EE 105 Feedback control systems

Control in state space: tracking the input

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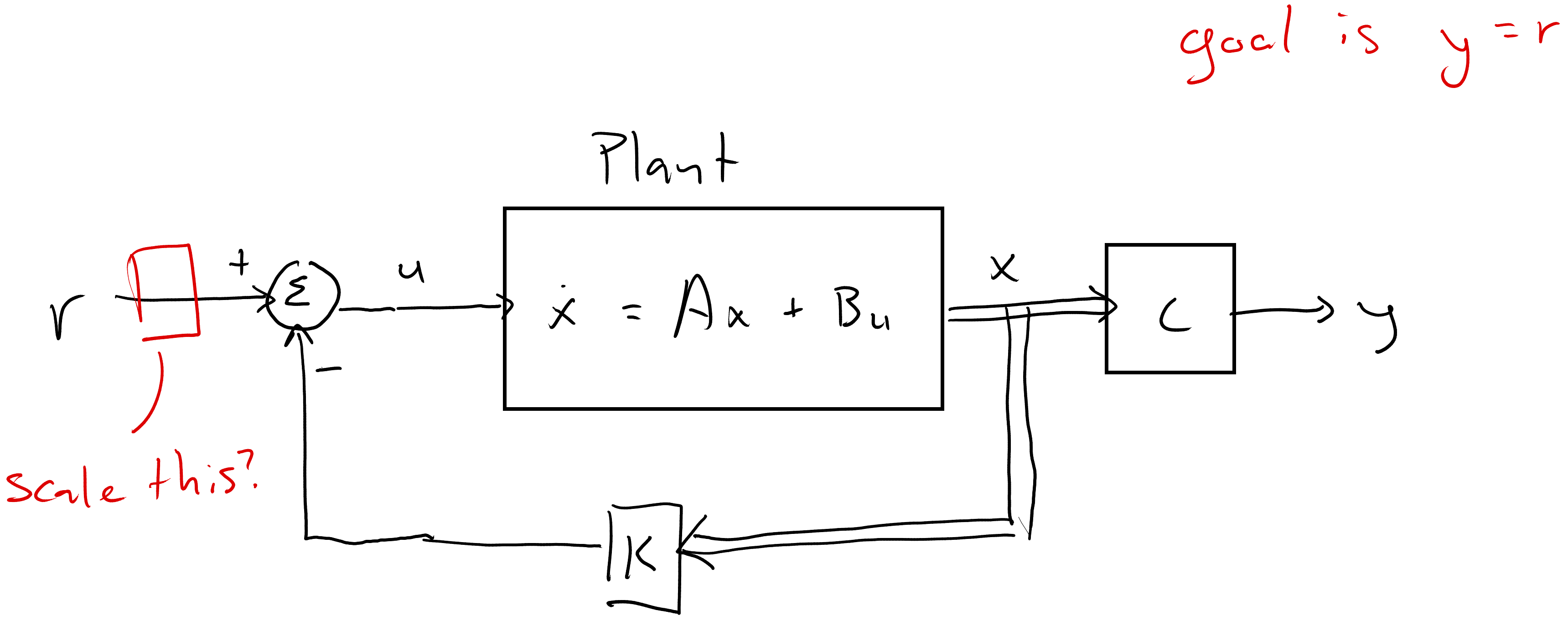
Control in state space

We can pick K to put the poles anywhere we want

- Subject to the actual gains we can apply
- And assuming the system is controllable



Let's try this with a step input in MATLAB

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If we simply add u = -**Kx** + r, there will be steady-state error

In TF-land, we would increase k_p or add an integral term

- But in state space, the poles are already where we want them...

Could we just scale the input?

plant state, plant input

$$\dot{x} = 0 = A x_{ss} + B u_{ss}$$

$$y_{ss} = r_{ss} = C x_{ss} + D u_{ss}$$

output tracks reference

$$\begin{bmatrix} 0 \\ r_{ss} \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} x_{ss} \\ u_{ss} \end{bmatrix}$$

factor out r_{ss}

$$\underbrace{\begin{bmatrix} 0 \\ 1 \end{bmatrix}}_{b} = \underbrace{\begin{bmatrix} A & B \\ C & D \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} \frac{x_{ss}}{r_{ss}} \\ \frac{u_{ss}}{r_{ss}} \end{bmatrix}}_{x} \Rightarrow \begin{bmatrix} N_x \\ N_u \end{bmatrix}$$

