EE 105 **Feedback control systems**

Linear algebra for controls

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By the end of class today, you should be able to:

- Define the range of a matrix, "full rank", and invertability
- Explain why eigenvalues and eigenvectors are special
- Use MATLAB to compute and eigenvalue decomposition

What is a matrix?

$$A = \begin{array}{cc} & \xleftrightarrow{\quad n \quad} \\ m \updownarrow & \begin{bmatrix} \quad & \quad & \quad \\ \quad & \quad & \quad \\ \quad & \quad & \quad \end{bmatrix} \end{array}$$

Vector $x \in \mathbb{R}^n$

$$y = A x$$

y is in $\mathbb{R}^m$

What is special about state-transition matrices?

$$\dot{x} = A \underbrace{x}_{\text{current state}} + B \underbrace{u}_{\text{input}}$$

A is square! $(n \times n)$

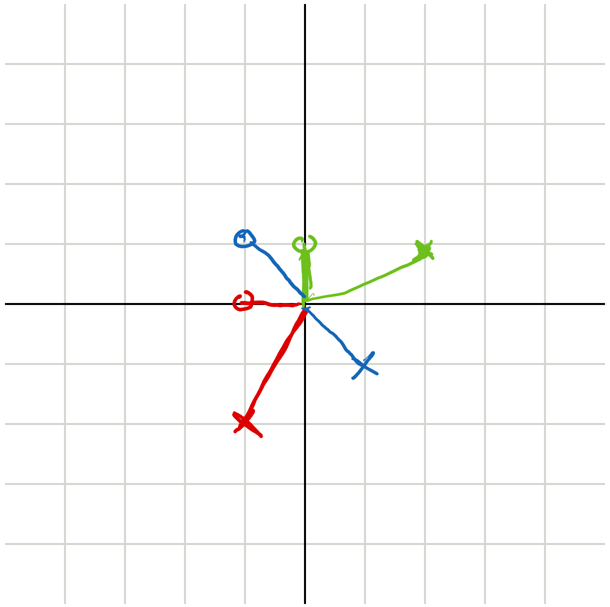
What does a matrix do to a vector?

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$$

$$x = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \qquad \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$x = \begin{bmatrix} -1 \\ 0 \end{bmatrix} \qquad Ax = \begin{bmatrix} -1 \\ -2 \end{bmatrix}$$

$$x = \begin{bmatrix} -1 \\ 1 \end{bmatrix} \qquad Ax = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$



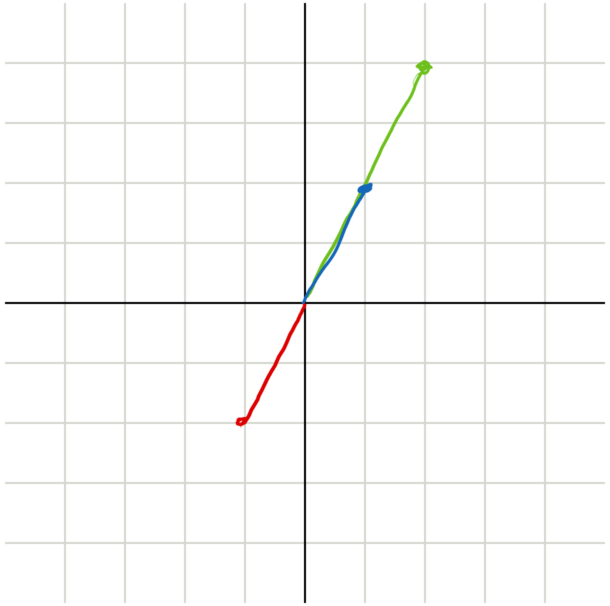
What about this matrix?

