\le I_4\\
R_{11}+R_{12} &\le I_5\\
R_{10}+R_{11}+R_{12} &\le I_6\\
R_{12}+R_{21} &\le I_7\\
R_{10}+R_{12}+R_{21} &\le I_8\\
R_{11}+R_{12}+R_{21} &\le I_9\\
R_{10}+R_{11}+R_{12}+R_{21} &\le I_{10}\\
R_{22} &\le I_{11}-I_1\\
R_{21}+R_{22} &\le I_{12}-I_1\\
R_{11}+R_{22} &\le I_{13}-I_1\\
R_{11}+R_{21}+R_{22} &\le I_{14}-I_1. \qquad (A.2)
\end{aligned}$$

Let $R_{12} = R_1 - R_{10} - R_{11}$, we get

$$\begin{aligned}
R_{10} &\le I_2\\
R_1-R_{10}-R_{11} &\le I_3\\
R_1-R_{11} &\le I_4\\
R_1-R_{10} &\le I_5\\
R_1 &\le I_6\\
R_1-R_{10}-R_{11}+R_{21} &\le I_7\\
R_1-R_{11}+R_{21} &\le I_8\\
R_1-R_{10}+R_{21} &\le I_9\\
R_1+R_{21} &\le I_{10}\\
R_{22} &\le I_{11}-I_1\\
R_{21}+R_{22} &\le I_{12}-I_1\\
R_{11}+R_{22} &\le I_{13}-I_1\\
R_{11}+R_{21}+R_{22} &\le I_{14}-I_1. \qquad (A.3)
\end{aligned}$$

Rearranging, we get

$$\begin{aligned}
R_1-R_{10}-R_{11} &\le I_3\\
R_1-R_{11} &\le I_4\\
R_1-R_{10}-R_{11}+R_{21} &\le I_7\\
R_1-R_{11}+R_{21} &\le I_8\\
R_{11}+R_{22} &\le I_{13}-I_1\\
R_{11}+R_{21}+R_{22} &\le I_{14}-I_1\\
R_{10} &\le I_2\\
R_1-R_{10} &\le I_5\\
R_1 &\le I_6\\
R_1-R_{10}+R_{21} &\le I_9\\
R_1+R_{21} &\le I_{10}\\
R_{22} &\le I_{11}-I_1\\
R_{21}+R_{22} &\le I_{12}-I_1. \qquad (A.4)
\end{aligned}$$

Eliminating R_{11} and rearranging, we get

$$\begin{aligned}
R_1 - R_{10} &\leq I_5 \\
R_1 - R_{10} + R_{21} &\leq I_9 \\
R_1 - R_{10} + R_{22} &\leq I_3 + I_{13} - I_1 \\
R_1 - R_{10} + R_{21} + R_{22} &\leq I_7 + I_{13} - I_1 \\
R_1 - R_{10} + R_{21} + R_{22} &\leq I_3 + I_{14} - I_1 \\
R_1 - R_{10} + 2R_{21} + R_{22} &\leq I_7 + I_{14} - I_1 \\
R_{10} &\leq I_2 \\
R_1 + R_{22} &\leq I_4 + I_{13} - I_1 \\
R_1 + R_{21} + R_{22} &\leq I_8 + I_{13} - I_1 \\
R_1 + R_{21} + R_{22} &\leq I_4 + I_{14} - I_1 \\
R_1 + 2R_{21} + R_{22} &\leq I_8 + I_{14} - I_1 \\
R_1 &\leq I_6 \\
R_1 + R_{21} &\leq I_{10} \\
R_{22} &\leq I_{11} - I_1 \\
R_{21} + R_{22} &\leq I_{12} - I_1. \qquad \text{(A.5)}
\end{aligned}$$

Eliminating R_{10}, we get

$$\begin{aligned}
R_1 &\leq I_2 + I_5 \\
R_1 + R_{21} &\leq I_2 + I_9 \\
R_1 + R_{22} &\leq I_2 + I_3 + I_{13} - I_1 \\
R_1 + R_{21} + R_{22} &\leq I_2 + I_7 + I_{13} - I_1 \\
R_1 + R_{21} + R_{22} &\leq I_2 + I_3 + I_{14} - I_1 \\
R_1 + 2R_{21} + R_{22} &\leq I_2 + I_7 + I_{14} - I_1 \\
R_1 + R_{22} &\leq I_4 + I_{13} - I_1 \\
R_1 + R_{21} + R_{22} &\leq I_8 + I_{13} - I_1 \\
R_1 + R_{21} + R_{22} &\leq I_4 + I_{14} - I_1 \\
R_1 + 2R_{21} + R_{22} &\leq I_8 + I_{14} - I_1 \\
R_1 &\leq I_6 \\
R_1 + R_{21} &\leq I_{10} \\
R_{22} &\leq I_{11} - I_1 \\
R_{21} + R_{22} &\leq I_{12} - I_1. \qquad \text{(A.6)}
\end{aligned}$$

Let $R_{22} = R_2 - R_{21}$, we get

$$\begin{aligned}
R_1 &\le I_2 + I_5 \\
R_1 + R_{21} &\le I_2 + I_9 \\
R_1 + R_2 - R_{21} &\le I_2 + I_3 + I_{13} - I_1 \\
R_1 + R_2 &\le I_2 + I_7 + I_{13} - I_1 \\
R_1 + R_2 &\le I_2 + I_3 + I_{14} - I_1 \\
R_1 + R_2 + R_{21} &\le I_2 + I_7 + I_{14} - I_1 \\
R_1 + R_2 - R_{21} &\le I_4 + I_{13} - I_1 \\
R_1 + R_2 &\le I_8 + I_{13} - I_1 \\
R_1 + R_2 &\le I_4 + I_{14} - I_1 \\
R_1 + R_2 + R_{21} &\le I_8 + I_{14} - I_1 \\
R_1 &\le I_6 \\
R_1 + R_{21} &\le I_{10} \\
R_2 - R_{21} &\le I_{11} - I_1 \\
R_2 &\le I_{12} - I_1.
\end{aligned} \tag{A.7}$$

Rearranging, we get

$$\begin{aligned}
R_1 + R_2 - R_{21} &\le I_2 + I_3 + I_{13} - I_1 \\
R_1 + R_2 - R_{21} &\le I_4 + I_{13} - I_1 \\
R_2 - R_{21} &\le I_{11} - I_1 \\
R_1 + R_{21} &\le I_2 + I_9 \\
R_1 + R_{21} &\le I_{10} \\
R_1 + R_2 + R_{21} &\le I_2 + I_7 + I_{14} - I_1 \\
R_1 + R_2 + R_{21} &\le I_8 + I_{14} - I_1 \\
R_1 &\le I_2 + I_5 \\
R_1 &\le I_6 \\
R_2 &\le I_{12} - I_1 \\
R_1 + R_2 &\le I_2 + I_7 + I_{13} - I_1 \\
R_1 + R_2 &\le I_2 + I_3 + I_{14} - I_1 \\
R_1 + R_2 &\le I_8 + I_{13} - I_1 \\
R_1 + R_2 &\le I_4 + I_{14} - I_1.
\end{aligned} \tag{A.8}$$

Simplifying, we get

$$\begin{aligned}
R_1 + R_2 - R_{21} &\le \min\{I_2 + I_3 + I_{13} - I_1, I_4 + I_{13} - I_1\} \\
R_2 - R_{21} &\le I_{11} - I_1 \\
R_1 + R_{21} &\le \min\{I_2 + I_9, I_{10}\} \\
R_1 + R_2 + R_{21} &\le \min\{I_2 + I_7 + I_{14} - I_1, I_8 + I_{14} - I_1\} \\
R_1 &\le \min\{I_2 + I_5, I_6\} \\
R_2 &\le I_{12} - I_1 \\
R_1 + R_2 &\le I_2 + I_7 + I_{13} - I_1 \\
R_1 + R_2 &\le I_2 + I_3 + I_{14} - I_1 \\
R_1 + R_2 &\le I_8 + I_{13} - I_1 \\
R_1 + R_2 &\le I_4 + I_{14} - I_1.
\end{aligned} \tag{A.9}$$

Eliminating R_{21}, we get

$$
\begin{aligned}
2R_1 + R_2 &\le \min\{I_2 + I_3 + I_{13} - I_1, I_4 + I_{13} - I_1\} + \min\{I_2 + I_9, I_{10}\} \\
R_1 + R_2 &\le I_{11} - I_1 + \min\{I_2 + I_9, I_{10}\} \\
2R_1 + 2R_2 &\le \min\{I_2 + I_3 + I_{13} - I_1, I_4 + I_{13} - I_1\} + \min\{I_2 + I_7 + I_{14} - I_1, I_8 + I_{14} - I_1\} \\
R_1 + 2R_2 &\le I_{11} - I_1 + \min\{I_2 + I_7 + I_{14} - I_1, I_8 + I_{14} - I_1\} \\
R_1 &\le \min\{I_2 + I_5, I_6\} \\
R_2 &\le I_{12} - I_1 \\
R_1 + R_2 &\le I_2 + I_7 + I_{13} - I_1 \\
R_1 + R_2 &\le I_2 + I_3 + I_{14} - I_1 \\
R_1 + R_2 &\le I_8 + I_{13} - I_1 \\
R_1 + R_2 &\le I_4 + I_{14} - I_1.
\end{aligned}
\tag{A.10}
$$

