Homework 3

Fall 2017 - EE 105, Feedback Control Systems (Prof. Khan)

Problem 1 (20 pts)

Consider the following matrix, $A \in \mathbb{R}^{2\times 2}$,

$$A = \begin{bmatrix} 1 & \alpha \\ 0 & 2 \end{bmatrix}. \tag{1}$$

Plot both its two-norm, $\|A\|_2$, defined as

$$\|A\|_2 \triangleq \sqrt{\lambda_{\max}(A^T A)},$$

where $\lambda_{\max}(\cdot)$ is the maximum eigenvalue of the matrix in the argument; and spectral radius, $\rho(A)$, defined as

$$\rho(A) \triangleq \max_i |\lambda_i(A)|.$$

as a function of $\alpha \in [-10, 10]$. Comment on the dependence of α on $\|A\|_2$ and $\rho(A)$. Repeat for

$$A = \begin{bmatrix} 1 & \alpha \\ \alpha & 2 \end{bmatrix}. \tag{2}$$

and comment.

From the plot, provide a relationship between $\|A\|_2$ and $\rho(A)$.

Problem 2 (20 pts)

Write the state-space representation of the parallel RLC circuit in Fig. 1. From the state space, find the following:

(a) internal, external and total response of the RLC circuit;

(b) write the eigenvalues in terms of R, L, and C;

(c) find the conditions under which the internal response is stable (goes to zero with out oscillations), unstable, stable-oscillations, oscillatory.

(d) For each of the cases: stable (goes to zero with out oscillations), unstable, stable-oscillations, oscillatory, use Matlab to plot the RLC internal response, i.e., the time evolution of $v_c(t)$ and $i_L(t)$.

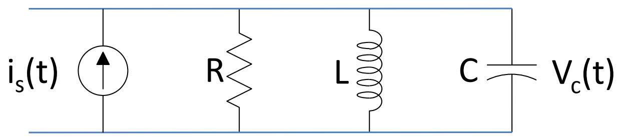

Fig. 1. A parallel RLC circuit

Problem 3 (20 pts)

(a) For $A \in \mathbb{R}^{2\times 2}$, let $\det(A) = -4$ and let trace$(A) = 3$. Find the eigenvalues of A. Is A positive-definite ?

(b) Is the eigenvalue information enough to conclude that A is symmetric ? If yes, justify, if not, give a counter-example.

In other words, we know that a (real-valued) symmetric matrix has real eigenvalues, but if a matrix has real eigenvalues, is it symmetric ?