EE 105 Feedback control systems

Pole locations and system response

Steven Bell

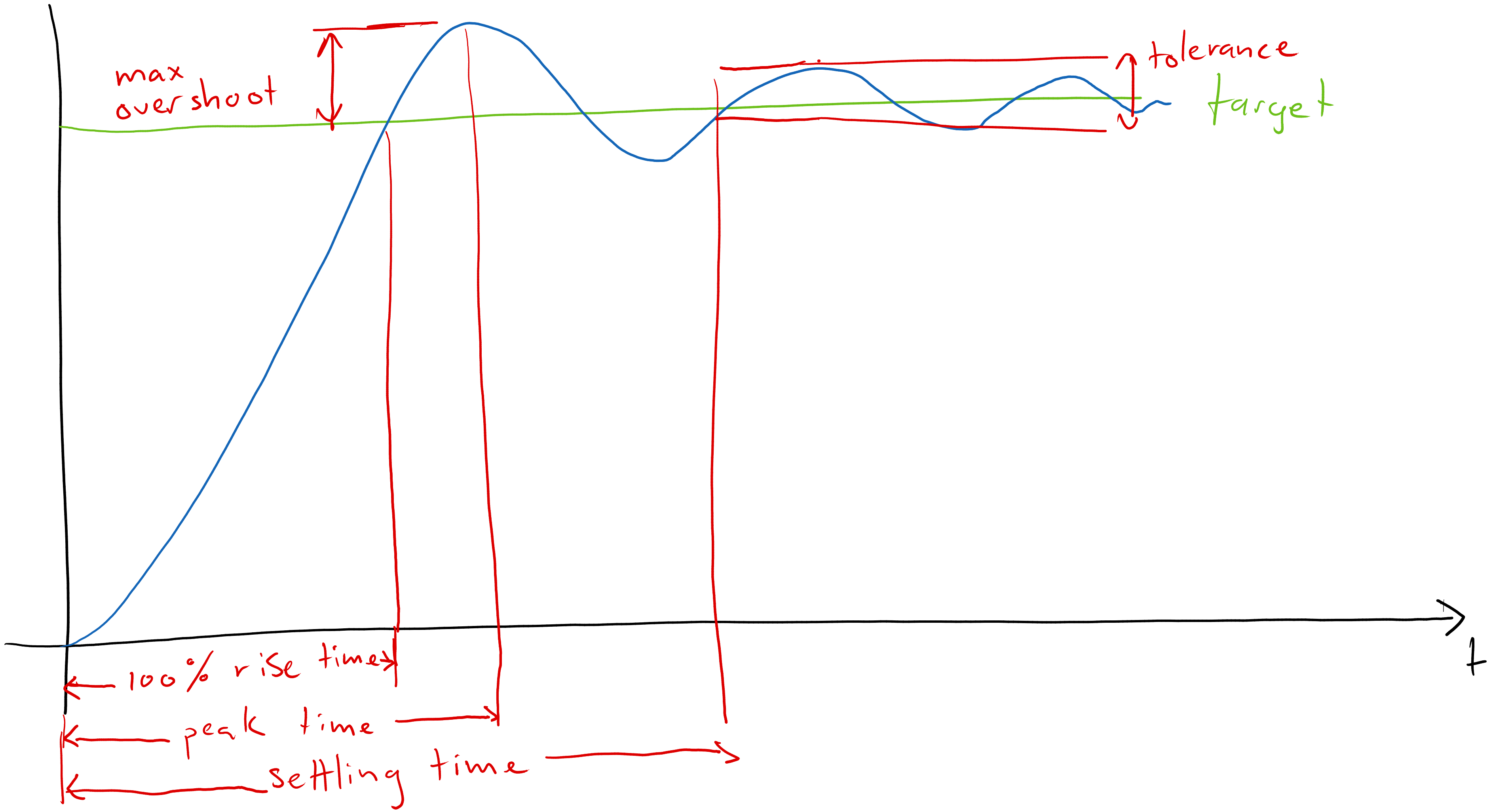
25 September 2024

Tufts UNIVERSITY

By the end of class today, you should be able to:

- Explain how ω and ζ describe a second-order system
- Specify desirable characteristics of a system in terms of is pole locations

Desirable response characteristics



Let's work with a 2nd-order system

$$H(s) = \frac{k}{s^2 + bs + c}$$

$$\omega_n = \sqrt{c}$$

$$\zeta = \frac{b}{2\omega_n}$$

Denominator

$$s^2 + 2\zeta\omega_n s + \omega_n^2$$

roots are

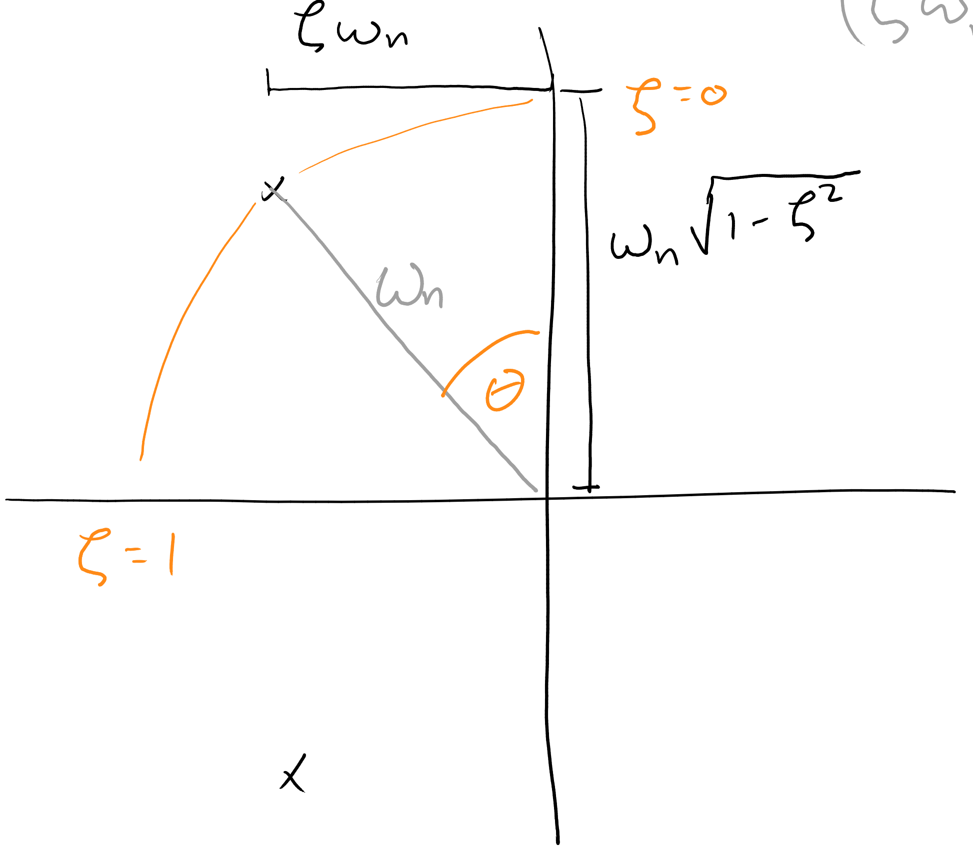
$$\frac{-2\zeta\omega_n \pm \sqrt{(2\zeta\omega_n)^2 - 4\omega_n^2}}{2} \Rightarrow -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1}$$

$$= \underbrace{-\zeta\omega_n}_{\text{real}} \pm \underbrace{j\omega_n\sqrt{1-\zeta^2}}_{\text{imaginary}}$$