Rearranging, we get

$$
\begin{aligned}
R_1 &\le \min\{I_2 + I_5, I_6\} \\
R_2 &\le I_{12} - I_1 \\
R_1 + R_2 &\le I_2 + I_7 + I_{13} - I_1 \\
R_1 + R_2 &\le I_2 + I_3 + I_{14} - I_1 \\
R_1 + R_2 &\le I_8 + I_{13} - I_1 \\
R_1 + R_2 &\le I_4 + I_{14} - I_1 \\
R_1 + R_2 &\le I_{11} - I_1 + \min\{I_2 + I_9, I_{10}\} \\
2R_1 + R_2 &\le \min\{I_2 + I_3 + I_{13} - I_1, I_4 + I_{13} - I_1\} + \min\{I_2 + I_9, I_{10}\} \\
R_1 + 2R_2 &\le I_{11} - I_1 + \min\{I_2 + I_7 + I_{14} - I_1, I_8 + I_{14} - I_1\} \\
2R_1 + 2R_2 &\le \min\{I_2 + I_3 + I_{13} - I_1, I_4 + I_{13} - I_1\} + \min\{I_2 + I_7 + I_{14} - I_1, I_8 + I_{14} - I_1\}.
\end{aligned}
\tag{A.11}
$$

Simplifying, we get

$$\begin{aligned}
R_1 &\le \min\{I_2 + I_5, I_6\} \\
R_2 &\le I_{12} - I_1 \\
R_1 + R_2 &\le \min\{I_2 + I_7, I_8\} + I_{13} - I_1 \\
R_1 + R_2 &\le \min\{I_2 + I_3, I_4\} + I_{14} - I_1 \\
R_1 + R_2 &\le \min\{I_2 + I_9, I_{10}\} + I_{11} - I_1 \\
2R_1 + R_2 &\le \min\{I_2 + I_3 + I_{13} - I_1, I_4 + I_{13} - I_1\} + \min\{I_2 + I_9, I_{10}\} \\
R_1 + 2R_2 &\le I_{11} - I_1 + \min\{I_2 + I_7 + I_{14} - I_1, I_8 + I_{14} - I_1\} \\
2R_1 + 2R_2 &\le \min\{I_2 + I_3 + I_{13} - I_1, I_4 + I_{13} - I_1\} + \min\{I_2 + I_7 + I_{14} - I_1, I_8 + I_{14} - I_1\}.
\end{aligned} \tag{A.12}$$

Comparing the last constraint with the sum of the third to fourth ones, we can find the last one is redundant.

$$\begin{aligned}
R_1 &\le \min\{I_2 + I_5, I_6\} \\
R_2 &\le I_{12} - I_1 \\
R_1 + R_2 &\le \min\{I_2 + I_7, I_8\} + I_{13} - I_1 \\
R_1 + R_2 &\le \min\{I_2 + I_3, I_4\} + I_{14} - I_1 \\
R_1 + R_2 &\le \min\{I_2 + I_9, I_{10}\} + I_{11} - I_1 \\
2R_1 + R_2 &\le \min\{I_2 + I_3, I_4\} + \min\{I_2 + I_9, I_{10}\} + I_{13} - I_1 \\
R_1 + 2R_2 &\le \min\{I_2 + I_7, I_8\} + I_{11} - I_1 + I_{14} - I_1.
\end{aligned} \tag{A.13}$$

A.2 Analysis for Prabhakaran and Viswanath scheme one

In [24], it shows the rate region for the first scheme (see [24] Theorem 4(a)). The following shows the complete rate regions for this scheme.

Given a distribution $p_W, p_{V_1,U_1,X_1|W}, p_{V_2,U_2,X_2|W}$. The rate pair (R_1, R_2) is achievable if there are nonnegative $r_{V_1}, r_{V_2}, r_{U_1}, r_{U_2}, r_{X_1}, r_{X_2}$, such that $R_1 = r_{V_1} + r_{U_1} + r_{X_1}$, $R_2 =$

$r_{V_2} + r_{U_2} + r_{X_2}$.

$$
\begin{aligned}
r_{V_1} &\le I(V_1; Y_2|W) \\
r_{X_1} &\le I(X_1; Y_3|V_1, V_2, W, U_1, U_2) \\
r_{U_1} + r_{X_1} &\le I(U_1, X_1; Y_3|V_1, V_2, W, U_2) \\
r_{U_2} + r_{X_1} &\le I(U_2, X_1; Y_3|V_1, V_2, W, U_1) \\
r_{U_1} + r_{U_2} + r_{X_1} &\le I(U_1, U_2, X_1; Y_3|V_1, V_2, W) \\
r_{V_1} + r_{V_2} + r_{U_1} + r_{U_2} + r_{X_1} &\le I(W, V_1, V_2, U_1, U_2, X_1; Y_3) \\
r_{V_2} &\le I(V_2; Y_1|W) \\
r_{X_2} &\le I(X_2; Y_4|V_1, V_2, W, U_1, U_2) \\
r_{U_2} + r_{X_2} &\le I(U_2, X_2; Y_4|V_1, V_2, W, U_1) \\
r_{U_1} + r_{X_2} &\le I(U_1, X_2; Y_4|V_1, V_2, W, U_2) \\
r_{U_1} + r_{U_2} + r_{X_2} &\le I(U_1, U_2, X_2; Y_4|V_1, V_2, W) \\
r_{V_1} + r_{V_2} + r_{U_1} + r_{U_2} + r_{X_2} &\le I(W, V_1, V_2, U_1, U_2, X_2; Y_4).
\end{aligned} \tag{A.14}
$$

A.2.1 Compare Prabhakaran and Viswanath scheme one with the HK region

In (A.14), let $r_{V_1} = r_{V_2} = 0$, and set the cooperative public messages at both sources to 0 ($V_1 = V_2 = W = \emptyset$). We get

$$
\begin{aligned}
r_{X_1} &\le I(X_1; Y_3|U_1, U_2) \\
r_{U_1} + r_{X_1} &\le I(U_1, X_1; Y_3|U_2) \\
r_{U_2} + r_{X_1} &\le I(U_2, X_1; Y_3|U_1) \\
r_{U_1} + r_{U_2} + r_{X_1} &\le I(U_1, U_2, X_1; Y_3) \\
r_{X_2} &\le I(X_2; Y_4|U_1, U_2) \\
r_{U_2} + r_{X_2} &\le I(U_2, X_2; Y_4|U_1) \\
r_{U_1} + r_{X_2} &\le I(U_1, X_2; Y_4|U_2) \\
r_{U_1} + r_{U_2} + r_{X_2} &\le I(U_1, U_2, X_2; Y_4).
\end{aligned} \tag{A.15}
$$

This is identical to the HK region.

A.2.2 Compare Prabhakaran and Viswanath scheme one with the PDF rate

The rate region for the PDF scheme (see [6]) is

$$\begin{aligned} R_1 &\le I(U;Y_2|X_2) + I(X_1;Y_3|X_2,U) \\ R_1 &\le I(X_1,X_2;Y_3). \end{aligned} \tag{A.16}$$

We only analyze for the source one (i.e. if source one can achieve the PDF rate), it is similar for source two. Let $r_{U_1} = 0$, set the HK public message to 0 ($U_1 = \emptyset$) and source two's messages parts to 0 ($V_2 = U_2 = \emptyset$). Then the cooperative message m_W is only the previous message of m_{V_1}.

$$\begin{aligned} r_{V_1} &\le I(V_1;Y_2|W) \\ r_{X_1} &\le I(X_1;Y_3|V_1,W) \\ r_{V_1} + r_{X_1} &\le I(W,V_1,X_1;Y_3) \\ r_{V_1} &\le I(W,V_1;Y_4). \end{aligned} \tag{A.17}$$

The first three constraints are identical to the PDF rate, but the last one is an extra constraint on r_{V_1}, the cooperative common message part. This is because the common message is decoded at both receivers, hence it leads to an extra constraint compared to the PDF rate.

In conclusion, Viswanth's first scheme includes HK rate region but not always the PDF rate. When node four doesn't decode anything, then node three can achieve the PDF rate, but if node four still decodes, node three will achieve a rate less than PDF rate.

A.3 Analysis for Prabhakaran and Viswanath scheme two

In [24], it shows the rate region for the second scheme (see [24] Theorem 4(b)). Applying Prabhakaran and Viswanath scheme two in our channel (set the cooperative public and private message to 0, $V_2 = \emptyset, S_2 = \emptyset$), we get:

$$r_{S_1} \leq I(X_1; Y_2|W, V_1, U_1, S_1, Z_1) = I_1 \tag{A.18}$$

$$r_{Z_1} + r_{S_1} \leq I(Z_1, X_1; Y_2|W, V_1, U_1, S_1) = I_2 \tag{A.19}$$

$$r_{U_1} + r_{Z_1} + r_{S_1} \leq I(U_1, Z_1, X_1; Y_2|W, V_1, S_1) = I_3 \tag{A.20}$$

$$r_{V_1} + r_{U_1} + r_{Z_1} + r_{S_1} \leq I(V_1, U_1, Z_1, X_1; Y_2|W, S_1) = I_4 \tag{A.21}$$

$$r_{Z_1} \leq I(Z_1; Y_3|W, V_1, U_1, U_2, S_1) = I_5 \tag{A.22}$$

$$r_{U_1} + r_{Z_1} \leq I(U_1, Z_1; Y_3|W, V_1, U_2, S_1) = I_6 \tag{A.23}$$

$$r_{S_1} + r_{Z_1} \leq I(S_1, Z_1; Y_3|W, V_1, U_1, U_2) = I_7 \tag{A.24}$$

$$r_{S_1} + r_{U_1} + r_{Z_1} \leq I(S_1, U_1, Z_1; Y_3|W, V_1, U_2) = I_8 \tag{A.25}$$

$$r_{U_2} + r_{Z_1} \leq I(U_2, Z_1; Y_3|W, V_1, U_1, S_1) = I_9 \tag{A.26}$$

$$r_{U_2} + r_{U_1} + r_{Z_1} \leq I(U_2, U_1, Z_1; Y_3|W, V_1, S_1) = I_{10} \tag{A.27}$$

$$r_{U_2} + r_{S_1} + r_{Z_1} \leq I(U_2, S_1, Z_1; Y_3|W, V_1, U_1) = I_{11} \tag{A.28}$$

$$r_{U_2} + r_{S_1} + r_{U_1} + r_{Z_1} \leq I(U_2, S_1, U_1, Z_1; Y_3|W, V_1) = I_{12} \tag{A.29}$$

$$r_{V_1} + r_{U_2} + r_{S_1} + r_{U_1} + r_{Z_1} \leq I(W, V_1, U_2, S_1, U_1, Z_1; Y_3) = I_{13} \tag{A.30}$$

$$r_{Z_2} \leq I(Z_2; Y_4|W, V_1, U_1, U_2) = I_{14} \tag{A.31}$$

$$r_{U_2} + r_{Z_2} \leq I(U_2, Z_2; Y_4|W, V_1, U_1) = I_{15} \tag{A.32}$$

$$r_{U_1} + r_{Z_2} \leq I(U_1, Z_2; Y_4|W, V_1, U_2) = I_{16} \tag{A.33}$$

$$r_{U_1} + r_{U_2} + r_{Z_2} \leq I(U_1, U_2, Z_2; Y_4|W, V_1) = I_{17} \tag{A.34}$$

$$r_{V_1} + r_{U_1} + r_{U_2} + r_{Z_2} \leq I(W, V_1, U_1, U_2, Z_2; Y_4) = I_{18}. \tag{A.35}$$

