EE 105 Feedback control systems

Rules for plotting root locus

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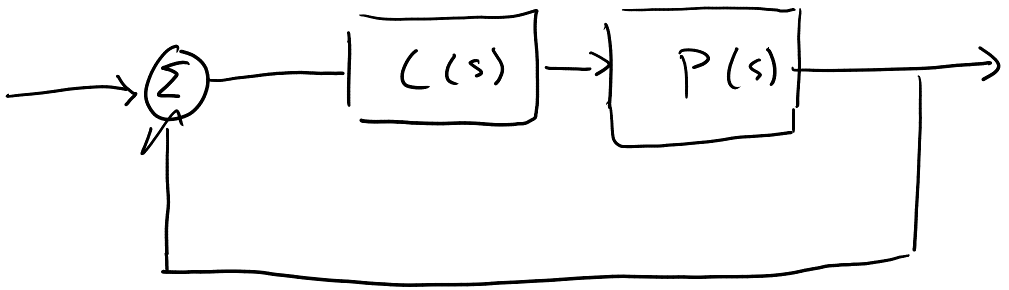
By the end of class today, you should be able to:

- Calculate the phase of a transfer function
- Sketch the asymptotes of a root locus (rules 1-3 in the book)
- Predict qualitatively what would happen to the root locus with the addition of poles or zeros

Root locus so far



$$H(s) = \frac{C(s)\,P(s)}{1 + C(s)\,P(s)}$$

$$\Downarrow$$

$$1 + K\,L(s) = 0$$

gives poles of $H(s)$

Pick a point on the s-plane

Evaluate $L(s)$

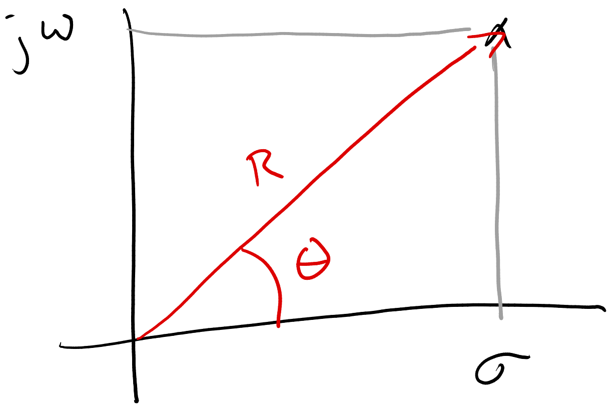
if $L(s)$ is real + negative,

then real + positive K exists ⇒

point is on root locus

Phase of L(s) !?

$$L(s) = \frac{s+2}{s(s+4)}$$



$$R = \sqrt{\sigma^2 + \omega^2}$$

$$\theta = \text{atan}\left(\frac{\omega}{\sigma}\right)$$

$$\sigma = R\cos(\theta)$$

$$\omega = R\sin(\theta)$$

Multiply complex numbers

$$A_1 A_2 = (\sigma_1 + j\omega_1)(\sigma_2 + j\omega_2)$$

$$= \sigma_1\sigma_2 + \sigma_1 j\omega_2 + \sigma_2 j\omega_1 + j^2\omega_1\omega_2$$

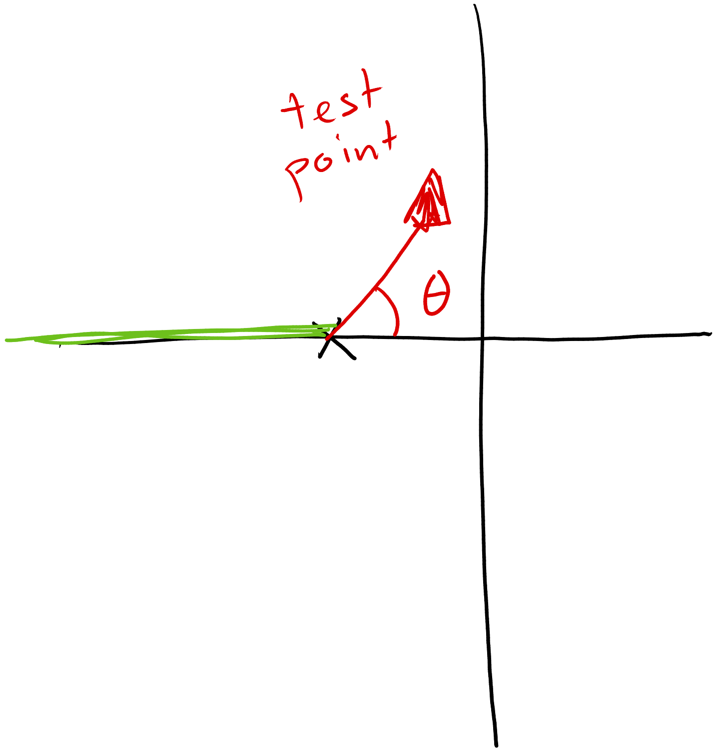
$$= \underbrace{(\sigma_1\sigma_2 - \omega_1\omega_2)}_{\text{real } \sigma} + j\underbrace{(\sigma_1\omega_2 + \sigma_2\omega_1)}_{\text{imag } \omega}$$

