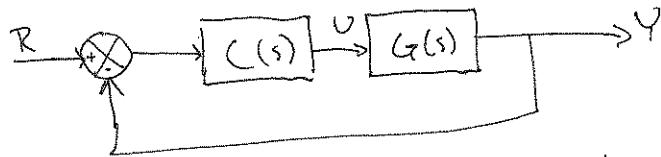
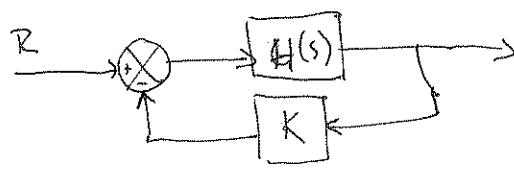


Root locus summary so far:

We have a feedback system:



This is how we've thought of controllers (esp PID so far), but we can redraw them in the form:



Combine $C(s)$ and $G(s)$, factor out some gain K of interest.

Note that $K = K_p$ for a pure proportional controller.

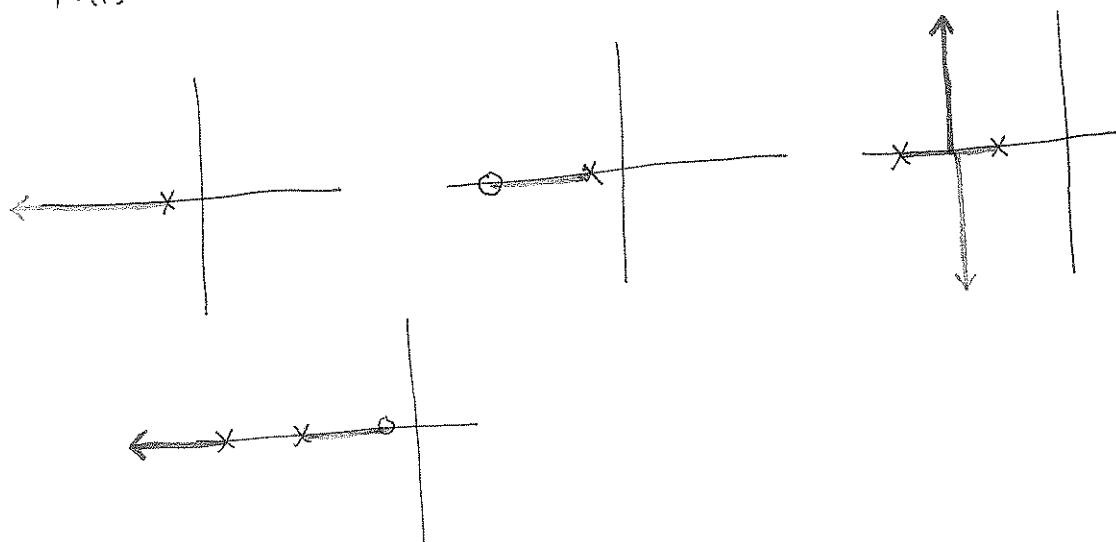
$$\frac{H(s)}{1 + KH(s)} \Rightarrow \text{roots of } 1 + KL(s)$$

Root locus plots poles of closed-loop (CL) system as K varies from 0 to ∞ .

Let there be n poles and m zeros.

- ① Locus branches start at poles of open-loop TF ($H(s)$), an m of them end on zeros. The rest go to infinity.
- ② Loci are on the real axis to the left of an odd number of poles/zeros.

This rule covers several scenarios:



Let's formulate #2 mathematically.

We have the equation
Use $L(s)$ to be consistent with textbooks.

$$1 + K H(s) = 0$$

Our approach so far is to pick K and solve for s .

But if we're thinking geometrically, it's easier to pick α 's,

and ask if some $K > 0$ makes the equation true.

We don't have to solve for K ; ~~only~~ proving its existence is good enough.

For $1 + K H(s) = 0$, and $K > 0$, $L(s)$ must be real and negative.

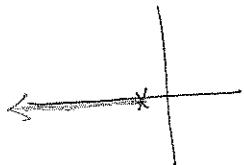
We don't care about the value; K can change.

We don't care about the value; $|L(s)|$ is irrelevant.

In other words, $\angle L(s) = 180^\circ$ (or π)

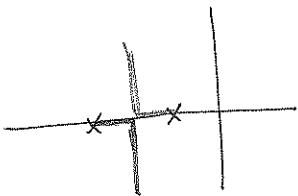
Consider $L(s) = \frac{1}{s+1}$

$L(s)$ is only real and negative if $s < -1$.



Let's try $L(s) = \frac{1}{(s+1)(s+2)}$

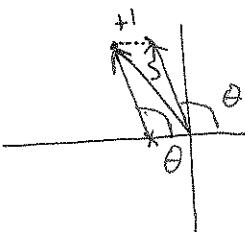
$L(s)$ is only real and negative is $-2 < s < -1$.



Said another way, $\angle L(s)$ only equals 180° ...

$$\angle L(s) = \angle \frac{1}{s+1} + \angle \frac{1}{s+2} \quad (\text{thanks to trig})$$

What is the phase of $\frac{1}{s+1}$?



$$A_n = \sigma_n + j\omega_n$$

$$A_1, A_2 = (\sigma_1, \sigma_2 - \omega_1, \omega_2) + j(\omega_1, \sigma_2 + \sigma_1, \omega_2)$$

$$\angle A_1 A_2 = \text{atan} \left(\frac{\omega_1, \sigma_2 + \sigma_1, \omega_2}{\sigma_1, \sigma_2 - \omega_1, \omega_2} \right) \cdot \text{divide by } \sigma_1, \sigma_2 \Rightarrow \text{atan} \left(\frac{\frac{\omega_1}{\sigma_1} + \frac{\omega_2}{\sigma_2}}{1 - \frac{\omega_1}{\sigma_1} \frac{\omega_2}{\sigma_2}} \right)$$

$$= \text{atan} \left(\frac{\omega_1}{\sigma_1} \right) + \text{atan} \left(\frac{\omega_2}{\sigma_2} \right) \quad \text{thanks to trig: } \text{atan}(x) + \text{atan}(y) = \text{atan} \left(\frac{x+y}{1-xy} \right)$$

Now that we're thinking in angles/phase, the next rule is easy

③ Loci radiate out at angles $\frac{\pm 180^\circ (2k+1)}{n-m}$

(Ogata)

FPE has a different
and less intuitive equation.

For 1 pole, $\rightarrow 180^\circ$

3 poles, $\rightarrow \pm 60^\circ, 180^\circ$

2 poles, $\rightarrow \pm 90^\circ$

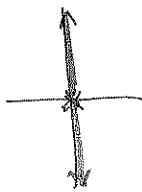
etc.

$$\frac{\sum p - \sum z}{n-m}$$

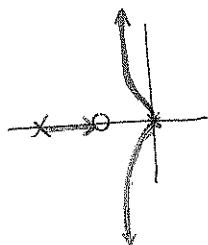
The center $\alpha =$

Let's try some examples!

$$\frac{1}{s^2}$$



$$\frac{s+1}{s^2(s+2)}$$



$$\frac{s^2 + 2s + 2}{s^2(s+2)}$$