Note: (A.18)-(A.21) is the rate for the decoding at source two. (A.22)-(A.30) is the rate for the decoding at destination three. (A.31)-(A.35) is the rate for the decoding at destination four.

The following comparisons with the HK region and the PDF rate are both based on the assumptions that Prabhakaran and Viswanath scheme two is applied in the CCIC (set the cooperative public and private message to 0, $V_2 = \emptyset, S_2 = \emptyset$).

A.3.1 Compare Prabhakaran and Viswanath scheme two with the HK region

In order to compare with HK scheme, we further set the cooperative public and private message at source one to 0 ($W = V_1 = S_1 = \emptyset$) in (A.18)-(A.35). (We already set $V_2 = \emptyset, S_2 = \emptyset$ when applying this scheme in the CCIC). And the rate region becomes

$$\begin{aligned}
r_{Z_1} &\leq I(Z_1; Y_2|U_1) \\
r_{U_1} + r_{Z_1} &\leq I(U_1, Z_1; Y_2) \\
r_{Z_1} &\leq I(Z_1; Y_3|U_1, U_2) \\
r_{U_1} + r_{Z_1} &\leq I(U_1, Z_1; Y_3|U_2) \\
r_{U_2} + r_{Z_1} &\leq I(U_2, Z_1; Y_3|U_1) \\
r_{U_2} + r_{U_1} + r_{Z_1} &\leq I(U_2, U_1, Z_1; Y_3) \\
r_{Z_2} &\leq I(Z_2; Y_4|U_1, U_2) \\
r_{U_2} + r_{Z_2} &\leq I(U_2, Z_2; Y_4|U_1) \\
r_{U_1} + r_{Z_2} &\leq I(U_1, Z_2; Y_4|U_2) \\
r_{U_1} + r_{U_2} + r_{Z_2} &\leq I(U_1, U_2, Z_2; Y_4).
\end{aligned} \tag{A.36}$$

Compared with the HK scheme, the first two constraints are the extra constraints because the source two needs to decode the HK public and HK private message from the source one, thus making its rate smaller than the HK rate.

A.3.2 Compare Prabhakaran and Viswanath scheme two with the PDF rate

When considering the maximum rate for R_1, we need to compare it with the PDF rate. S_2 only forwards the message it decodes from S_1, and doesn't have any new message of its own. In other words, we need to set $U_2 = Z_2 = \emptyset$ (we already set $V_2 = \emptyset, S_2 = \emptyset$ when applying this scheme in the CCIC). After this setting, the constraints in (A.18)-(A.35) containing

$I_9 - I_{12}$, $I_{14} - I_{15}$ and I_{17} are redundant. The rate region becomes

$$\begin{aligned}
r_{S_1} &\leq I(X_1; Y_2|W, V_1, U_1, S_1, Z_1)\\
r_{Z_1} + r_{S_1} &\leq I(Z_1, X_1; Y_2|W, V_1, U_1, S_1)\\
r_{U_1} + r_{Z_1} + r_{S_1} &\leq I(U_1, Z_1, X_1; Y_2|W, V_1, S_1)\\
r_{V_1} + r_{U_1} + r_{Z_1} + r_{S_1} &\leq I(V_1, U_1, Z_1, X_1; Y_2|W, S_1)\\
r_{Z_1} &\leq I(Z_1; Y_3|W, V_1, U_1, S_1)\\
r_{U_1} + r_{Z_1} &\leq I(U_1, Z_1; Y_3|W, V_1, S_1)\\
r_{S_1} + r_{Z_1} &\leq I(S_1, Z_1; Y_3|W, V_1, U_1)\\
r_{S_1} + r_{U_1} + r_{Z_1} &\leq I(S_1, U_1, Z_1; Y_3|W, V_1)\\
r_{V_1} + r_{S_1} + r_{U_1} + r_{Z_1} &\leq I(W, V_1, S_1, U_1, Z_1; Y_3)\\
r_{U_1} &\leq I(U_1; Y_4|W, V_1)\\
r_{V_1} + r_{U_1} &\leq I(W, V_1, U_1; Y_4).
\end{aligned} \tag{A.37}$$

Moreover, since destination four also decodes U_1, we set the public message part of source one to 0 (set $r_{U_1} = 0$, $U_1 = \emptyset$), which can reduce some of the constraints at destination four. After this setting, we get

$$\begin{aligned}
r_{S_1} &\leq I(X_1; Y_2|W, V_1, S_1, Z_1)\\
r_{Z_1} + r_{S_1} &\leq I(Z_1, X_1; Y_2|W, V_1, S_1)\\
r_{V_1} + r_{Z_1} + r_{S_1} &\leq I(V_1, Z_1, X_1; Y_2|W, S_1)\\
r_{Z_1} &\leq I(Z_1; Y_3|W, V_1, S_1)\\
r_{S_1} + r_{Z_1} &\leq I(S_1, Z_1; Y_3|W, V_1)\\
r_{V_1} + r_{S_1} + r_{Z_1} &\leq I(W, V_1, S_1, Z_1; Y_3)\\
r_{V_1} &\leq I(W, V_1; Y_4).
\end{aligned} \tag{A.38}$$

Since in the PDF rate, there is only one forwarding message part, we can set either the cooperative public (V_1) or the cooperative private message (S_1) in the Prabhakaran and Viswanath scheme two to 0. These two cases include all the possibilities when comparing with the PDF rate.

First, setting the cooperative public message to 0 ($V_1 = \emptyset$ and $W = \emptyset$), we get

$$\begin{aligned} r_{S_1} &\leq I(X_1; Y_2|S_1, Z_1) \\ r_{Z_1} + r_{S_1} &\leq I(Z_1, X_1; Y_2|S_1) \\ r_{Z_1} &\leq I(Z_1; Y_3|S_1) \\ r_{S_1} + r_{Z_1} &\leq I(S_1, Z_1; Y_3). \end{aligned} \tag{A.39}$$

Prabhakaran and Viswanath scheme two can now be simplied as follows: the non-cooperative private message part Z_1 is superimposed on the previous cooperative message part S_1, then the current cooperative private message X_1 is superimposed on both S_1 and Z_1. However, in the partial decode-forward scheme, the current forwarding message part is superimposed on the previous forwarding message part. And the non-cooperative message is superimposed on both the previous and the current cooperative private messages. Because of this difference in the order of superposition, source two needs to decode both the current cooperative and the non-cooperative message parts (see the first two constraints in (A.39)). But, in the partial decode-forward scheme, source two decodes only the current cooperative message part. Since source two decodes two message parts, but can only forward one part, this can make the rate less than the PDF rate.

Second, setting the cooperative private message to 0 ($S_1 = \emptyset$), we get

$$\begin{aligned} r_{Z_1} &\leq I(Z_1, X_1; Y_2|W, V_1) \\ r_{V_1} + r_{Z_1} &\leq I(V_1, Z_1, X_1; Y_2|W) \\ r_{Z_1} &\leq I(Z_1; Y_3|W, V_1) \\ r_{V_1} + r_{Z_1} &\leq I(W, V_1, Z_1; Y_3) \\ r_{V_1} &\leq I(W, V_1; Y_4). \end{aligned} \tag{A.40}$$

The last constraint is an extra constraint compared with the PDF rate (the first two constraints are also different). The last constraint exists because the destination four needs to decode the cooperative public message V_1, which can reduce the rate to below the PDF rate.

In conclusion, Prabhakaran and Viswanath scheme two does not always achieve the PDF rate.

