

(3)

Total response is Internal + External

$$y(t) = \underbrace{(e^{At}x(0))}_{\text{Internal}} + C \int_0^t e^{A(t-\tau)} B \cdot u(\tau) d\tau \quad (\text{ignoring } D)$$

Is this stable?

Convergence to 0 doesn't help much...  $u(t)$  could be anything and go on forever.

BIBO: given a finite input, is the output bounded?

Only if integral of impulse response is finite.

$$\int_{-\infty}^{\infty} |h(t)| dt < \infty$$

What is the impulse response for a state-space formulation?

$u(t)$  is an impulse at  $t=0$ ,  $x(0) = 0$

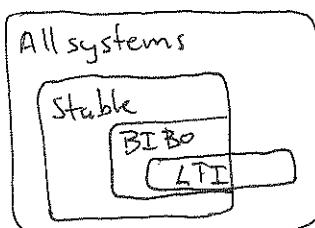
$$y(t) = C e^{At} \cdot B, \text{ designated } h(t)$$

Let's take the integral:

$$\begin{aligned} & \int_{-\infty}^{\infty} |h(t)| dt \\ &= \int_{-\infty}^{\infty} |C e^{At} B| dt \\ &= \int_0^{\infty} |C A^{-1} e^{At} B| dt \\ &= 0 - C A^{-1} I B \quad \text{if all } \lambda < 0 \\ & \qquad \qquad \qquad \text{otherwise} \\ & \qquad \qquad \qquad \infty \end{aligned}$$

Draw a Venn diagram

All systems, stable systems, BIBO, LTI



No stable LTI systems that are not also BIBO.

(4)

This is a perfect segue...

$h(t)$  is the impulse response, so if we took the Laplace transform, we'd have the transfer function.

$\mathcal{L}\{e^t\} = \frac{1}{s-1}$ , so  $\mathcal{L}\{e^{At}\}$  with a matrix  $A$  is  $(sI - A)^{-1}$

$$H(s) = C(sI - A)^{-1} B$$

We could write this out as

$$H(s) = \frac{\text{some polynomial of } s}{|sI - A|} \quad \text{because } P^{-1} = \frac{1}{|P|} \text{adj}(P)$$

$\searrow$  determinant

Remember how we calculate eigenvalues?

$|\lambda I - A|$ , find the roots

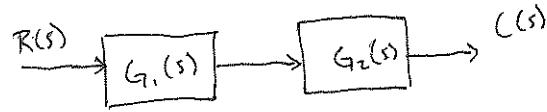
Roots of  $|sI - A|$  are the poles, which are the eigenvalues.

We're back in TF-land for now.

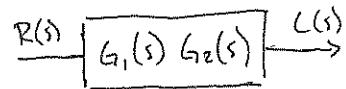
Today: manipulating / simplifying block diagrams - Why?

Wednesday: What happens in the time domain

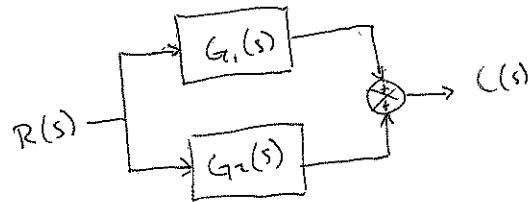
Next week: Feedback (finally!)



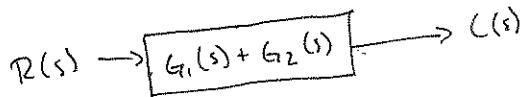
We can just multiply these TFs,



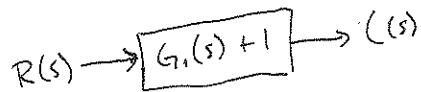
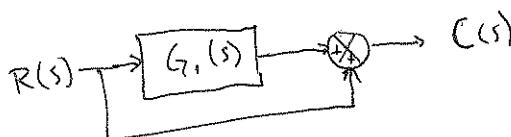
Parallel + sum



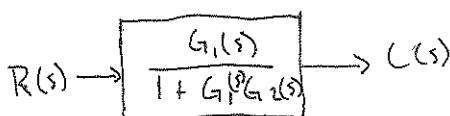
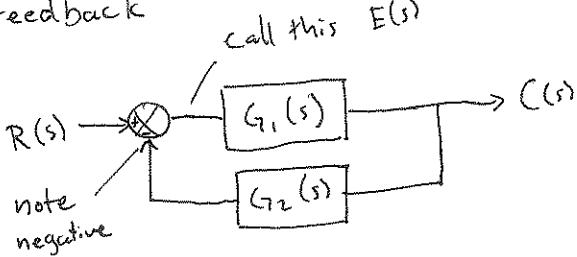
$$\begin{aligned} C(s) &= R(s) G_1(s) + R(s) G_2(s) \\ &= R(s) (G_1(s) + G_2(s)) \end{aligned}$$



Special case where  $G_2 = 1$ .



Feedback



$$E(s) = R(s) - G_2(s) C(s)$$

$$C(s) = E(s) G_1(s)$$

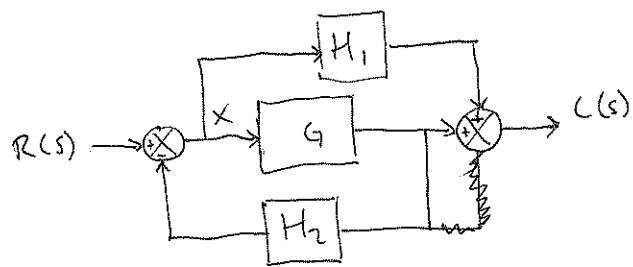
$$C(s) = G_1(s) (R(s) - G_2(s) C(s))$$

$$C(s) = G_1(s) R(s) - G_1(s) G_2(s) C(s)$$

$$C(s) (1 + G_1(s) G_2(s)) = G_1(s) R(s)$$

$$H(s) = \frac{C(s)}{R(s)} = \frac{G_1(s)}{1 + G_1(s) G_2(s)}$$

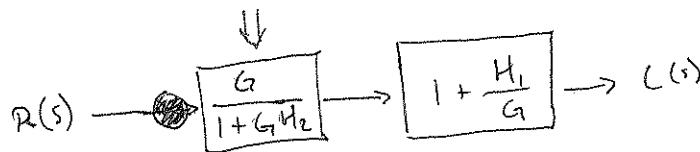
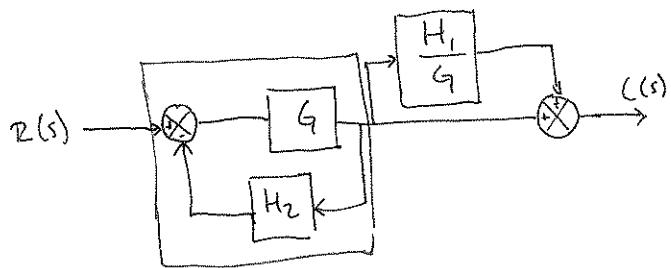
Now let's do something *yucky* (Ogata A-2-1)



$$X(s) = -G(s)H_2(s)x(s) + R(s)$$

$$\begin{aligned} C(s) &= G(s)x(s) + H_1(s)x(s) \\ &= x(s)(G(s) + H_1(s)) \end{aligned}$$

$$R(s) = \frac{1}{1 + G(s)H_2(s)}$$



$$\left( \frac{G}{1 + GH_2} \right) \left( \frac{G + H_1}{G} \right) = \frac{G + H_1}{1 + GH_2}$$