# EE 105 Feedback control systems

## Modeling LTI systems

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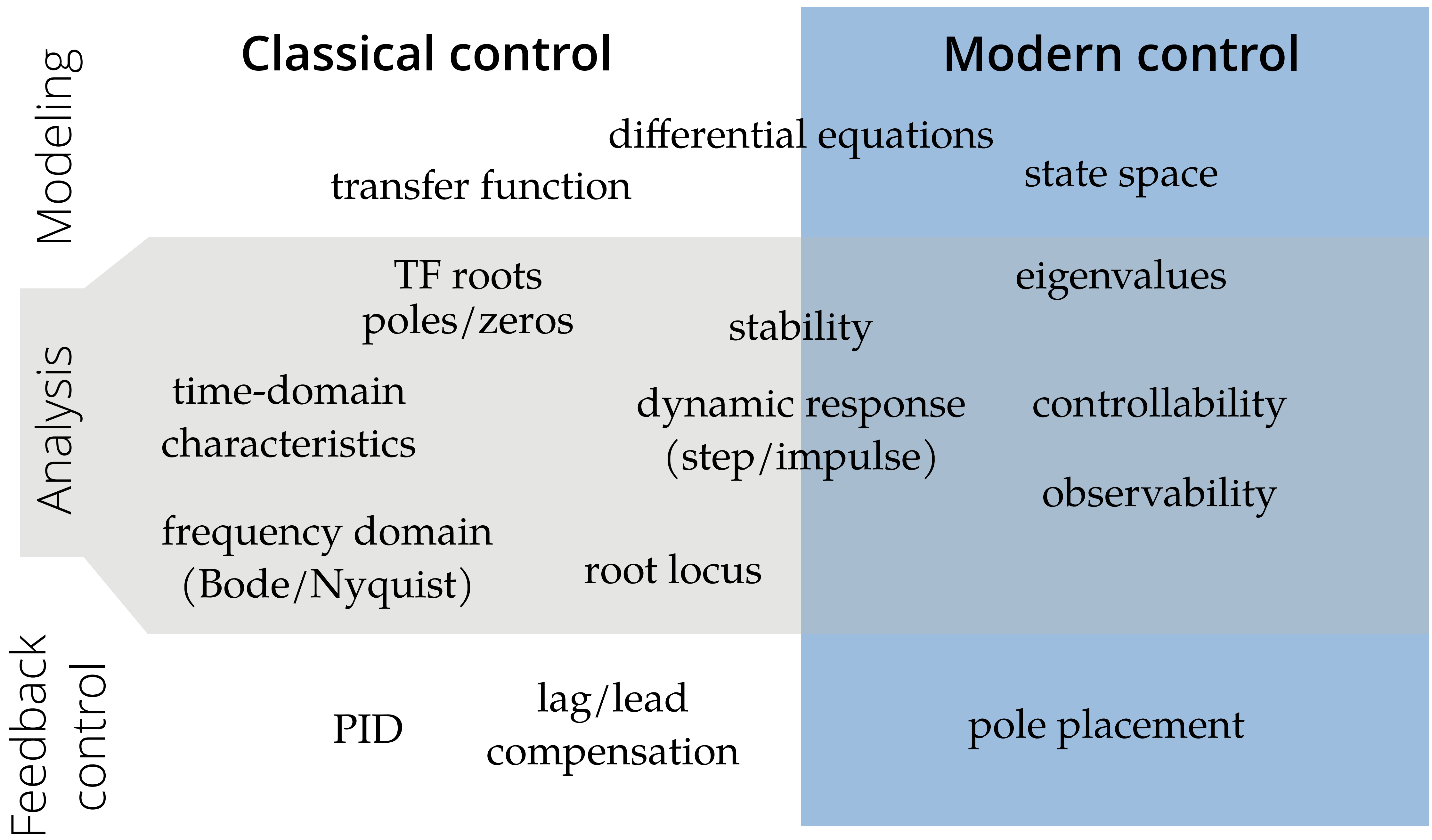
# By the end of class today, you should be able to:

- Write differential equations for a system with known dynamics
- Use the Laplace transform to find the transfer function
- Formulate a state-space matrix equation for a system


Classical control
Modern control
Modeling
differential equations
transfer function
state space
Analysis
TF roots
poles/zeros
eigenvalues
stability
time-domain
characteristics
dynamic response
(step/impulse)
controllability
observability
frequency domain
(Bode/Nyquist)
root locus
Feedback
control
PID
lag/lead
compensation
pole placement

# Where we are

LTI systems (last time)