A.3.3 Fourier-Motzkin Elimination for Prabhakaran and Viswanath scheme two

Let $R_2 = r_{U_2} + r_{Z_2}$, and $R_1 = r_{V_1} + r_{U_1} + r_{Z_1} + r_{S_1}$, we get

$$\begin{aligned}
r_{S_1} &\le I_1 \\
r_{Z_1} + r_{S_1} &\le I_2 \\
r_{U_1} + r_{Z_1} + r_{S_1} &\le I_3 \\
r_{V_1} + r_{U_1} + r_{Z_1} + r_{S_1} &\le I_4 \\
r_{Z_1} &\le I_5 \\
r_{U_1} + r_{Z_1} &\le I_6 \\
r_{S_1} + r_{Z_1} &\le I_7 \\
r_{S_1} + r_{U_1} + r_{Z_1} &\le I_8 \\
r_{U_2} + r_{Z_1} &\le I_9 \\
r_{U_2} + r_{U_1} + r_{Z_1} &\le I_{10} \\
r_{U_2} + r_{S_1} + r_{Z_1} &\le I_{11} \\
r_{U_2} + r_{S_1} + r_{U_1} + r_{Z_1} &\le I_{12} \\
r_{V_1} + r_{U_2} + r_{S_1} + r_{U_1} + r_{Z_1} &\le I_{13} \\
r_{Z_2} &\le I_{14} \\
r_{U_2} + r_{Z_2} &\le I_{15} \\
r_{U_1} + r_{Z_2} &\le I_{16} \\
r_{U_1} + r_{U_2} + r_{Z_2} &\le I_{17} \\
r_{V_1} + r_{U_1} + r_{U_2} + r_{Z_2} &\le I_{18}.
\end{aligned} \tag{A.41}$$

Let $r_{S_1} = R_1 - r_{V_1} - r_{U_1} - r_{Z_1}$, we get

$$\begin{aligned}
R_1 - r_{V_1} - r_{U_1} - r_{Z_1} &\le I_1 \\
R_1 - r_{V_1} - r_{U_1} &\le I_2 \\
R_1 - r_{V_1} &\le I_3 \\
R_1 &\le I_4 \\
r_{Z_1} &\le I_5 \\
r_{U_1} + r_{Z_1} &\le I_6 \\
R_1 - r_{V_1} - r_{U_1} &\le I_7 \\
R_1 - r_{V_1} &\le I_8 \\
r_{U_2} + r_{Z_1} &\le I_9 \\
r_{U_2} + r_{U_1} + r_{Z_1} &\le I_{10} \\
r_{U_2} + R_1 - r_{V_1} - r_{U_1} &\le I_{11} \\
r_{U_2} + R_1 - r_{V_1} &\le I_{12} \\
R_1 + r_{U_2} &\le I_{13} \\
r_{Z_2} &\le I_{14} \\
r_{U_2} + r_{Z_2} &\le I_{15} \\
r_{U_1} + r_{Z_2} &\le I_{16} \\
r_{U_1} + r_{U_2} + r_{Z_2} &\le I_{17} \\
r_{V_1} + r_{U_1} + r_{U_2} + r_{Z_2} &\le I_{18}.
\end{aligned} \tag{A.42}$$

Rearranging, we get

$$
\begin{aligned}
R_1 - r_{V_1} - r_{U_1} - r_{Z_1} &\le I_1 \\
r_{Z_1} &\le I_5 \\
r_{U_1} + r_{Z_1} &\le I_6 \\
r_{U_2} + r_{Z_1} &\le I_9 \\
r_{U_2} + r_{U_1} + r_{Z_1} &\le I_{10} \\
R_1 - r_{V_1} - r_{U_1} &\le I_2 \\
R_1 - r_{V_1} - r_{U_1} &\le I_7 \\
R_1 - r_{V_1} &\le I_3 \\
R_1 - r_{V_1} &\le I_8 \\
R_1 &\le I_4 \\
r_{U_2} + R_1 - r_{V_1} - r_{U_1} &\le I_{11} \\
r_{U_2} + R_1 - r_{V_1} &\le I_{12} \\
R_1 + r_{U_2} &\le I_{13} \\
r_{Z_2} &\le I_{14} \\
r_{U_2} + r_{Z_2} &\le I_{15} \\
r_{U_1} + r_{Z_2} &\le I_{16} \\
r_{U_1} + r_{U_2} + r_{Z_2} &\le I_{17} \\
r_{V_1} + r_{U_1} + r_{U_2} + r_{Z_2} &\le I_{18}.
\end{aligned} \tag{A.43}
$$

Simplifying, we get

$$
\begin{aligned}
R_1 - r_{V_1} - r_{U_1} - r_{Z_1} &\le I_1 \\
r_{Z_1} &\le I_5 \\
r_{U_1} + r_{Z_1} &\le I_6 \\
r_{U_2} + r_{Z_1} &\le I_9 \\
r_{U_2} + r_{U_1} + r_{Z_1} &\le I_{10} \\
R_1 - r_{V_1} - r_{U_1} &\le \min\{I_2, I_7\} \\
R_1 - r_{V_1} &\le \min\{I_3, I_8\} \\
R_1 &\le I_4 \\
r_{U_2} + R_1 - r_{V_1} - r_{U_1} &\le I_{11} \\
r_{U_2} + R_1 - r_{V_1} &\le I_{12} \\
R_1 + r_{U_2} &\le I_{13} \\
r_{Z_2} &\le I_{14} \\
r_{U_2} + r_{Z_2} &\le I_{15} \\
r_{U_1} + r_{Z_2} &\le I_{16} \\
r_{U_1} + r_{U_2} + r_{Z_2} &\le I_{17} \\
r_{V_1} + r_{U_1} + r_{U_2} + r_{Z_2} &\le I_{18}.
\end{aligned} \tag{A.44}
$$

Eliminating r_{Z_1}, we get

$$\begin{aligned}
R_1 - r_{V_1} - r_{U_1} &\le I_1 + I_5 \\
R_1 - r_{V_1} &\le I_1 + I_6 \\
r_{U_2} + R_1 - r_{V_1} - r_{U_1} &\le I_1 + I_9 \\
r_{U_2} + R_1 - r_{V_1} &\le I_1 + I_{10} \\
R_1 - r_{V_1} - r_{U_1} &\le \min\{I_2, I_7\} \\
R_1 - r_{V_1} &\le \min\{I_3, I_8\} \\
R_1 &\le I_4 \\
r_{U_2} + R_1 - r_{V_1} - r_{U_1} &\le I_{11} \\
r_{U_2} + R_1 - r_{V_1} &\le I_{12} \\
R_1 + r_{U_2} &\le I_{13} \\
r_{Z_2} &\le I_{14} \\
r_{U_2} + r_{Z_2} &\le I_{15} \\
r_{U_1} + r_{Z_2} &\le I_{16} \\
r_{U_1} + r_{U_2} + r_{Z_2} &\le I_{17} \\
r_{V_1} + r_{U_1} + r_{U_2} + r_{Z_2} &\le I_{18}. \qquad \text{(A.45)}
\end{aligned}$$

Rearranging, we get

$$\begin{aligned}
R_1 - r_{V_1} - r_{U_1} &\le \min\{I_1 + I_5, I_2, I_7\} \\
r_{U_2} + R_1 - r_{V_1} - r_{U_1} &\le \min\{I_1 + I_9, I_{11}\} \\
r_{Z_2} + r_{U_1} &\le I_{16} \\
r_{U_2} + r_{Z_2} + r_{U_1} &\le I_{17} \\
r_{U_2} + r_{Z_2} + r_{V_1} + r_{U_1} &\le I_{18} \\
R_1 - r_{V_1} &\le \min\{I_1 + I_6, I_3, I_8\} \\
r_{U_2} + R_1 - r_{V_1} &\le \min\{I_1 + I_{10}, I_{12}\} \\
R_1 &\le I_4 \\
R_1 + r_{U_2} &\le I_{13} \\
r_{Z_2} &\le I_{14} \\
r_{U_2} + r_{Z_2} &\le I_{15}. \qquad \text{(A.46)}
\end{aligned}$$

Adding a constraint $-r_{U_1} \le 0$, we get

$$\begin{aligned}
R_1 - r_{V_1} - r_{U_1} &\le \min\{I_1 + I_5, I_2, I_7\} \\
r_{U_2} + R_1 - r_{V_1} - r_{U_1} &\le \min\{I_1 + I_9, I_{11}\} \\
r_{Z_2} + r_{U_1} &\le I_{16} \\
r_{U_2} + r_{Z_2} + r_{U_1} &\le I_{17} \\
r_{U_2} + r_{Z_2} + r_{V_1} + r_{U_1} &\le I_{18} \\
-r_{U_1} &\le 0 \\
R_1 - r_{V_1} &\le \min\{I_1 + I_6, I_3, I_8\} \\
r_{U_2} + R_1 - r_{V_1} &\le \min\{I_1 + I_{10}, I_{12}\} \\
R_1 &\le I_4 \\
R_1 + r_{U_2} &\le I_{13} \\
r_{Z_2} &\le I_{14} \\
r_{U_2} + r_{Z_2} &\le I_{15}. \qquad \text{(A.47)}
\end{aligned}$$

Eliminating r_{U_1}, we get

$$\begin{aligned}
r_{Z_2} + R_1 - r_{V_1} &\leq \min\{I_1 + I_5, I_2, I_7\} + I_{16} \\
r_{Z_2} + r_{U_2} + R_1 - r_{V_1} &\leq \min\{I_1 + I_9, I_{11}\} + I_{16} \\
r_{U_2} + r_{Z_2} + R_1 - r_{V_1} &\leq \min\{I_1 + I_5, I_2, I_7\} + I_{17} \\
r_{U_2} + r_{Z_2} + r_{U_2} + R_1 - r_{V_1} &\leq \min\{I_1 + I_9, I_{11}\} + I_{17} \\
r_{U_2} + r_{Z_2} + R_1 &\leq \min\{I_1 + I_5, I_2, I_7\} + I_{18} \\
r_{U_2} + r_{Z_2} + r_{U_2} + R_1 &\leq \min\{I_1 + I_9, I_{11}\} + I_{18} \\
R_1 - r_{V_1} &\leq \min\{I_1 + I_6, I_3, I_8\} \\
r_{Z_2} &\leq I_{16} \\
r_{U_2} + r_{Z_2} &\leq I_{17} \\
r_{U_2} + r_{Z_2} + r_{V_1} &\leq I_{18} \\
r_{U_2} + R_1 - r_{V_1} &\leq \min\{I_1 + I_{10}, I_{12}\} \\
R_1 &\leq I_4 \\
R_1 + r_{U_2} &\leq I_{13} \\
r_{Z_2} &\leq I_{14} \\
r_{U_2} + r_{Z_2} &\leq I_{15}.
\end{aligned} \tag{A.48}$$

