EE 105 Feedback control systems

Control in state space: using an estimator

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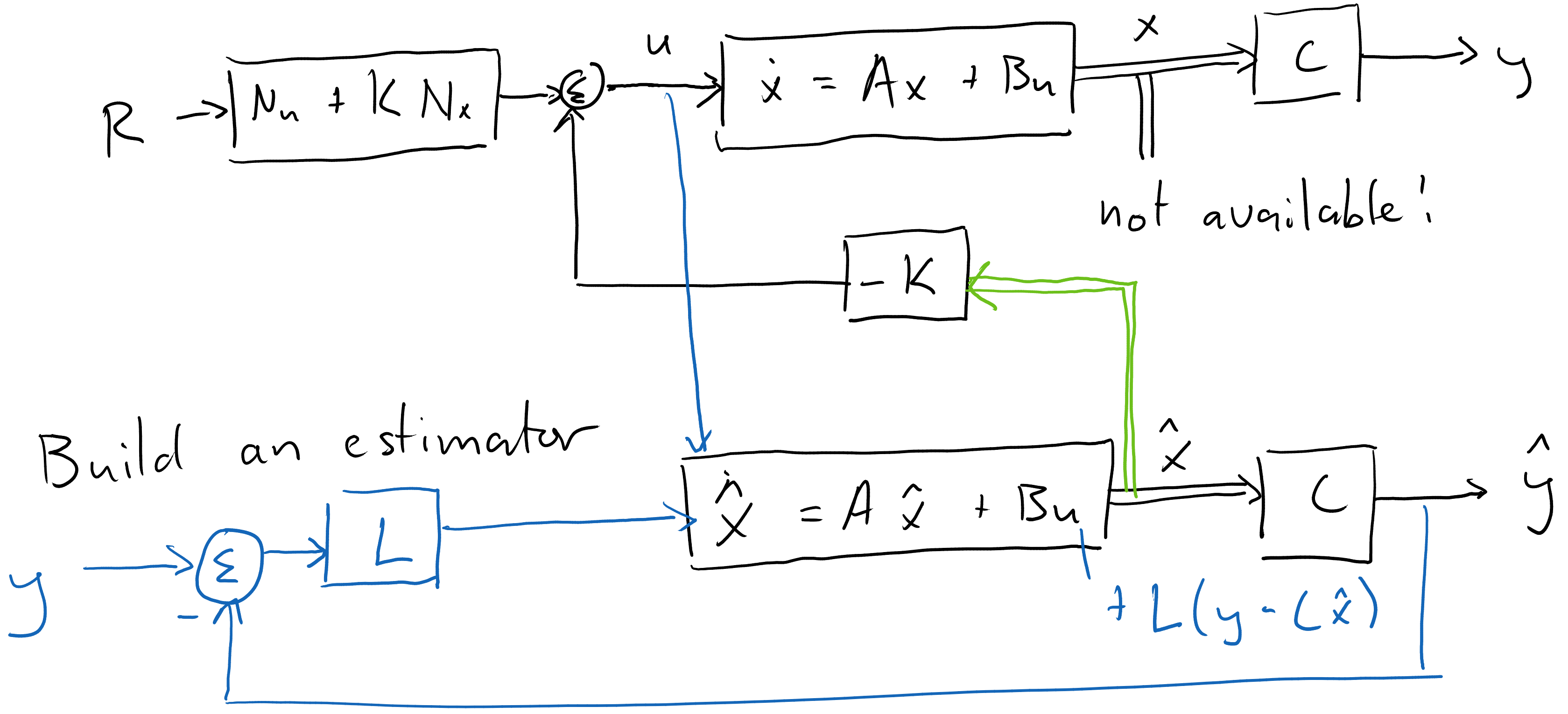
Control in state space

We want to implement feedback control



But we can't practically measure all dimensions of x.

Building an estimator

error

$$\tilde{x} = x - \hat{x}$$

$$\dot{\hat{x}} = A\hat{x} + Bu + L(y - C\hat{x})$$

$$\dot{\tilde{x}} = \dot{x} - \dot{\hat{x}}$$

$$= (Ax + \cancel{Bu}) - (A\hat{x} + \cancel{Bu} + L(y - C\hat{x}))$$

$$y = Cx$$

$$= Ax - (A\hat{x} + L(Cx - C\hat{x}))$$

$$= (Ax - LCx) - (A\hat{x} - LC\hat{x})$$

$$= (A - LC)(x - \hat{x})$$

$$= (A - LC)\tilde{x}$$

$$A - BK$$

Picking L

We want to choose gains so that the error goes to 0 "quickly"

For control

$$A - BK$$

$$[n \times n] - [n \times 1][1 \times n]$$

$$K = \text{acker}(A, B, \text{poles})$$

For estimation

$$A - LC$$

$$[n \times n] - [n \times 1][1 \times n]$$

$$L = \text{acker}(A', C', \text{poles})'$$

Using MATLAB

See handout

$$\begin{bmatrix} \dot{x} \\ \dot{\hat{x}} \end{bmatrix} = \begin{bmatrix} A - BK & 0 \\ LC - BK & A - LC \end{bmatrix} \begin{bmatrix} x \\ \hat{x} \end{bmatrix}$$

$$y = \begin{bmatrix} C & 0 \end{bmatrix}$$

$$\hat{y} = \begin{bmatrix} 0 & C \end{bmatrix}$$

$$\tilde{y} = \begin{bmatrix} C & -C \end{bmatrix}$$

$$\begin{bmatrix} \Theta \\ \dot{\Theta} \\ \hat{\Theta} \\ \hat{\dot{\Theta}} \end{bmatrix}$$

Observability

$$\mathcal{O} = \begin{bmatrix} C \\ CA \\ CA^2 \\ \vdots \\ CA^{n-1} \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & 1 \\ -\omega^2 & 0 \end{bmatrix} \qquad x = \begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

or

$$\begin{bmatrix} 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \dot{\theta} \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ \alpha \end{bmatrix} u$$

Wait, is that combined matrix right?

Not quite... that's our last bit to finish up!