EE 105 Feedback control systems

Root locus

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Root locus

In geometry, a **locus** (plural: loci)
(Latin word for "place", "location")
is a **set of all points**
(commonly, a line, a line segment, a curve or a surface),
whose location satisfies or is determined by
one or more specified conditions.

Root locus

A plot showing where the roots of the characteristic equation
(aka poles of the system) are, as a function of gain (or other parameter).

Who cares?

We're going to do some mathematical gymnastics,
and then discover some rules for plotting this easily.

1) If a time machine ever takes you back to before 1980, you will still be able to build a control system!

2) You'll be able to tell when MATLAB is giving you nonsense

3) You'll have some tools for thoughfully picking a controller

Let's start with proportional control

Plant $P(s)$, controller $C(s)$, le's use $C(s) = k_p$

Overall TF is

$$H(s) = \frac{k_p P(s)}{1 + k_p P(s)} \Rightarrow 1 + K L(s) = 0$$

scalar

sometimes easier to say

$$L(s) = \frac{b(s)}{a(s)} \qquad 1 + K \frac{b(s)}{a(s)} = 0$$

$$a(s) + K b(s) = 0$$

Let's start with proportional control

Let $P(s) = \frac{1}{s(s+c)}$

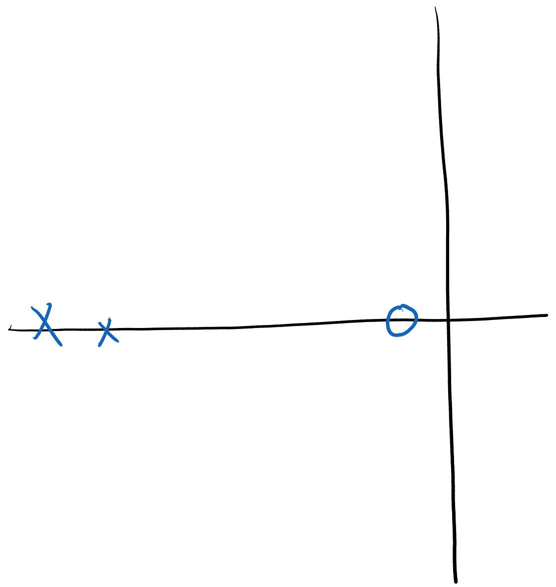
What about $\frac{1}{s(s+10)}$

$$\frac{1}{s^2}$$

$$\frac{1}{s^2+s+1}$$

Let's start with proportional control

$$\frac{s+1}{(s+10)(s+12)}$$



Why does this have steady-state error?

$$H(s) = \frac{\frac{s+1}{(s+10)(s+12)} K}{1 + \frac{s+1}{(s+10)(s+12)} K}$$

DC / steady-state $s=0$

$$= \frac{(s+1)K}{(s+10)(s+12)+(s+1)K} \Rightarrow \frac{1}{121}$$

$$\Rightarrow \frac{1+K}{120+K}$$

Let's start with proportional control

PI control

$$C(s) = K_p + \frac{1}{s} K_i$$

$$L(s) = \left(1 + \frac{K_i}{K_p} \cdot \frac{1}{s}\right) P(s)$$

Let's try to nail down the definition