Rearranging, we get

$$\begin{aligned}
R_1 - r_{V_1} &\leq \min\{I_1 + I_6, I_3, I_8\} \\
r_{Z_2} + R_1 - r_{V_1} &\leq \min\{I_1 + I_5, I_2, I_7\} + I_{16} \\
r_{U_2} + R_1 - r_{V_1} &\leq \min\{I_1 + I_{10}, I_{12}\} \\
r_{Z_2} + r_{U_2} + R_1 - r_{V_1} &\leq \min\{I_1 + I_9, I_{11}\} + I_{16} \\
r_{U_2} + r_{Z_2} + R_1 - r_{V_1} &\leq \min\{I_1 + I_5, I_2, I_7\} + I_{17} \\
r_{U_2} + r_{Z_2} + r_{U_2} + R_1 - r_{V_1} &\leq \min\{I_1 + I_9, I_{11}\} + I_{17} \\
r_{U_2} + r_{Z_2} + r_{V_1} &\leq I_{18} \\
r_{U_2} + r_{Z_2} + R_1 &\leq \min\{I_1 + I_5, I_2, I_7\} + I_{18} \\
r_{U_2} + r_{Z_2} + r_{U_2} + R_1 &\leq \min\{I_1 + I_9, I_{11}\} + I_{18}
\end{aligned}$$

$$
\begin{aligned}
r_{Z_2} &\leq I_{16} \\
r_{U_2} + r_{Z_2} &\leq I_{17} \\
R_1 &\leq I_4 \\
R_1 + r_{U_2} &\leq I_{13} \\
r_{Z_2} &\leq I_{14} \\
r_{U_2} + r_{Z_2} &\leq I_{15}.
\end{aligned} \tag{A.49}
$$

Eliminating r_{V_1}, we get

$$
\begin{aligned}
R_1 + r_{U_2} + r_{Z_2} &\leq \min\{I_1 + I_6, I_3, I_8\} + I_{18} \\
r_{Z_2} + R_1 + r_{U_2} + r_{Z_2} &\leq \min\{I_1 + I_5, I_2, I_7\} + I_{16} + I_{18} \\
r_{U_2} + R_1 + r_{U_2} + r_{Z_2} &\leq \min\{I_1 + I_{10}, I_{12}\} + I_{18} \\
r_{Z_2} + r_{U_2} + R_1 + r_{U_2} + r_{Z_2} &\leq \min\{I_1 + I_9, I_{11}\} + I_{16} + I_{18} \\
r_{U_2} + r_{Z_2} + R_1 + r_{U_2} + r_{Z_2} &\leq \min\{I_1 + I_5, I_2, I_7\} + I_{17} + I_{18} \\
r_{U_2} + r_{Z_2} + r_{U_2} + R_1 + r_{U_2} + r_{Z_2} &\leq \min\{I_1 + I_9, I_{11}\} + I_{17} + I_{18} \\
r_{U_2} + r_{Z_2} + R_1 &\leq \min\{I_1 + I_5, I_2, I_7\} + I_{18} \\
r_{U_2} + r_{Z_2} + r_{U_2} + R_1 &\leq \min\{I_1 + I_9, I_{11}\} + I_{18} \\
r_{Z_2} &\leq I_{16} \\
r_{U_2} + r_{Z_2} &\leq I_{17} \\
R_1 &\leq I_4 \\
R_1 + r_{U_2} &\leq I_{13} \\
r_{Z_2} &\leq I_{14} \\
r_{U_2} + r_{Z_2} &\leq I_{15}.
\end{aligned} \tag{A.50}
$$

Let $r_{Z_2} = R_2 - r_{U_2}$, we get

$$
\begin{aligned}
R_1 + R_2 &\leq \min\{I_1 + I_6, I_3, I_8\} + I_{18} \\
R_1 + 2R_2 - r_{U_2} &\leq \min\{I_1 + I_5, I_2, I_7\} + I_{16} + I_{18} \\
R_1 + R_2 + r_{U_2} &\leq \min\{I_1 + I_{10}, I_{12}\} + I_{18} \\
R_1 + 2R_2 &\leq \min\{I_1 + I_9, I_{11}\} + I_{16} + I_{18}
\end{aligned}
$$

$$\begin{aligned}
R_1 + 2R_2 &\le \min\{I_1 + I_5, I_2, I_7\} + I_{17} + I_{18} \\
R_1 + 2R_2 + r_{U_2} &\le \min\{I_1 + I_9, I_{11}\} + I_{17} + I_{18} \\
R_1 + R_2 &\le \min\{I_1 + I_5, I_2, I_7\} + I_{18} \\
R_1 + R_2 + r_{U_2} &\le \min\{I_1 + I_9, I_{11}\} + I_{18} \\
R_2 - r_{U_2} &\le I_{16} \\
R_2 &\le I_{17} \\
R_1 &\le I_4 \\
R_1 + r_{U_2} &\le I_{13} \\
R_2 - r_{U_2} &\le I_{14} \\
R_2 &\le I_{15}.
\end{aligned} \tag{A.51}$$

Rearranging, we get

$$\begin{aligned}
R_2 - r_{U_2} &\le I_{14} \\
R_2 - r_{U_2} &\le I_{16} \\
R_1 + 2R_2 - r_{U_2} &\le \min\{I_1 + I_5, I_2, I_7\} + I_{16} + I_{18} \\
R_1 + r_{U_2} &\le I_{13} \\
R_1 + R_2 + r_{U_2} &\le \min\{I_1 + I_{10}, I_{12}\} + I_{18} \\
R_1 + R_2 + r_{U_2} &\le \min\{I_1 + I_9, I_{11}\} + I_{18} \\
R_1 + 2R_2 + r_{U_2} &\le \min\{I_1 + I_9, I_{11}\} + I_{17} + I_{18} \\
R_1 &\le I_4 \\
R_2 &\le I_{15} \\
R_2 &\le I_{17} \\
R_1 + R_2 &\le \min\{I_1 + I_6, I_3, I_8\} + I_{18} \\
R_1 + R_2 &\le \min\{I_1 + I_5, I_2, I_7\} + I_{18} \\
R_1 + 2R_2 &\le \min\{I_1 + I_9, I_{11}\} + I_{16} + I_{18} \\
R_1 + 2R_2 &\le \min\{I_1 + I_5, I_2, I_7\} + I_{17} + I_{18}.
\end{aligned} \tag{A.52}$$

Since $I_{16} \ge I_{14}$, the second constraint is redundant. Since $\min\{I_1 + I_{10}, I_{12}\} + I_{18} \ge \min\{I_1+I_9, I_{11}\}+I_{18}$, the fifth constraint is redundant. Since $I_{17} \ge I_{15}$, the tenth constraint is redundant. And $\min\{I_1+I_6, I_3, I_8\}+I_{18} \geq \min\{I_1+I_5, I_2, I_7\}+I_{18}$, the eleven's constraint is redundant. Eliminating these, we get

$$\begin{aligned}
R_2 - r_{U_2} &\leq I_{14} \\
R_1 + 2R_2 - r_{U_2} &\leq \min\{I_1 + I_5, I_2, I_7\} + I_{16} + I_{18} \\
R_1 + r_{U_2} &\leq I_{13} \\
R_1 + R_2 + r_{U_2} &\leq \min\{I_1 + I_9, I_{11}\} + I_{18} \\
R_1 + 2R_2 + r_{U_2} &\leq \min\{I_1 + I_9, I_{11}\} + I_{17} + I_{18} \\
R_1 &\leq I_4 \\
R_2 &\leq I_{15} \\
R_1 + R_2 &\leq \min\{I_1 + I_5, I_2, I_7\} + I_{18} \\
R_1 + 2R_2 &\leq \min\{I_1 + I_9, I_{11}\} + I_{16} + I_{18} \\
R_1 + 2R_2 &\leq \min\{I_1 + I_5, I_2, I_7\} + I_{17} + I_{18}.
\end{aligned} \tag{A.53}$$

Since $\min\{I_1 + I_5, I_2, I_7\} + I_{17} + I_{18} \geq \min\{I_1 + I_5, I_2, I_7\} + I_{18} + I_{15}$, the last constraint is redundant.

$$\begin{aligned}
R_2 - r_{U_2} &\leq I_{14} \\
R_1 + 2R_2 - r_{U_2} &\leq \min\{I_1 + I_5, I_2, I_7\} + I_{16} + I_{18} \\
R_1 + r_{U_2} &\leq I_{13} \\
R_1 + R_2 + r_{U_2} &\leq \min\{I_1 + I_9, I_{11}\} + I_{18} \\
R_1 + 2R_2 + r_{U_2} &\leq \min\{I_1 + I_9, I_{11}\} + I_{17} + I_{18} \\
R_1 &\leq I_4 \\
R_2 &\leq I_{15} \\
R_1 + R_2 &\leq \min\{I_1 + I_5, I_2, I_7\} + I_{18} \\
R_1 + 2R_2 &\leq \min\{I_1 + I_9, I_{11}\} + I_{16} + I_{18}.
\end{aligned} \tag{A.54}$$

Since $\min\{I_1 + I_5, I_2, I_7\} + I_{16} + I_{18} \geq I_{14} + \min\{I_1 + I_5, I_2, I_7\} + I_{18}$, the second constraint is redundant.

