

Can we determine stability (under feedback/gain) using only the Bode plot?

③

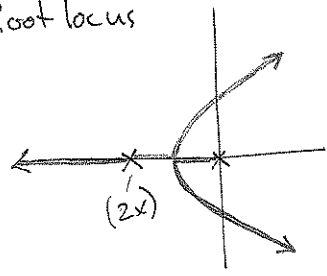
The pole will be on the  $j\omega$ -axis if  $|H(j\omega)| = 1$  when  $\angle = -180^\circ$ .

For a single crossing, pole is left of axis if  $|H(j\omega)| < 1$  when  $\angle = -180^\circ$  or phase  $> -180^\circ$  when  $|H(j\omega)| = 1$ .

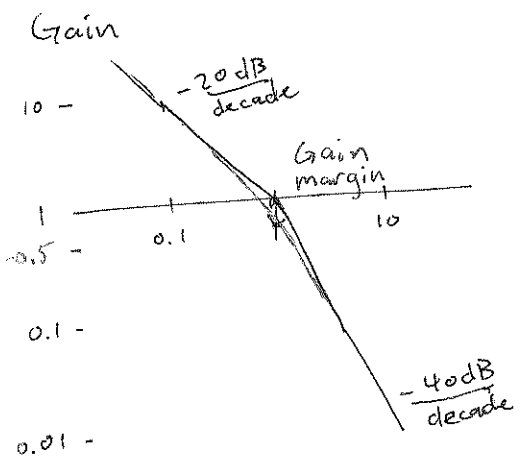
We can quantify "how close" we are to instability by looking at gain/phase margin

$H(s) = \frac{1}{s(s+1)^2}$  (following FPE)

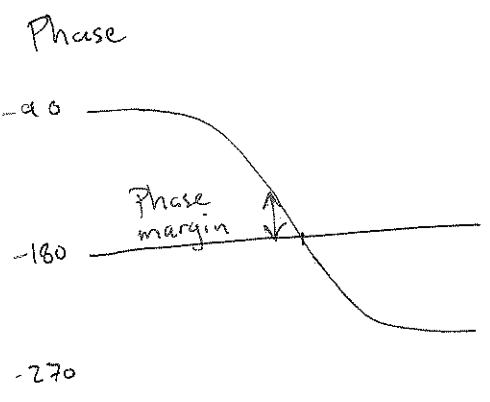
Root locus



This is stable if the gain is small.



Note what would happen if we increased the gain by 2x.



Why phase margin?

## Damping ratio and phase margin

Given the system

$$G = \frac{\omega_n}{s(s + 2\zeta\omega_n)}$$

which under unity feedback produces an overall TF of

$$H = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

Use Matlab to draw a Bode plot, and examine how the phase margin changes as a function of  $\zeta$ . To start, use a fixed  $\omega_n$  of 1.

If you have time, examine both the gain and phase margin, and consider dependance on  $\omega_n$ .

## Controllers and compensation

For each of the following controllers, make a Bode plot with Matlab and draw some conclusions about what it does in frequency-land. Does it filter/amplify certain frequencies? How does it affect the phase margin, and therefore the stability?

- Proportional control
- PD control
- PI control
- Lead compensation:  $\left(\frac{s+z}{s+p}\right)$  with  $z < p$  or  $\left(\frac{T_D s+1}{\alpha T_D s+1}\right)$ , with  $\alpha < 1$
- Lag compensation:  $\left(\frac{s+z}{s+p}\right)$  with  $z > p$  or  $\left(\alpha \frac{T_D s+1}{T_D s+1}\right)$ , with  $\alpha > 1$