Name: ______________________________

EE 105 Homework 2

Due in class, September 18 2019

Problem 1: State space formulation

Write a state-space representation for the following differential equations:

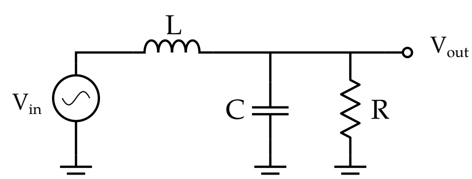
a) $\ddot{x}(t) = a_1\dot{x}(t) + a_2x(t) + bu(t)$

b) $\dot{y}(t) = k_1y(t) + k_2u(t)$ *Don't overthink this one.*

c) $\dddot{y}(t) = bu(t)$

Problem 2: Circuit model

The circuit below represents a basic buck converter, which converts a DC voltage to some lower voltage. We'll analyze the transient behavior in future homework problems, but for now we just need a model.



a) Find the transfer function for this circuit.

b) Develop a state-space model for this circuit.

c) Find the eigenvalues of your state transition matrix, A.

Problem 3: Matrix theory

a) Let $A \in \mathbb{R}^{2\times 2}$ be symmetric with eigen-pairs $(\lambda_1, \mathbf{v_1})$ and $(\lambda_2, \mathbf{v_2})$. If

$$\det(A) = 3,$$

$$\operatorname{trace}(A) = 2,$$

$$\mathbf{v_1} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

find A. Is the solution unique?

b) Are the following statements true or false? If true, prove the result; if false, show a counter-example.

- Symmetric matrices are invertible.
- For all matrices A where all eigenvalues are zero, A is a zero matrix (all elements are zero).
- If the matrix Λ is diagonal, then $V\Lambda V^T$ is symmetric for any square matrix V.

Problem 4: A fun matrix

Consider the following matrix:

$$A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & -\cos\theta \end{bmatrix}$$

where $0 \leq \theta \leq 2\pi$.

a) Find the eigenvalues of A.

b) What does this matrix do to a vector in $\mathbb{R}^2$?

Problem 5: Reflection

a) Approximately how long did you spend on this problem set?

b) What questions do you have about this problem set, or about the course material so far?