$$\begin{aligned}
R_2 - r_{U_2} &\leq I_{14} \\
R_1 + r_{U_2} &\leq I_{13} \\
R_1 + R_2 + r_{U_2} &\leq \min\{I_1 + I_9, I_{11}\} + I_{18} \\
R_1 + 2R_2 + r_{U_2} &\leq \min\{I_1 + I_9, I_{11}\} + I_{17} + I_{18} \\
R_1 &\leq I_4 \\
R_2 &\leq I_{15} \\
R_1 + R_2 &\leq \min\{I_1 + I_5, I_2, I_7\} + I_{18} \\
R_1 + 2R_2 &\leq \min\{I_1 + I_9, I_{11}\} + I_{16} + I_{18}.
\end{aligned} \tag{A.55}$$

Since $\min\{I_1 + I_9, I_{11}\} + I_{17} + I_{18} \geq \min\{I_1 + I_9, I_{11}\} + I_{18} + I_{15}$, the fourth constraint is redundant.

$$\begin{aligned}
R_2 - r_{U_2} &\leq I_{14} \\
R_1 + r_{U_2} &\leq I_{13} \\
R_1 + R_2 + r_{U_2} &\leq \min\{I_1 + I_9, I_{11}\} + I_{18} \\
R_1 &\leq I_4 \\
R_2 &\leq I_{15} \\
R_1 + R_2 &\leq \min\{I_1 + I_5, I_2, I_7\} + I_{18} \\
R_1 + 2R_2 &\leq \min\{I_1 + I_9, I_{11}\} + I_{16} + I_{18}.
\end{aligned} \tag{A.56}$$

Eliminating r_{U_2}, we get

$$\begin{aligned}
R_1 + R_2 &\leq I_{13} + I_{14} \\
R_1 + 2R_2 &\leq \min\{I_1 + I_9, I_{11}\} + I_{18} + I_{14} \\
R_1 &\leq I_4 \\
R_2 &\leq I_{15} \\
R_1 + R_2 &\leq \min\{I_1 + I_5, I_2, I_7\} + I_{18} \\
R_1 + 2R_2 &\leq \min\{I_1 + I_9, I_{11}\} + I_{16} + I_{18}.
\end{aligned} \tag{A.57}$$

Since $\min\{I_1 + I_9, I_{11}\} + I_{16} + I_{18} \geq \min\{I_1 + I_9, I_{11}\} + I_{18} + I_{14}$, the last constraint is redundant.

$$\begin{aligned} R_1 + R_2 &\leq I_{13} + I_{14} \\ R_1 + 2R_2 &\leq \min\{I_1 + I_9, I_{11}\} + I_{18} + I_{14} \\ R_1 &\leq I_4 \\ R_2 &\leq I_{15} \\ R_1 + R_2 &\leq \min\{I_1 + I_5, I_2, I_7\} + I_{18}. \end{aligned} \tag{A.58}$$

Rearranging, we get

$$\begin{aligned} R_1 &\leq I_4 \\ R_2 &\leq I_{15} \\ R_1 + R_2 &\leq I_{13} + I_{14} \\ R_1 + R_2 &\leq \min\{I_1 + I_5, I_2, I_7\} + I_{18} \\ R_1 + 2R_2 &\leq \min\{I_1 + I_9, I_{11}\} + I_{14} + I_{18}. \end{aligned} \tag{A.59}$$

A.3.4 Gaussian rate region for Prabhakaran and Viswanath scheme two

The Gaussian signal representation for the codewords are:

$$\begin{aligned} W &= \alpha_1 w \\ V_1 &= \alpha_1 w + \beta_1 v_1 \\ U_1 &= \alpha_1 w + \beta_1 v_1 + \gamma_1 u_1 \\ S_1 &= \alpha_2 w + \delta_1 s_1 \\ Z_1 &= \alpha_1 w + \beta_1 v_1 + \gamma_1 u_1 + \alpha_2 w + \delta_1 s_1 + \mu_1 z_1 \\ &= (\alpha_1 + \alpha_2) w + \beta_1 v_1 + \gamma_1 u_1 + \delta_1 s_1 + \mu_1 z_1 \\ X_1 &= (\alpha_1 + \alpha_2) w + \beta_1 v_1 + \gamma_1 u_1 + \delta_1 s_1 + \mu_1 z_1 + \theta_1 x_1 \\ U_2 &= \alpha_3 w + \gamma_2 u_2 \\ Z_2 &= \alpha_3 w + \gamma_2 u_2 + \mu_2 z_2, \end{aligned} \tag{A.60}$$

where w, v_1, u_1, s_1, z_1, x_1 are independent $\mathcal{N}(0,1)$ random variables that encode m_W, m_{V_1}, m_{U_1}, m_{S_1}, m_{Z_1},m_{X_1}. Similar u_2, z_2 are independent $\mathcal{N}(0,1)$ random variables that encode m_{U_2}, m_{Z_2}. Note that the final codeword at node two is Z_2. ($X_2 = Z_2$)

The power constraints are

$$\begin{aligned}(\alpha_1+\alpha_2)^2+\beta_1^2+\gamma_1^2+\delta_1^2+\mu_1^2+\theta_1^2 &\leq P_1,\\ \alpha_3^2+\gamma_2^2+\mu_2^2 &\leq P_2.\end{aligned} \tag{A.61}$$

The received signal are

$$\begin{aligned}Y_2 &= c_{21}X_1+N_2\\ Y_3 &= X_1+c_{32}X_2+N_3\\ Y_4 &= c_{41}X_1+X_2+N_4,\end{aligned} \tag{A.62}$$

where N_2, N_3, N_4 are independent $\mathcal{N}(0,1)$ Gaussian noise at node two, three and four. Substituting back, we get

$$\begin{aligned}Y_2 &= c_{21}(\alpha_1+\alpha_2)w+c_{21}\beta_1v_1+c_{21}\gamma_1u_1+c_{21}\delta_1s_1+c_{21}\mu_1z_1+c_{21}\theta_1x_1+N_2\\ Y_3 &= (\alpha_1+\alpha_2)w+\beta_1v_1+\gamma_1u_1+\delta_1s_1+\mu_1z_1+\theta_1x_1+c_{32}\alpha_3w+c_{32}\gamma_2u_2+c_{32}\mu_2z_2+N_3\\ &= (\alpha_1+\alpha_2+c_{32}\alpha_3)w+\beta_1v_1+\gamma_1u_1+\delta_1s_1+\mu_1z_1+\theta_1x_1+c_{32}\gamma_2u_2+c_{32}\mu_2z_2+N_3\\ Y_4 &= c_{41}(\alpha_1+\alpha_2)w+c_{41}\beta_1v_1+c_{41}\gamma_1u_1+c_{41}\delta_1s_1+c_{41}\mu_1z_1+c_{41}\theta_1x_1+\alpha_3w+\gamma_2u_2+\mu_2z_2+N_4\\ &= (c_{41}(\alpha_1+\alpha_2)+\alpha_3)w+c_{41}\beta_1v_1+c_{41}\gamma_1u_1+c_{41}\delta_1s_1+c_{41}\mu_1z_1+c_{41}\theta_1x_1+\gamma_2u_2+\mu_2z_2+N_4.\end{aligned} \tag{A.63}$$

We are going to calculate the terms I_1-I_{18} in Gaussian case. The term I_1 is

$$\begin{aligned}I_1 &= I(X_1;Y_2|W,V_1,U_1,S_1,Z_1)\\ &= H(Y_2|W,V_1,U_1,S_1,Z_1)-H(Y_2|W,V_1,U_1,S_1,Z_1,X_1)\\ &= H(c_{21}\theta_1x_1+N_2)-H(N_2)\\ &= C(c_{21}^2\theta_1^2).\end{aligned} \tag{A.64}$$

The term I_2 is

$$\begin{aligned} I_2 &= I(Z_1, X_1; Y_2|W, V_1, U_1, S_1) \\ &= H(Y_2|W, V_1, U_1, S_1) - H(Y_2|W, V_1, U_1, S_1, Z_1, X_1) \\ &= H(c_{21}\mu_1 z_1 + c_{21}\theta_1 x_1 + N_2) - H(N_2) \\ &= C(c_{21}^2\mu_1^2 + c_{21}^2\theta_1^2). \end{aligned} \tag{A.65}$$

The term I_3 is

$$\begin{aligned} I_3 &= I(U_1, Z_1, X_1; Y_2|W, V_1, S_1) \\ &= H(Y_2|W, V_1, S_1) - H(Y_2|W, V_1, U_1, S_1, Z_1, X_1) \\ &= H(c_{21}\gamma_1 u_1 + c_{21}\mu_1 z_1 + c_{21}\theta_1 x_1 + N_2) - H(N_2) \\ &= C(c_{21}^2\gamma_1^2 + c_{21}^2\mu_1^2 + c_{21}^2\theta_1^2). \end{aligned} \tag{A.66}$$

The term I_4 is

$$\begin{aligned} I_4 &= I(V_1, U_1, Z_1, X_1; Y_2|W, S_1) \\ &= H(Y_2|W, S_1) - H(Y_2|W, V_1, U_1, S_1, Z_1, X_1) \\ &= H(c_{21}\beta_1 v_1 + c_{21}\gamma_1 u_1 + c_{21}\mu_1 z_1 + c_{21}\theta_1 x_1 + N_2) - H(N_2) \\ &= C(c_{21}^2\beta_1^2 + c_{21}^2\gamma_1^2 + c_{21}^2\mu_1^2 + c_{21}^2\theta_1^2). \end{aligned} \tag{A.67}$$

The term I_5 is

$$\begin{aligned} I_5 &= I(Z_1; Y_3|W, V_1, U_1, U_2, S_1) \\ &= H(Y_3|W, V_1, U_1, U_2, S_1) - H(Y_3|W, V_1, U_1, U_2, S_1, Z_1) \\ &= H(\mu_1 z_1 + \theta_1 x_1 + c_{32}\mu_2 z_2 + N_3) - H(\theta_1 x_1 + c_{32}\mu_2 z_2 + N_3) \\ &= C\left(\frac{\mu_1^2}{\theta_1^2 + c_{32}^2\mu_2^2 + 1}\right). \end{aligned} \tag{A.68}$$