Let's work with a 2nd-order system

$$(\zeta\omega_n)^2 + \left(\omega_n\sqrt{1-\zeta^2}\right)^2 = \zeta^2\omega_n^2 + \omega_n^2 - \omega_n^2\zeta^2$$

e^{st}



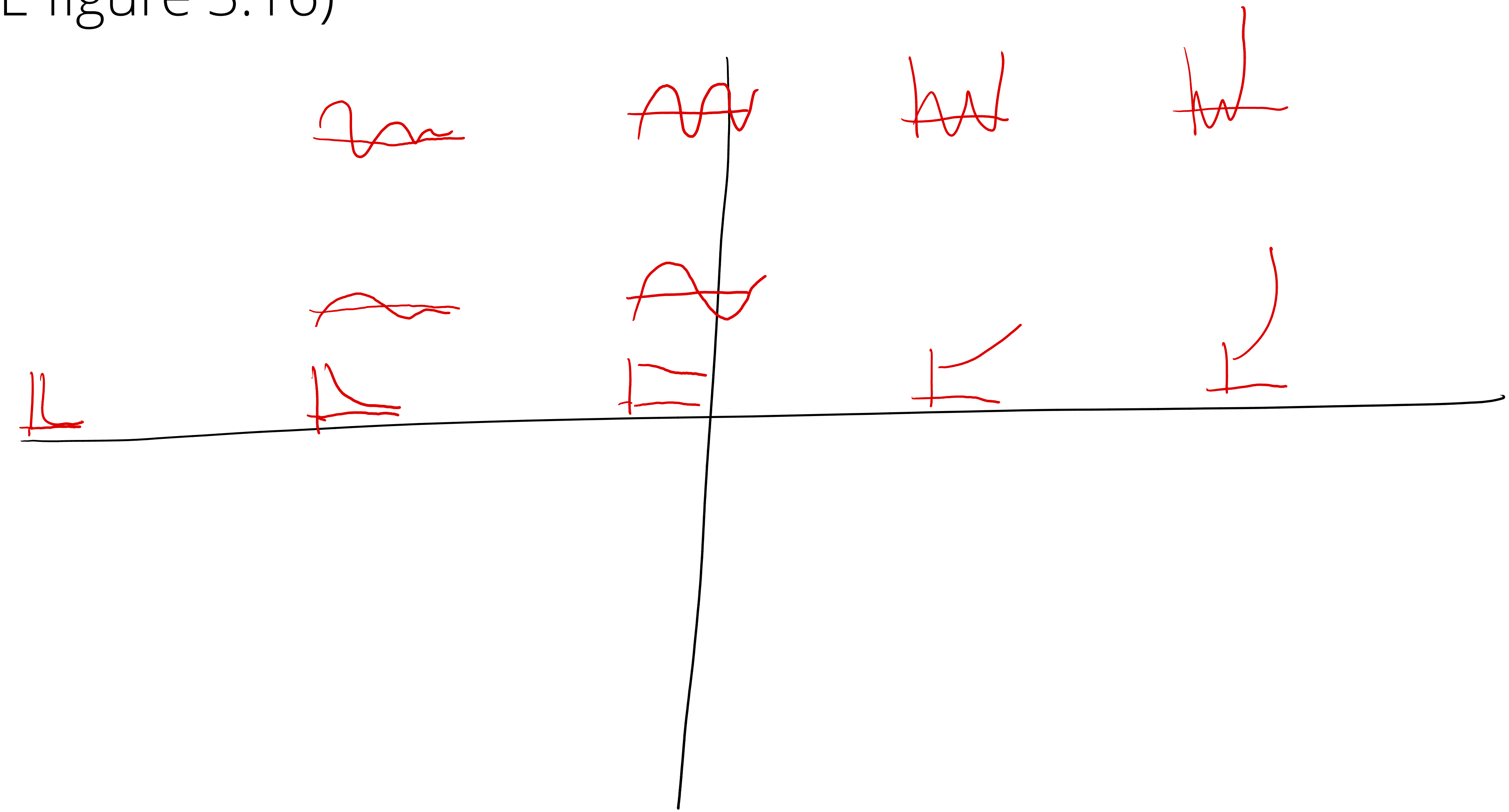
$$\theta = \operatorname{asin}\left(\frac{\zeta\omega_n}{\omega_n}\right) = \operatorname{asin}(\zeta)$$

Let's plot this for some values

(FPE figure 3.19)

"Commit this to memory"

(FPE figure 3.16)



Back to our specs: Rise time

$$t_r = \frac{1.8}{\omega_n}$$

Back to our specs: Rise time

Back to our specs: Settling time