Differential equations

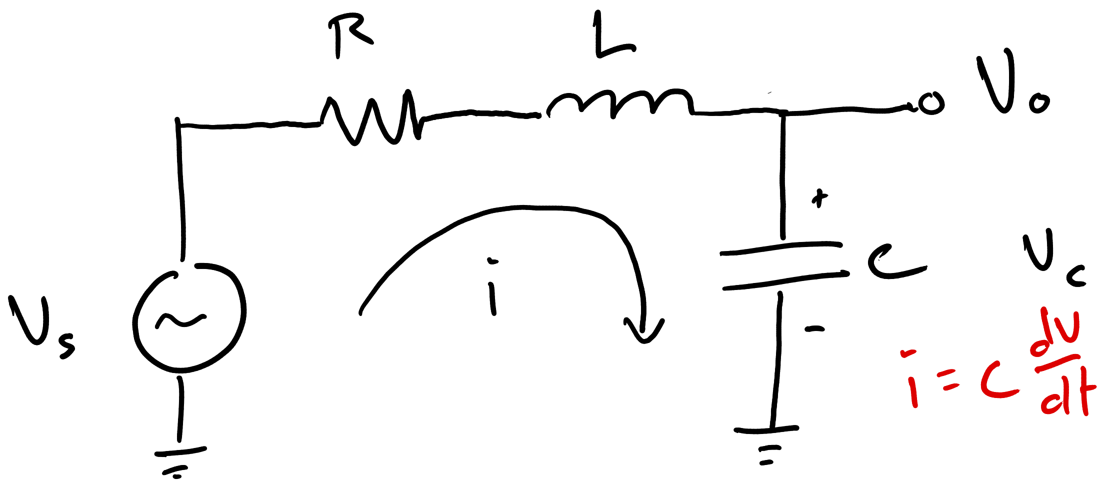
today

Transfer functions next week State space

wednesday

# Let's start with a circuit

$$v = L\frac{di}{dt}$$




$$i = C\frac{dv}{dt}$$

$$V_s = Ri + L\dot{i} + V_o$$

$$i = C\dot{V}_o$$

$$V_s = RC\dot{V}_o + LC\ddot{V}_o + V_o$$

# Refresh: Laplace transform

Converts a time-domain representation into the "s-domain"