The term I_6 is

$$\begin{aligned} I_6 &= I(U_1, Z_1; Y_3|W, V_1, U_2, S_1) \\ &= H(Y_3|W, V_1, U_2, S_1) - H(Y_3|W, V_1, U_1, U_2, S_1, Z_1) \\ &= H(\gamma_1 u_1 + \mu_1 z_1 + \theta_1 x_1 + c_{32}\mu_2 z_2 + N_3) - H(\theta_1 x_1 + c_{32}\mu_2 z_2 + N_3) \\ &= C\left(\frac{\gamma_1^2 + \mu_1^2}{\theta_1^2 + c_{32}^2\mu_2^2 + 1}\right). \end{aligned} \tag{A.69}$$

The term I_7 is

$$\begin{aligned} I_7 &= I(S_1, Z_1; Y_3|W, V_1, U_1, U_2) \\ &= H(Y_3|W, V_1, U_1, U_2) - H(Y_3|W, V_1, U_1, U_2, S_1, Z_1) \\ &= H(\delta_1 s_1 + \mu_1 z_1 + \theta_1 x_1 + c_{32}\mu_2 z_2 + N_3) - H(\theta_1 x_1 + c_{32}\mu_2 z_2 + N_3) \\ &= C\left(\frac{\delta_1^2 + \mu_1^2}{\theta_1^2 + c_{32}^2\mu_2^2 + 1}\right). \end{aligned} \tag{A.70}$$

The term I_8 is

$$\begin{aligned} I_8 &= I(S_1, U_1, Z_1; Y_3|W, V_1, U_2) \\ &= H(Y_3|W, V_1, U_2) - H(Y_3|W, V_1, U_1, U_2, S_1, Z_1) \\ &= H(\gamma_1 u_1 + \delta_1 s_1 + \mu_1 z_1 + \theta_1 x_1 + c_{32}\mu_2 z_2 + N_3) - H(\theta_1 x_1 + c_{32}\mu_2 z_2 + N_3) \\ &= C\left(\frac{\gamma_1^2 + \delta_1^2 + \mu_1^2}{\theta_1^2 + c_{32}^2\mu_2^2 + 1}\right). \end{aligned} \tag{A.71}$$

The term I_9 is

$$\begin{aligned} I_9 &= I(U_2, Z_1; Y_3|W, V_1, U_1, S_1) \\ &= H(Y_3|W, V_1, U_1, S_1) - H(Y_3|W, V_1, U_1, S_1, Z_1, U_2) \\ &= H(\mu_1 z_1 + \theta_1 x_1 + c_{32}\gamma_2 u_2 + c_{32}\mu_2 z_2 + N_3) - H(\theta_1 x_1 + c_{32}\mu_2 z_2 + N_3) \\ &= C\left(\frac{\mu_1^2 + c_{32}^2\gamma_2^2}{\theta_1^2 + c_{32}^2\mu_2^2 + 1}\right). \end{aligned} \tag{A.72}$$

The term I_{10} is

$$\begin{aligned}
I_{10} &= I(U_2, U_1, Z_1; Y_3|W, V_1, S_1) \\
&= H(Y_3|W, V_1, S_1) - H(Y_3|W, V_1, U_1, S_1, Z_1, U_2) \\
&= H(\gamma_1 u_1 + \mu_1 z_1 + \theta_1 x_1 + c_{32}\gamma_2 u_2 + c_{32}\mu_2 z_2 + N_3) - H(\theta_1 x_1 + c_{32}\mu_2 z_2 + N_3) \\
&= C\left(\frac{\gamma_1^2 + \mu_1^2 + c_{32}^2\gamma_2^2}{\theta_1^2 + c_{32}^2\mu_2^2 + 1}\right).
\end{aligned} \tag{A.73}$$

The term I_{11} is

$$\begin{aligned}
I_{11} &= I(U_2, S_1, Z_1; Y_3|W, V_1, U_1) \\
&= H(Y_3|W, V_1, U_1) - H(Y_3|W, V_1, U_1, S_1, Z_1, U_2) \\
&= H(\delta_1 s_1 + \mu_1 z_1 + \theta_1 x_1 + c_{32}\gamma_2 u_2 + c_{32}\mu_2 z_2 + N_3) - H(\theta_1 x_1 + c_{32}\mu_2 z_2 + N_3) \\
&= C\left(\frac{\delta_1^2 + \mu_1^2 + c_{32}^2\gamma_2^2}{\theta_1^2 + c_{32}^2\mu_2^2 + 1}\right).
\end{aligned} \tag{A.74}$$

The term I_{12} is

$$\begin{aligned}
I_{12} &= I(U_2, S_1, U_1, Z_1; Y_3|W, V_1) \\
&= H(Y_3|W, V_1) - H(Y_3|W, V_1, U_1, S_1, Z_1, U_2) \\
&= H(\gamma_1 u_1 + \delta_1 s_1 + \mu_1 z_1 + \theta_1 x_1 + c_{32}\gamma_2 u_2 + c_{32}\mu_2 z_2 + N_3) - H(\theta_1 x_1 + c_{32}\mu_2 z_2 + N_3) \\
&= C\left(\frac{\gamma_1^2 + \delta_1^2 + \mu_1^2 + c_{32}^2\gamma_2^2}{\theta_1^2 + c_{32}^2\mu_2^2 + 1}\right).
\end{aligned} \tag{A.75}$$

The term I_{13} is

$$\begin{aligned}
I_{13} &= I(W, V_1, U_2, S_1, U_1, Z_1; Y_3) \\
&= H(Y_3) - H(Y_3|W, V_1, U_1, S_1, Z_1, U_2) \\
&= H((\alpha_1 + \alpha_2 + c_{32}\alpha_3)w + \beta_1 v_1 + \gamma_1 u_1 + \delta_1 s_1 + \mu_1 z_1 + \theta_1 x_1 + c_{32}\gamma_2 u_2 + c_{32}\mu_2 z_2 + N_3) \\
&\quad - H(\theta_1 x_1 + c_{32}\mu_2 z_2 + N_3) \\
&= C\left(\frac{(\alpha_1 + \alpha_2 + c_{32}\alpha_3)^2 + \beta_1^2 + \gamma_1^2 + \delta_1^2 + \mu_1^2 + c_{32}^2\gamma_2^2}{\theta_1^2 + c_{32}^2\mu_2^2 + 1}\right).
\end{aligned} \tag{A.76}$$

The term I_{14} is

$$\begin{aligned} I_{14} &= I(Z_2; Y_4|W, V_1, U_1, U_2) \\ &= H(Y_4|W, V_1, U_1, U_2) - H(Y_4|W, V_1, U_1, U_2, Z_2) \\ &= H(c_{41}\delta_1 s_1 + c_{41}\mu_1 z_1 + c_{41}\theta_1 x_1 + \mu_2 z_2 + N_4) - H(c_{41}\delta_1 s_1 + c_{41}\mu_1 z_1 + c_{41}\theta_1 x_1 + N_4) \\ &= C\left(\frac{\mu_2^2}{c_{41}^2\delta_1^2 + c_{41}^2\mu_1^2 + c_{41}^2\theta_1^2 + 1}\right). \end{aligned} \tag{A.77}$$

The term I_{15} is

$$\begin{aligned} I_{15} &= I(U_2, Z_2; Y_4|W, V_1, U_1) \\ &= H(Y_4|W, V_1, U_1) - H(Y_4|W, V_1, U_1, U_2, Z_2) \\ &= H(c_{41}\delta_1 s_1 + c_{41}\mu_1 z_1 + c_{41}\theta_1 x_1 + \gamma_2 u_2 + \mu_2 z_2 + N_4) - H(c_{41}\delta_1 s_1 + c_{41}\mu_1 z_1 + c_{41}\theta_1 x_1 + N_4) \\ &= C\left(\frac{\gamma_2^2 + \mu_2^2}{c_{41}^2\delta_1^2 + c_{41}^2\mu_1^2 + c_{41}^2\theta_1^2 + 1}\right). \end{aligned} \tag{A.78}$$

The term I_{16} is

$$\begin{aligned} I_{16} &= I(U_1, Z_2; Y_4|W, V_1, U_2) \\ &= H(Y_4|W, V_1, U_2) - H(Y_4|W, V_1, U_1, U_2, Z_2) \\ &= H(c_{41}\gamma_1 u_1 + c_{41}\delta_1 s_1 + c_{41}\mu_1 z_1 + c_{41}\theta_1 x_1 + \mu_2 z_2 + N_4) - H(c_{41}\delta_1 s_1 + c_{41}\mu_1 z_1 + c_{41}\theta_1 x_1 + N_4) \\ &= C\left(\frac{c_{41}^2\gamma_1^2 + \mu_2^2}{c_{41}^2\delta_1^2 + c_{41}^2\mu_1^2 + c_{41}^2\theta_1^2 + 1}\right). \end{aligned} \tag{A.79}$$

The term I_{17} is

$$\begin{aligned} I_{17} &= I(U_1, U_2, Z_2; Y_4|W, V_1) \\ &= H(Y_4|W, V_1) - H(Y_4|W, V_1, U_1, U_2, Z_2) \\ &= H(c_{41}\gamma_1 u_1 + c_{41}\delta_1 s_1 + c_{41}\mu_1 z_1 + c_{41}\theta_1 x_1 + \gamma_2 u_2 + \mu_2 z_2 + N_4) \\ &\quad - H(c_{41}\delta_1 s_1 + c_{41}\mu_1 z_1 + c_{41}\theta_1 x_1 + N_4) \\ &= C\left(\frac{c_{41}^2\gamma_1^2 + \gamma_2^2 + \mu_2^2}{c_{41}^2\delta_1^2 + c_{41}^2\mu_1^2 + c_{41}^2\theta_1^2 + 1}\right). \end{aligned} \tag{A.80}$$

