EE 105 Feedback control systems

Frequency response & Bode plots

Steven Bell

21 October 2024

Tufts UNIVERSITY

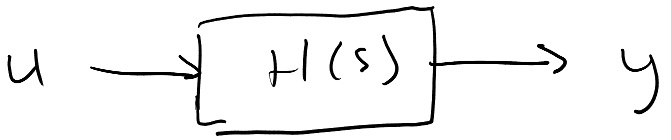
Why frequency response?

Easy to measure, even without knowing poles/zeros

We already have a toolbox for thinking about frequency!

Drawing a Bode plot

Some system $H(s)$

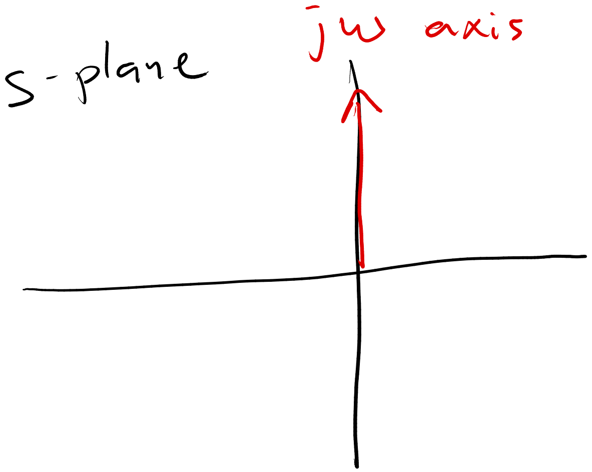


Let u be a sine wave

y is a wave of the same freq

with gain + phase shift

S-plane



Drawing a Bode plot

Let $H(s) = \frac{s+1}{s(s+100)} \Rightarrow \frac{j\omega + 1}{j\omega(j\omega + 100)}$

$\omega = 0, \quad |H(j\omega)| \to \infty$

$\omega = \infty, \quad |H(j\omega)| \to 0$

Factor out 100

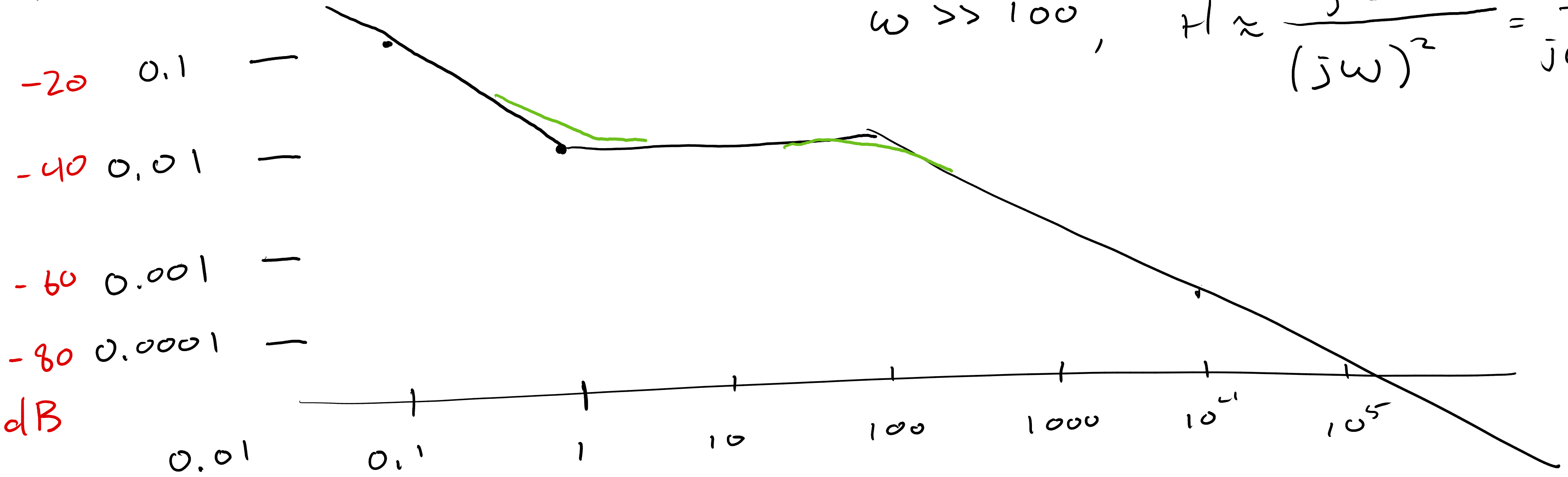
$\frac{1}{100} \cdot \frac{1}{j\omega} \cdot \frac{j\omega + 1}{\frac{j\omega}{100} + 1}$