$$\theta = \text{atan}\left(\frac{\sigma_1\omega_2 + \sigma_2\omega_1}{\sigma_1\sigma_2 - \omega_1\omega_2}\right)$$

$$\theta = \text{atan}\left(\frac{\cos(\theta_1)\sin(\theta_2) + \cos(\theta_2)\sin(\theta_1)}{\cos(\theta_1)\cos(\theta_2) - \sin(\theta_1)\sin(\theta_2)}\right)$$

$$\theta = \text{atan}\left(\frac{\sin(\theta_1 + \theta_2)}{\cos(\theta_1 + \theta_2)}\right) = \theta_1 + \theta_2$$

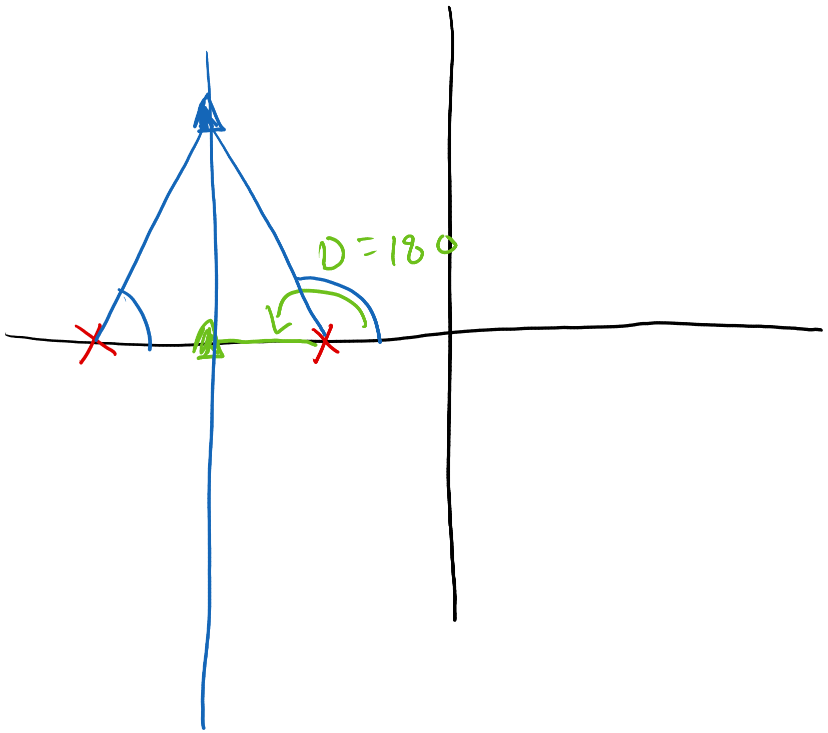
Calculating the phase of a transfer function



$$\frac{1}{s+2}$$

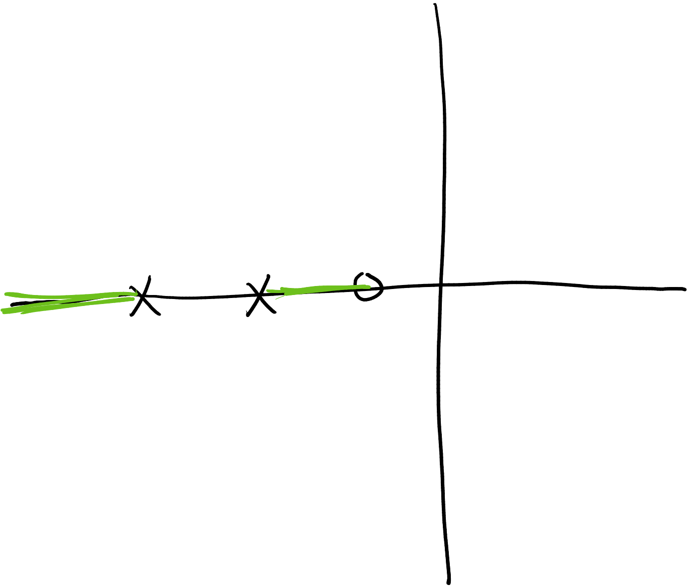
Calculating the phase of a transfer function