The term I_{18} is

$$\begin{aligned}
I_{18} &= I(W, V_1, U_1, U_2, Z_2; Y_4) \\
&= H(Y_4) - H(Y_4|W, V_1, U_1, U_2, Z_2) \\
&= H((c_{41}(\alpha_1+\alpha_2)+\alpha_3)w + c_{41}\beta_1 v_1 + c_{41}\gamma_1 u_1 + c_{41}\delta_1 s_1 + c_{41}\mu_1 z_1 + c_{41}\theta_1 x_1 + \gamma_2 u_2 + \mu_2 z_2 + N_4) \\
&- H(c_{41}\delta_1 s_1 + c_{41}\mu_1 z_1 + c_{41}\theta_1 x_1 + N_4) \\
&= C\left(\frac{(c_{41}(\alpha_1+\alpha_2)+\alpha_3)^2 + c_{41}^2\beta_1^2 + c_{41}^2\gamma_1^2 + \gamma_2^2 + \mu_2^2}{c_{41}^2\delta_1^2 + c_{41}^2\mu_1^2 + c_{41}^2\theta_1^2 + 1}\right). \qquad (A.81)
\end{aligned}$$

In summary, the DMC rate region is

$$\begin{aligned}
R_1 &\le I_4 \\
R_2 &\le I_{15} \\
R_1 + R_2 &\le I_{13} + I_{14} \\
R_1 + R_2 &\le \min\{I_1 + I_5, I_2, I_7\} + I_{18} \\
R_1 + 2R_2 &\le \min\{I_1 + I_9, I_{11}\} + I_{14} + I_{18}. \qquad (A.82)
\end{aligned}$$

The Gaussian rate is

$$\begin{aligned}
R_1 &\le C(c_{21}^2\beta_1^2 + c_{21}^2\gamma_1^2 + c_{21}^2\mu_1^2 + c_{21}^2\theta_1^2) \\
R_2 &\le C\left(\frac{\gamma_2^2 + \mu_2^2}{c_{41}^2\delta_1^2 + c_{41}^2\mu_1^2 + c_{41}^2\theta_1^2 + 1}\right) \\
R_1 + R_2 &\le C\left(\frac{(\alpha_1+\alpha_2+c_{32}\alpha_3)^2 + \beta_1^2 + \gamma_1^2 + \delta_1^2 + \mu_1^2 + c_{32}^2\gamma_2^2}{\theta_1^2 + c_{32}^2\mu_2^2 + 1}\right) + C\left(\frac{\mu_2^2}{c_{41}^2\delta_1^2 + c_{41}^2\mu_1^2 + c_{41}^2\theta_1^2 + 1}\right) \\
R_1 + R_2 &\le \min\left\{C(c_{21}^2\theta_1^2) + C\left(\frac{\mu_1^2}{\theta_1^2 + c_{32}^2\mu_2^2 + 1}\right), C(c_{21}^2\mu_1^2 + c_{21}^2\theta_1^2), C\left(\frac{\delta_1^2 + \mu_1^2}{\theta_1^2 + c_{32}^2\mu_2^2 + 1}\right)\right\} \\
&+ C\left(\frac{(c_{41}(\alpha_1+\alpha_2)+\alpha_3)^2 + c_{41}^2\beta_1^2 + c_{41}^2\gamma_1^2 + \gamma_2^2 + \mu_2^2}{c_{41}^2\delta_1^2 + c_{41}^2\mu_1^2 + c_{41}^2\theta_1^2 + 1}\right) \\
R_1 + 2R_2 &\le \min\{C(c_{21}^2\theta_1^2) + C\left(\frac{\mu_1^2 + c_{32}^2\gamma_2^2}{\theta_1^2 + c_{32}^2\mu_2^2 + 1}\right), C\left(\frac{\delta_1^2 + \mu_1^2 + c_{32}^2\gamma_2^2}{\theta_1^2 + c_{32}^2\mu_2^2 + 1}\right)\} \\
&+ C\left(\frac{\gamma_2^2 + \mu_2^2}{c_{41}^2\delta_1^2 + c_{41}^2\mu_1^2 + c_{41}^2\theta_1^2 + 1}\right) + C\left(\frac{(c_{41}(\alpha_1+\alpha_2)+\alpha_3)^2 + c_{41}^2\beta_1^2 + c_{41}^2\gamma_1^2 + \gamma_2^2 + \mu_2^2}{c_{41}^2\delta_1^2 + c_{41}^2\mu_1^2 + c_{41}^2\theta_1^2 + 1}\right). \\
&\qquad (A.83)
\end{aligned}$$

A.4 Proof of the optimal binning parameter λ^* for full-duplex PDF-binning

The optimal λ^* is obtained by minimizing the determinant of the covariance matrix in (3.26),

$$\begin{aligned}
|\mathrm{cov}(U_2, Y_2)| &= \mathrm{var}(U_2)\mathrm{var}(Y_2) - \mathrm{E}(U_2, Y_2)^2 \\
&= (\mu^2 + \lambda^2 + 2\mu\rho\lambda)\left((a\alpha + \mu\rho)^2 + a^2\beta^2 + a^2\gamma^2 + \mu^2(1-\rho^2) + 1\right) \\
&\quad - \left((\mu\rho + \lambda)(a\alpha + \mu\rho) + \mu^2(1-\rho^2)\right)^2 \\
&= (\lambda^2 + 2\mu\rho\lambda + \mu^2)\left(a^2(\alpha^2 + \beta^2 + \gamma^2) + \mu^2 + 2a\alpha\mu\rho + 1\right) \\
&\quad - \left(\lambda(a\alpha + \mu\rho) + \mu^2 + a\alpha\mu\rho\right)^2. \qquad \text{(A.84)}
\end{aligned}$$

Let $f(\lambda) = |\mathrm{cov}(U_2, Y_2)| = c_2\lambda^2 + c_1\lambda + c_0$. We calculate the parameters c_2, c_1 and c_0 in the following. The parameter c_2 for λ^2 is

$$\begin{aligned}
c_2 &= \left(a^2(\alpha^2 + \beta^2 + \gamma^2) + \mu^2 + 2a\alpha\mu\rho + 1\right) - (a\alpha + \mu\rho)^2 \\
&= a^2(\beta^2 + \gamma^2) + \mu^2(1-\rho^2) + 1. \qquad \text{(A.85)}
\end{aligned}$$

The parameter c_1 for λ is

$$\begin{aligned}
c_1 &= 2\mu\rho\left(a^2(\alpha^2 + \beta^2 + \gamma^2) + \mu^2 + 2a\alpha\mu\rho + 1\right) - 2(a\alpha + \mu\rho)(\mu^2 + a\alpha\mu\rho) \\
&= 2\mu\rho\left(a^2(\alpha^2 + \beta^2 + \gamma^2) + \mu^2 + 2a\alpha\mu\rho + 1 - \mu^2 - a\alpha\mu\rho\right) - 2a\alpha(\mu^2 + a\alpha\mu\rho) \\
&= 2\mu\rho\left(a^2(\alpha^2 + \beta^2 + \gamma^2) + a\alpha\mu\rho + 1\right) - 2a\alpha(\mu^2 + a\alpha\mu\rho) \\
&= 2\mu\rho\left(a^2(\alpha^2 + \beta^2 + \gamma^2) + a\alpha\mu\rho + 1 - a^2\alpha^2\right) - 2a\alpha\mu^2 \\
&= 2\mu\rho\left(a^2(\beta^2 + \gamma^2) + 1\right) + 2a\alpha\mu^2(\rho^2 - 1). \qquad \text{(A.86)}
\end{aligned}$$

The constant term c_0 is

$$\begin{aligned}
c_0 &= \mu^2\left(a^2(\alpha^2 + \beta^2 + \gamma^2) + \mu^2 + 2a\alpha\mu\rho + 1\right) - (\mu^2 + a\alpha\mu\rho)^2 \\
&= \mu^2\left(a^2(\alpha^2 + \beta^2 + \gamma^2) + \mu^2 + 2a\alpha\mu\rho + 1 - \mu^2 - 2a\alpha\mu\rho - a^2\alpha^2\rho^2\right) \\
&= \mu^2\left(a^2(\alpha^2 + \beta^2 + \gamma^2) + 1 - a^2\alpha^2\rho^2\right) \\
&= \mu^2\left(a^2\alpha^2(1-\rho^2) + a^2(\beta^2 + \gamma^2) + 1\right). \qquad \text{(A.87)}
\end{aligned}$$

For quadratic function $f(\lambda) = c_2\lambda^2 + c_1\lambda + c_0$, since $c_2 > 0$, the $f(\lambda)$ achieves its minimum when

$$\begin{aligned}
\lambda^* &= -\frac{c_1}{2c_2} \\
&= -\frac{2\mu\rho\left(a^2(\beta^2+\gamma^2)+1\right) + 2a\alpha\mu^2(\rho^2-1)}{2\left(a^2(\beta^2+\gamma^2)+\mu^2(1-\rho^2)+1\right)} \\
&= -\frac{\mu\rho\left(a^2(\beta^2+\gamma^2)+1\right) + a\alpha\mu^2(\rho^2-1)}{a^2(\beta^2+\gamma^2)+\mu^2(1-\rho^2)+1} \\
&= \frac{a\alpha\mu^2(1-\rho^2) - \mu\rho(a^2\beta^2+a^2\gamma^2+1)}{a^2\beta^2+a^2\gamma^2+\mu^2(1-\rho^2)+1}.
\end{aligned} \tag{A.88}$$

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