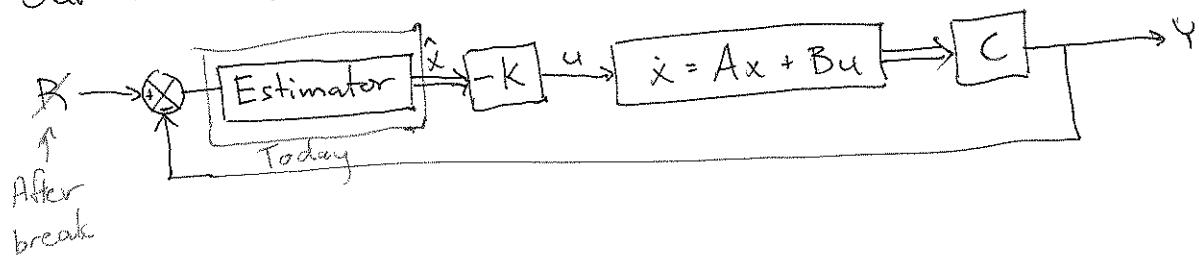


Our eventual model of state-space control:



Problem: given y , can we reliably estimate \hat{x} ?
 $\hat{x} - x$ goes to zero in a "short" time

Let's define the error: Warning! Ogata uses \tilde{x} for estimate,
 and 'e' for error.

$$\tilde{x} = x - \hat{x}$$

Our estimate evolves along with the plant:

$$\dot{\tilde{x}} = A\tilde{x} + Bu$$

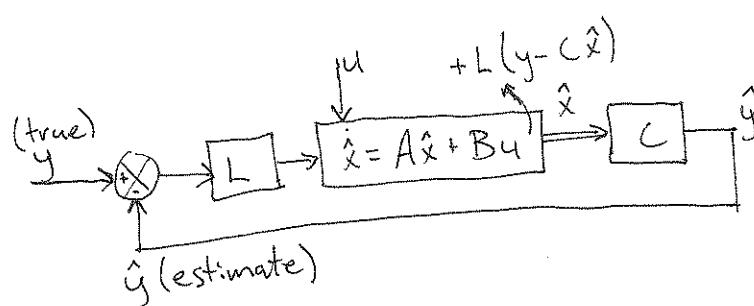
And so does the error:

$$\begin{aligned}\dot{\tilde{x}} &= \dot{x} - \dot{\hat{x}} \\ &= (Ax + Bu) - (A\hat{x} + Bu) \\ &= A(x - \hat{x}) \\ &= A\tilde{x}\end{aligned}$$

Which is a major bummer. We're totally at the mercy of the dynamics of A , which could even be unstable! only works if $\hat{x}(0) = x(0)$ and model of A is perfect.

Let's fix this with feedback! But how?

We can measure a reference input, y , and use the error relative to \hat{y} .



This looks eerily similar to the control problem...

Want to pick gain vector L such that \hat{y} converges to y as quickly as possible.

¹Ogata uses K_E , because it's a dual of K_C

How does \tilde{x} behave as a function of this new system?

$$\dot{\tilde{x}} = \dot{x} - \dot{\hat{x}}$$

$$= (Ax + Bu) - (A\hat{x} + Bu + L(y - (\hat{x}))$$

But $y = Cx$, so

$$\dot{\tilde{x}} = Ax - A\hat{x} - L(Cx - (\hat{x}))$$

$$= A(x - \hat{x}) - L(C(x - \hat{x}))$$

$$= (A - LC)\tilde{x}$$

So the error evolves according to $(A - LC)$, which we can (hopefully) control.

FPE Example 7.24 (dual of 7.14)

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ -w_0^2 & 0 \end{bmatrix}x + \begin{bmatrix} 0 \\ 1 \end{bmatrix}u \quad y = \begin{bmatrix} 1 & 0 \end{bmatrix}x$$

$$\text{Want to put the poles at } -w_0 \cdot 10 \quad (s + 10w_0)^2 = s^2 + 20w_0s + 100w_0^2$$

$$|sI - (A - LC)|$$

$$\left| \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -w_0^2 & 0 \end{bmatrix} + \begin{bmatrix} L_1 \\ L_2 \end{bmatrix} \begin{bmatrix} 1 & 0 \end{bmatrix} \right|$$

$$(s + L_1)s + (w_0^2 + L_2) \cdot 1 \\ s^2 + L_1s + L_2 + w_0^2 = s^2 + 20w_0s + 100w_0^2$$

$$L_1 = 20w_0$$

$$L_2 = 99w_0^2$$

For control:

$$A - BK$$

$$\begin{bmatrix} n \times n \\ n \times 1 \end{bmatrix} - \begin{bmatrix} n \times 1 \\ 1 \times n \end{bmatrix}$$

$$K = \text{acker}(A, B, \text{poles})$$

For estimation:

$$A - LC$$

$$\begin{bmatrix} n \times n \\ n \times 1 \end{bmatrix} - \begin{bmatrix} n \times 1 \\ 1 \times n \end{bmatrix}$$

$$L = \text{acker}(A^T, C^T, \text{poles})^T$$