Inverse Laplace transform goes back

Generalization of the Fourier transform

$$\int_{-\infty}^{\infty} f(t)\, e^{-\underline{j\omega} t}\, dt$$

$j\omega$ is imaginary

$$\int_{0}^{\infty} f(t)\, e^{-\underline{s} t}\, dt$$

$s$ is complex

1) Laplace is linear

$$\alpha f_1(t) + \beta f_2(t) \iff \alpha F_1(s) + \beta F_2(s)$$

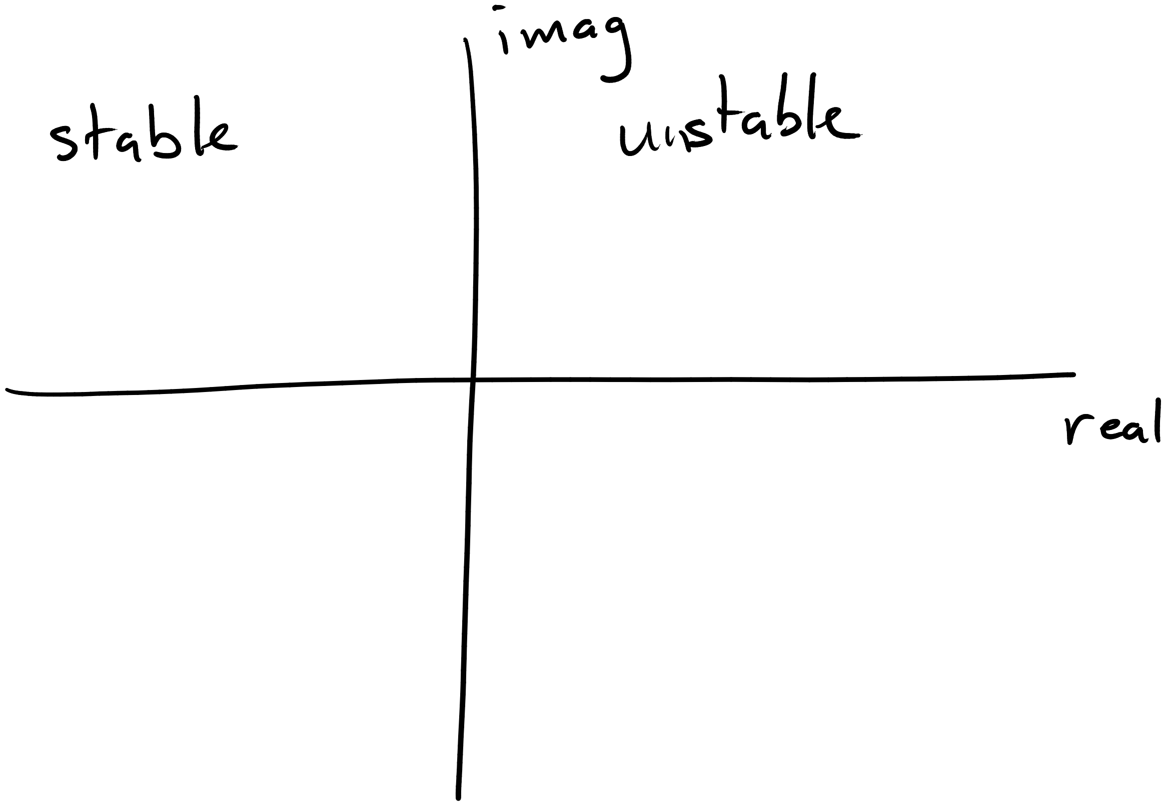
2 Derivatives in t become powers of s

$$f(t) \iff F(s)$$

$$\dot{f}(t) \iff sF(s)$$

$$\ddot{f}(t) \iff s^2 F(s) - s f(0)$$

S - plane (complex)