$$x = A \backslash b$$

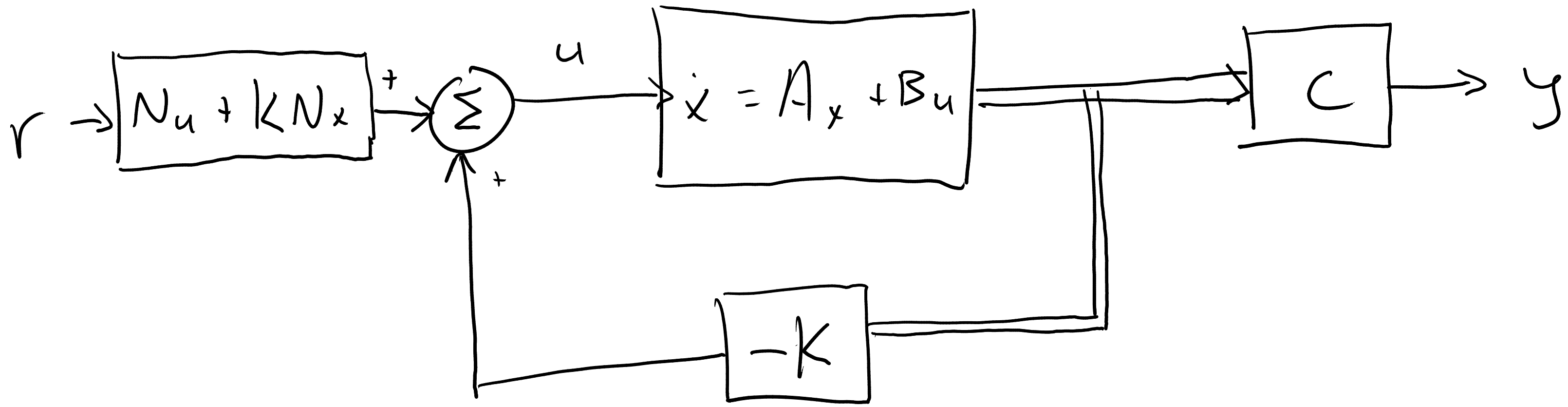
Using MATLAB

Previously, $u = -Kx + r$

We still have $\dot{x} = Ax + Bu$

But now $u = N_u \cdot r - K(x - N_x r)$

$u = -Kx + r(N_u + KN_x)$



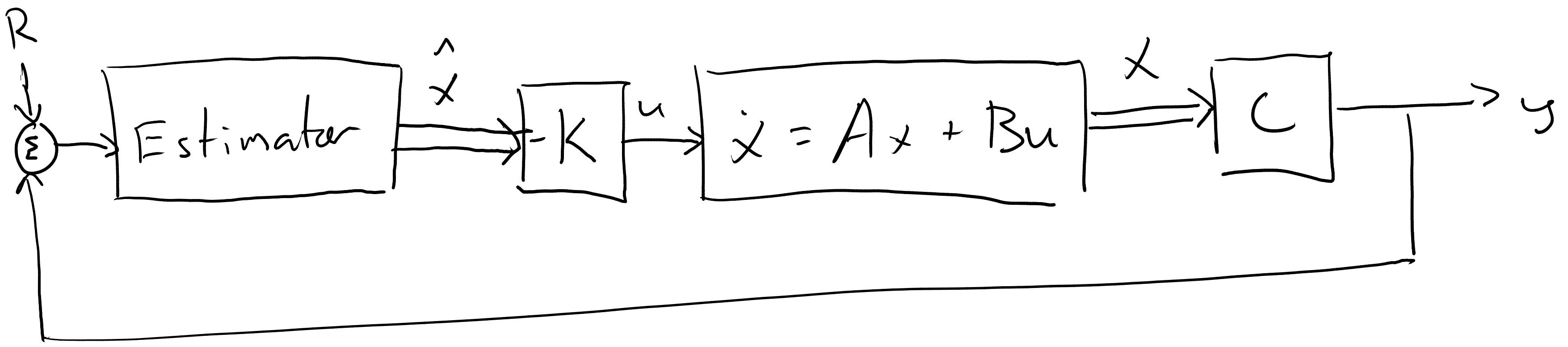
Where to put the poles?

Option 1: Pick a couple of dominant poles, push the others left

Option 2: LQR (more on this later)

But what happens when we don't know x?

How can we build a feedback system if we don't know the state?



Error $\tilde{x} = x - \hat{x}$

Estimate $\dot{\hat{x}} = A\hat{x} + Bu$

$$\dot{\tilde{x}} = \dot{x} - \dot{\hat{x}}$$

$$= (Ax + \sout{Bu}) - (A\hat{x} + \sout{Bu})$$

$$= A(x - \hat{x})$$

$$\dot{\tilde{x}} = A\tilde{x}$$

But what happens when we don't know x?

How can we build a feedback system if we don't know the state?