$$\frac{1}{(s+2)(s+4)}$$

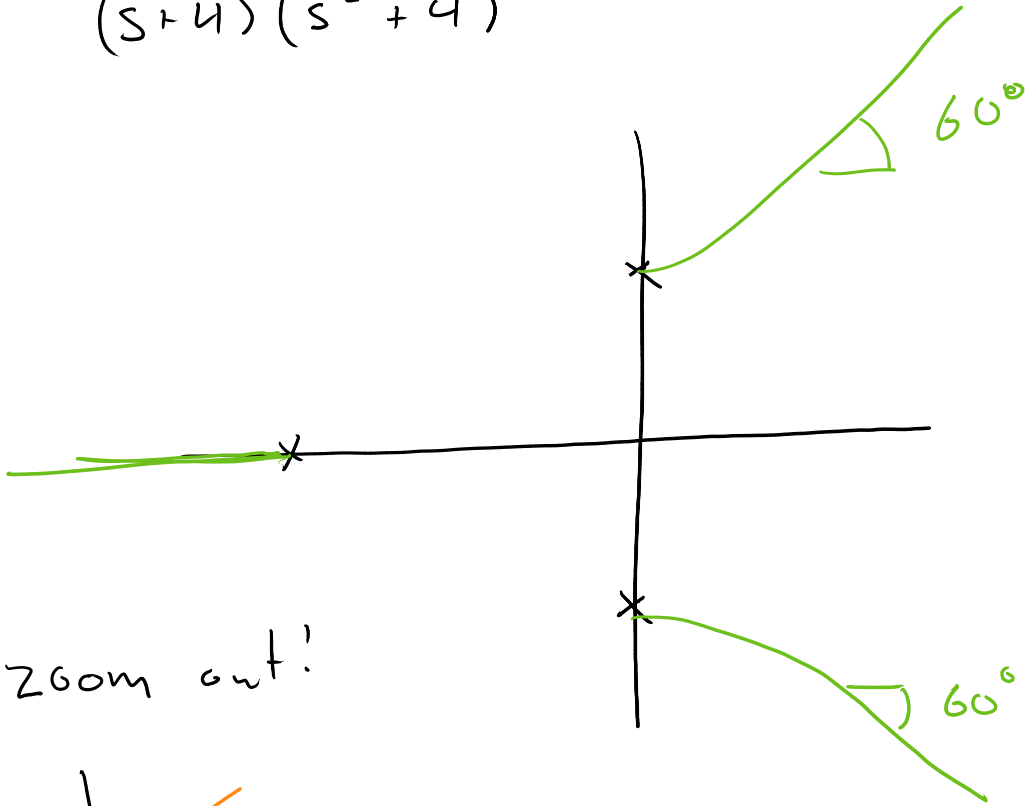


Calculating the phase of a transfer function

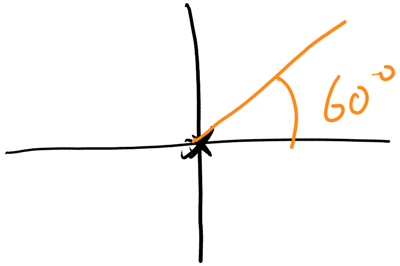
$$\frac{s+1}{(s+2)(s+4)}$$



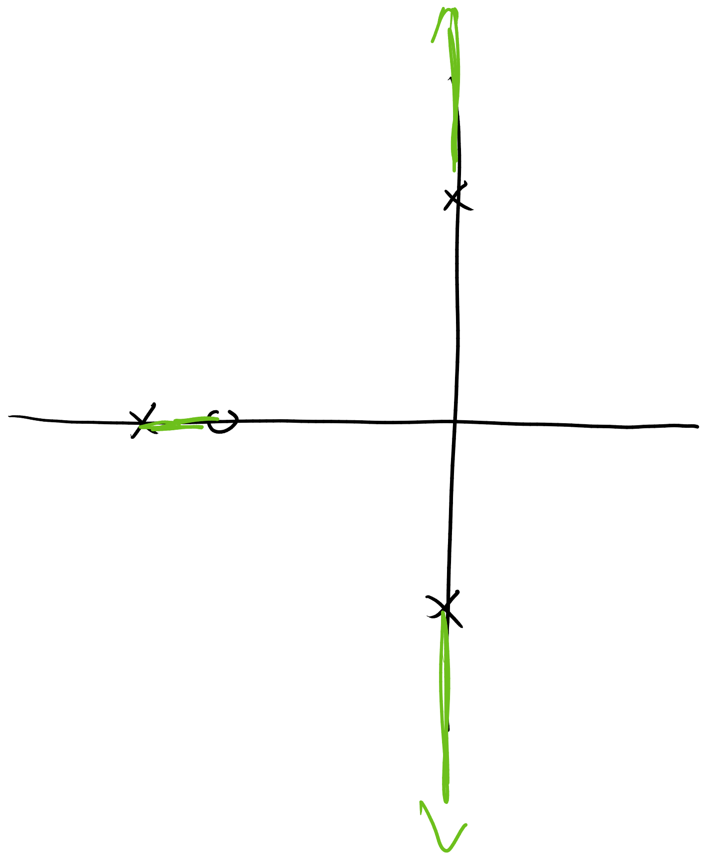
$$\frac{1}{(s+4)(s^2+4)}$$



zoom out!



Calculating the phase of a transfer function



Rule 1: (from last time)

The branches of the locus start on the poles

m of them will end on the m zeros.
The rest go off to infinity.

Rule 2:

The loci are on the real axis when **left** of an **odd number** of poles/zeros.

Rule 3

When K gets big, the loci radiate out following…

Let's experiment!

Write a TF with 3 poles.
Sketch its root locus, and then plot with MATLAB.