$A = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$

$x = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ $\quad Ax = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$

$x = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$ $\quad Ax = \begin{bmatrix} -1 \\ -2 \end{bmatrix}$

$x = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$ $\quad Ax = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$



Rank of a matrix

$y = Ax$ is always 0 $\quad$ rank $= 0$

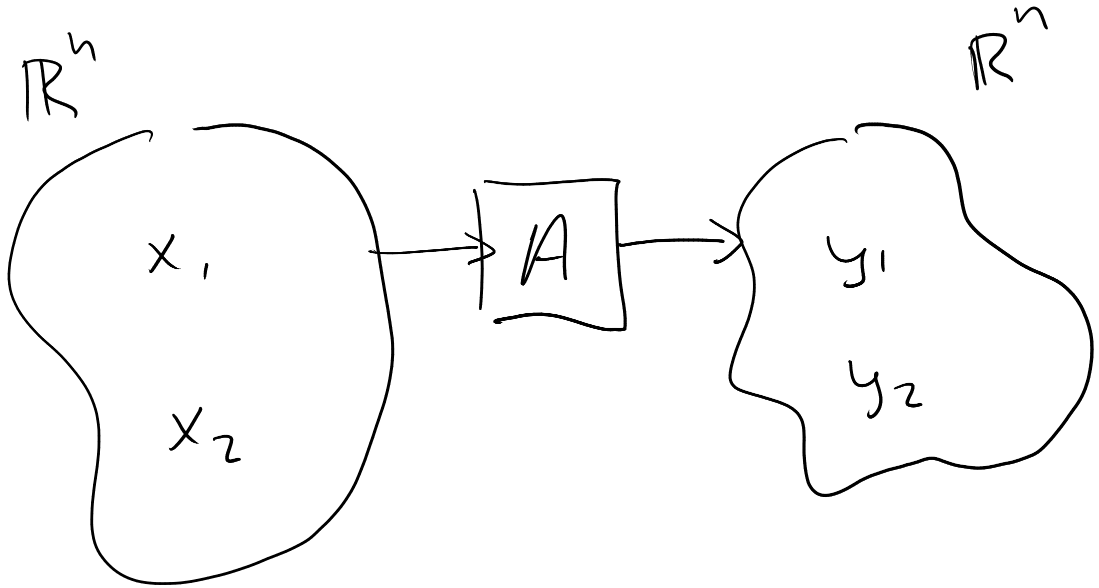
$y = Ax$ is in $\mathbb{R}^n$, where $n < m$

$y = Ax$ is anything in $\mathbb{R}^n$

Nullspace

The basis for vectors that give Ax=0

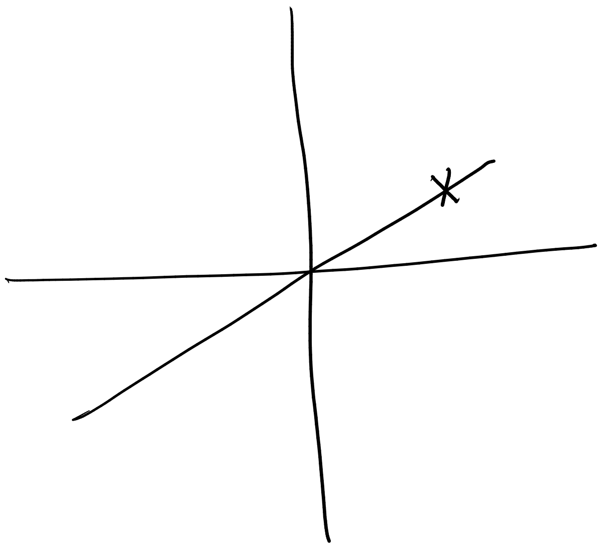
Invertibility



What is an eigenvector, again?

vector x so that

$Ax = \lambda x$ (scalar λ)



$A x = \lambda x$

$$A \begin{bmatrix} | & | \\ v_1 & v_2 \\ | & | \end{bmatrix} = \begin{bmatrix} \lambda_1 v_1 & \lambda_2 v_2 \end{bmatrix}$$

$$= \begin{bmatrix} | & | \\ v_1 & v_2 \\ | & | \end{bmatrix} \begin{bmatrix} \lambda_1 & \\ & \lambda_2 \end{bmatrix}$$

$$A = \begin{bmatrix} v_1 & v_2 \end{bmatrix} \begin{bmatrix} \lambda_1 & \\ & \lambda_2 \end{bmatrix} \begin{bmatrix} v_1 & v_2 \end{bmatrix}^{-1}$$

$$A = V D V^{-1} \quad (\text{assuming } V^{-1} \text{ exists})$$

MATLAB: exploring eigenvalues

What do the eigenvalues of a state-transition matrix A tell us about the behavior of the system?