EE 105 Homework 2

Due on Gradescope, November 1 2024

Problem 1: State space formulation

Write a state-space representation for the following differential equations:

a) $\ddot{x}(t) = a_1\dot{x}(t) + a_2 x(t) + bu(t)$

b) $\dot{y}(t) = k_1 y(t) + k_2 u(t)$ *Don't overthink this one.*

c) $\dddot{y}(t) = bu(t)$

Problem 2: State space to transfer function

Consider the system described by

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -4 & 1 \\ 3 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

a) Write the transfer function for this system. You should work this out by hand, but you can (and should) check your result with MATLAB.

b) Is this system internally stable? Show how you know.

Problem 3: Door closer

A basic automatic door closer uses a torsion spring to pull the door closed and a damper to prevent the door from swinging too fast and slamming shut.

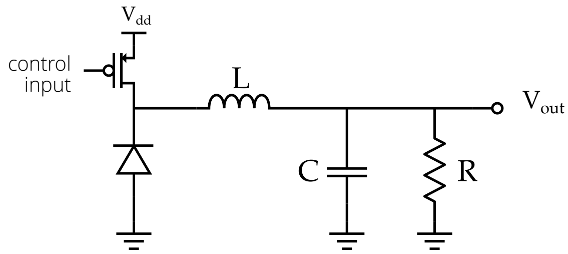
Assume the door has a moment of inertia, I, of $10\,\mathrm{kg \cdot m^2}$. This roughly represents a $30\,\mathrm{kg}$ door. The rotational damper has a damping force proportional to the angular velocity of the door, $B = 30\,\mathrm{N \cdot m}/\frac{\mathrm{rad}}{\mathrm{sec}}$, and the spring constant K is $22.5\,\mathrm{N \cdot m/rad}$.

a) The door is opened to an angle of $60°$ and released from a standstill. Use MATLAB to plot the angle for $t = 0$ to $5\,\mathrm{s}$.

b) Can you improve the performance of the door closer, so that it closes faster without slamming? If so, explain how; if not, provide some mathematical justification as to why the current performance is optimal.

Problem 4: Switching power supply

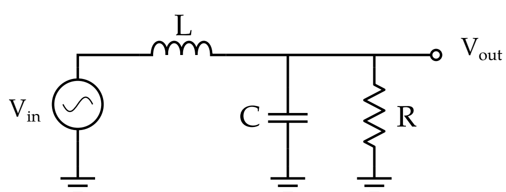
The schematic below shows a simple buck converter, which steps a voltage down from V_{dd} to V_{out} without wasting much energy[1] Other power supplies (including 110 V power adapters for charging phones and such) work on a similar principle.



We can approximate the diode and transistor with a voltage source that has a constant DC offset (V_{dd} · duty cycle) and a sine wave with peak-to-peak amplitude of V_{dd}.[2] For simplicity, let's assume the duty cycle is 50 %; i.e., we're stepping the voltage down from V_{dd} down to $V_{dd}/2$. Thus, we can write the input

$$V_{in} = V_{dd}(0.5 + \cos(2\pi f_{switch} \cdot t))$$

where f_{switch} is the switching frequency in Hz.



a) Find the transfer function for this circuit.

b) Develop a state-space model for this circuit.

c) Find the eigenvalues of your state matrix, A. Assuming the resistor has a positive and real value, it should be stable!

d) Find the forced/external response (i.e., the response to the input, assuming zero initial state). This will have two components, corresponding to the DC and sine wave components of the input. *Hint: You can use Matlab's* `step` *command to sanity-check the constant-input part of your answer.*

e) Plot the external response for $V_{dd} = 5\,\text{V}$, $f_{switch} = 10\,\text{kHz}$, $C = 50\,\mu\text{F}$, $L = 5\,\text{mH}$, $R = 5\,\Omega$, for at least 10 ms. Include your code and plot in your submission.

f) Use Simulink to check your result, and include a plot from Simulink. Note that you can use MATLAB workspace variables as parameters for your Simulink simulation.

g) Experiment with some different values of C and L, and comment on what happens.[3] Can you speed up the transient response so the power supply turns on faster? What are the tradeoffs?