# Our circuit in Laplace space

$$V_s = LC\ddot{V}_o + RC\dot{V}_o + V_o$$

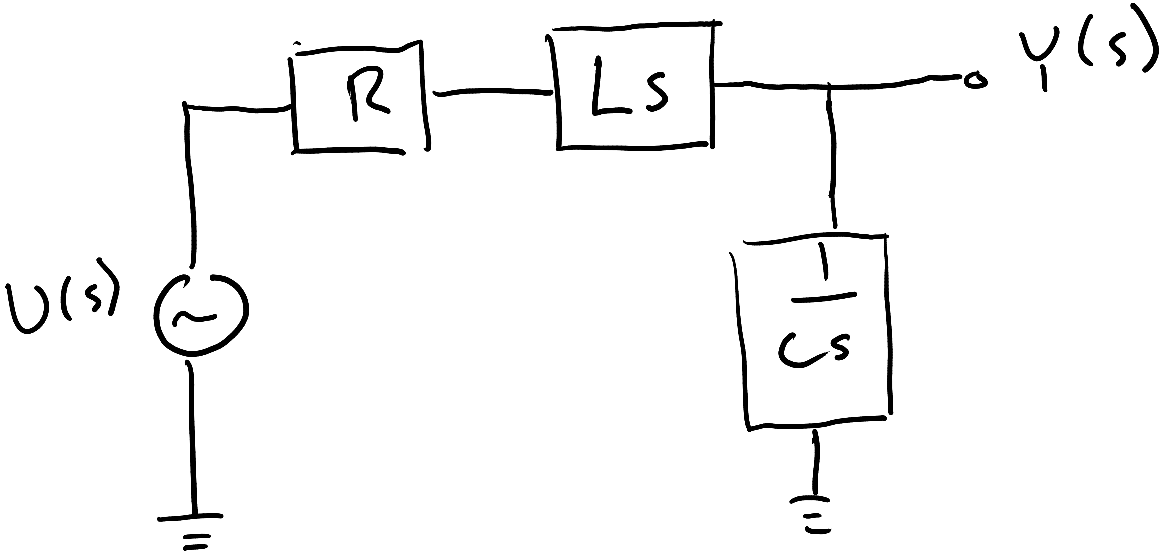
$$U(s) = (LCs^2 + RCs + 1)\,Y(s)$$

input

output

$$\frac{Y(s)}{U(s)} = \frac{1}{LCs^2 + RCs + 1}$$

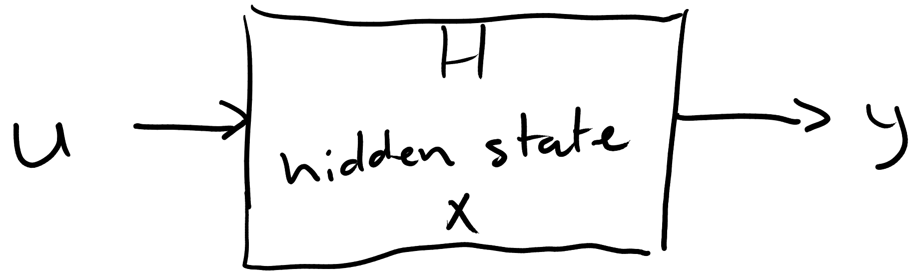
# Our circuit in Laplace space



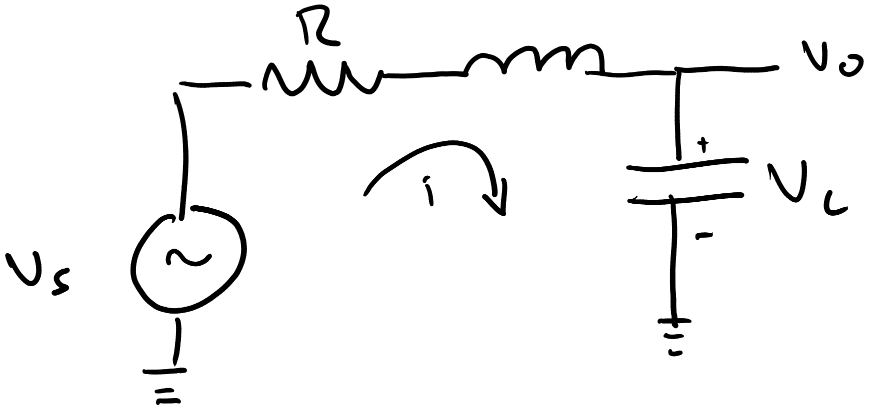

$$Y(s) = \frac{\frac{1}{Cs}}{R + Ls + \frac{1}{Cs}} \cdot U(s)$$

$$= \frac{1}{CLs^2 + CRs + 1} U(s)$$

# State space: another representation



# State space: another representation




$$V_s = iR + L\dot{i} + V_c$$

$$\dot{i} = -\frac{1}{L}V_c - \frac{R}{L}i + \frac{1}{L}V_s$$

state variable  input

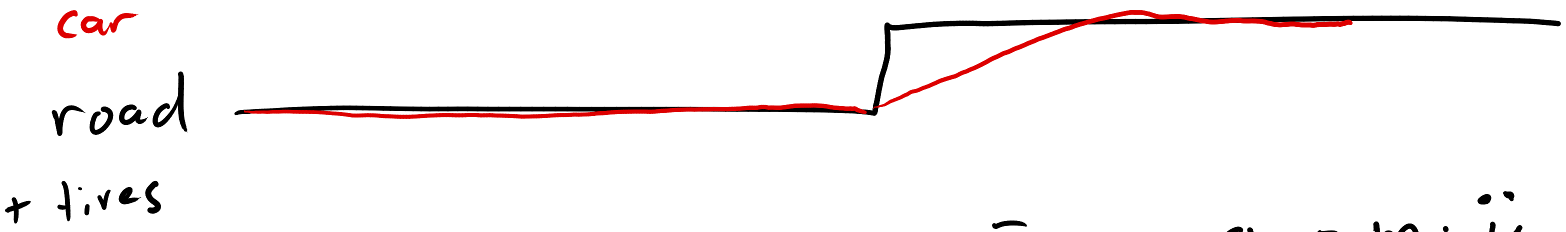
$$\dot{V}_c = \frac{1}{C}i \quad \left(\text{from } i = C\frac{dV}{dt}\right)$$

state variable

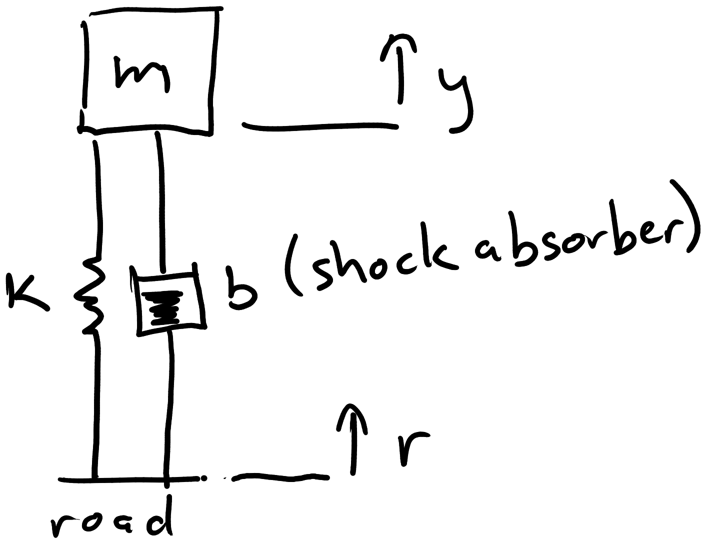
$$\dot{x} = [A]x + [B]u$$

$$\begin{bmatrix} \dot{i} \\ \dot{V}_c \end{bmatrix} = \underbrace{\begin{bmatrix} -\frac{R}{L} & -\frac{1}{L} \\ \frac{1}{C} & 0 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} i \\ V_c \end{bmatrix}}_{x} + \underbrace{\begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix}}_{B} V_s$$

FPE Example 7.2




$$F = ma = m \cdot \ddot{y} = -k(y-r) - b(\dot{y} - \dot{r})$$




$$m\ddot{y} = -ky + kr - b\dot{y} + b\dot{r}$$

$$m\ddot{y} + b\dot{y} + ky = b\dot{r} + kr$$

$$(ms^2 + bs + k)Y(s) = (bs + k)R(s)$$

$$\frac{Y(s)}{R(s)} = \frac{bs + k}{ms^2 + bs + k}$$