$|H(1)| \approx \frac{1}{100}$

For $\omega \ll 1$, $H \approx \frac{1}{100 \cdot j\omega}$

$1 < \omega < 100$, $H \approx \frac{j\omega}{100 \cdot j\omega} = \frac{1}{100}$

$\omega \gg 100$, $H \approx \frac{j\omega}{(j\omega)^2} = \frac{1}{j\omega}$



dB is Power, $P = IV$, $I = \frac{V}{R} \Rightarrow P = \frac{V^2}{R}$ $\quad 10 \cdot \log_{10}(P) = 10 \cdot \log_{10}(V^2) = 20 \log_{10}(V)$

Drawing a Bode plot

Let $H(s) = \dfrac{s+100}{s(s+1)}$

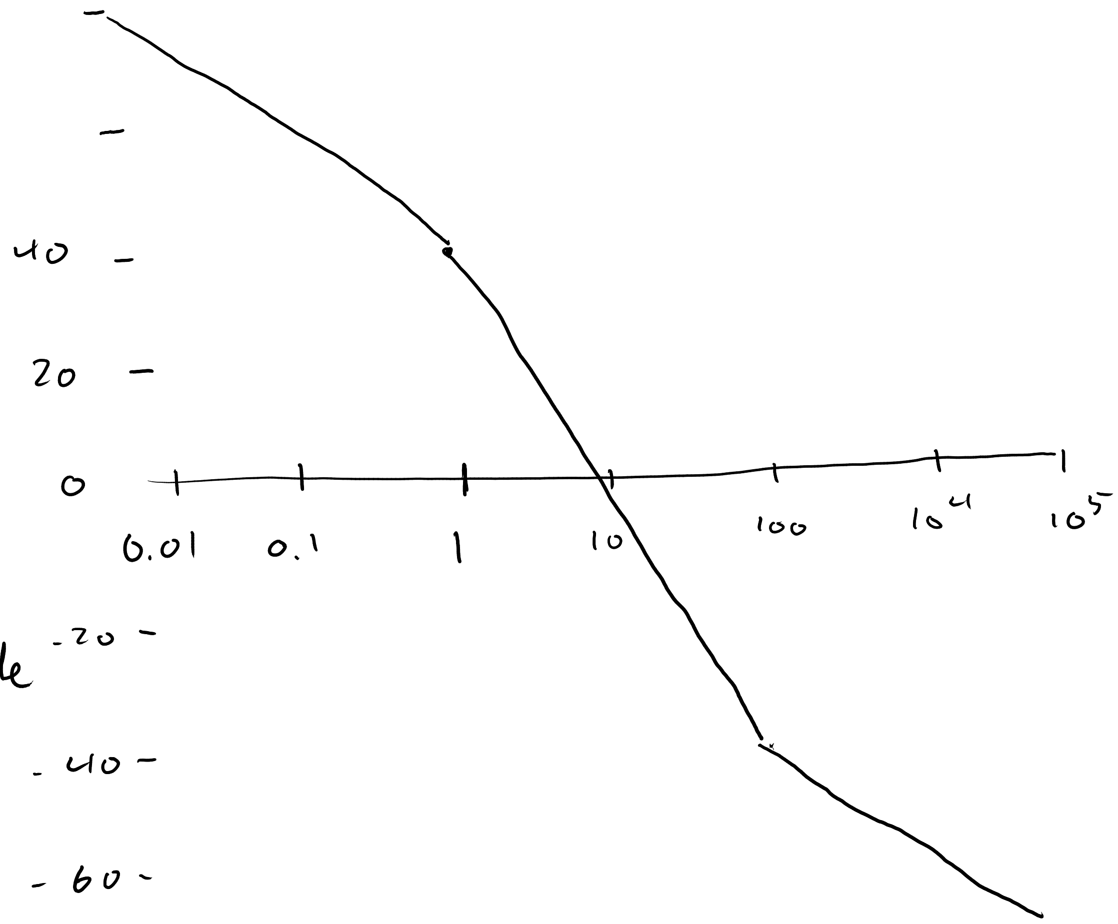
$$H(s) = 100 \frac{1}{j\omega} \frac{\frac{j\omega}{100}+1}{j\omega+1}$$

$|H(1)| \approx 100 = 40\text{dB}$

for $\omega \ll 1$, $H \approx \dfrac{100}{j\omega} \Rightarrow -20\,\text{dB/decade}$

for $1 < \omega < 100$ $H \approx \dfrac{100}{(j\omega)^2} \Rightarrow -40\,\text{dB/decade}$

for $\omega \gg 100$, $H \approx \dfrac{1}{j\omega} \Rightarrow -20\,\text{dB/decade}$



Let's experiment!

Write a TF with 3 terms (poles and/or zeros)
Sketch its Bode plot, and then plot with MATLAB.

Relationship to damping ratio?