It turns out this is a major challenge in power supply design and control — check out "The Kat Kim show" on YouTube for videos on both control theory and power electronics. (`https://www.youtube.com/user/katkimshow/videos`)

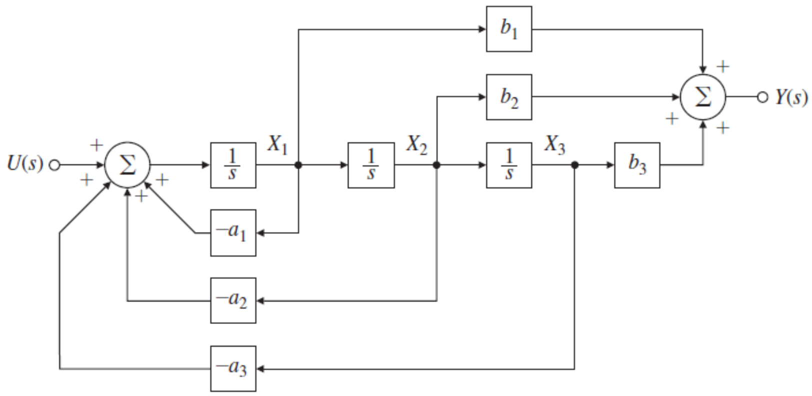
[1] as compared to a linear regulator, which is designed to waste the excess voltage.

[2] The actual diode/transistor combination will produce a square wave, but the constant and sine wave are the first two terms if we take the Fourier transform.

[3] Assume R is fixed, because this represents the load.

Problem 5: Block diagrams

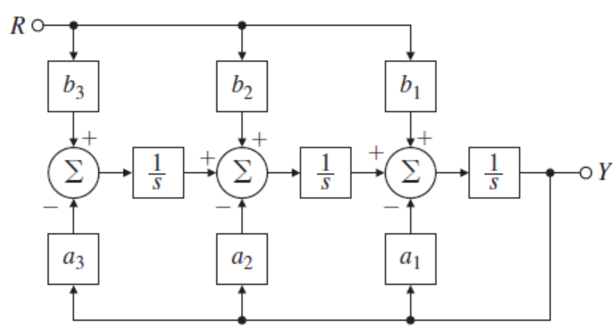
The block diagram below represents a system in "control canonical form".



a) Find the transfer function for the system. Note that a_n and b_n are constants.

b) Find a state-space representation of this system. This should be easy.

The block diagram below represents a system in "observer canonical form".

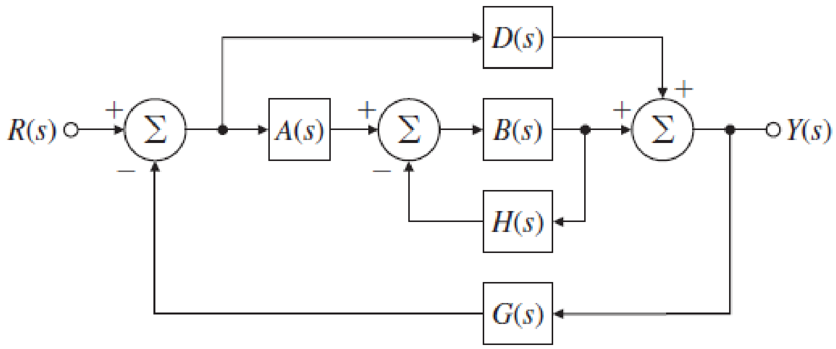


(b)

c) Find the transfer function for the system.

d) Find a state-space representation of this system. Again, this should be easy.

e) Write the transfer function for the (admittedly gross and contrived) block diagram with multiple feedback paths below.



(d)

Problem 6: Reflection

a) Approximately how long did you spend on this problem set?

b) What questions do you have about this problem set, or about the course material so far?