EE 105 Feedback control systems

Just a pinch of digital control

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Difference equations

$$y(k) = \alpha\, y(k-1) + u(k)$$

stable iff $|\alpha| < 1$

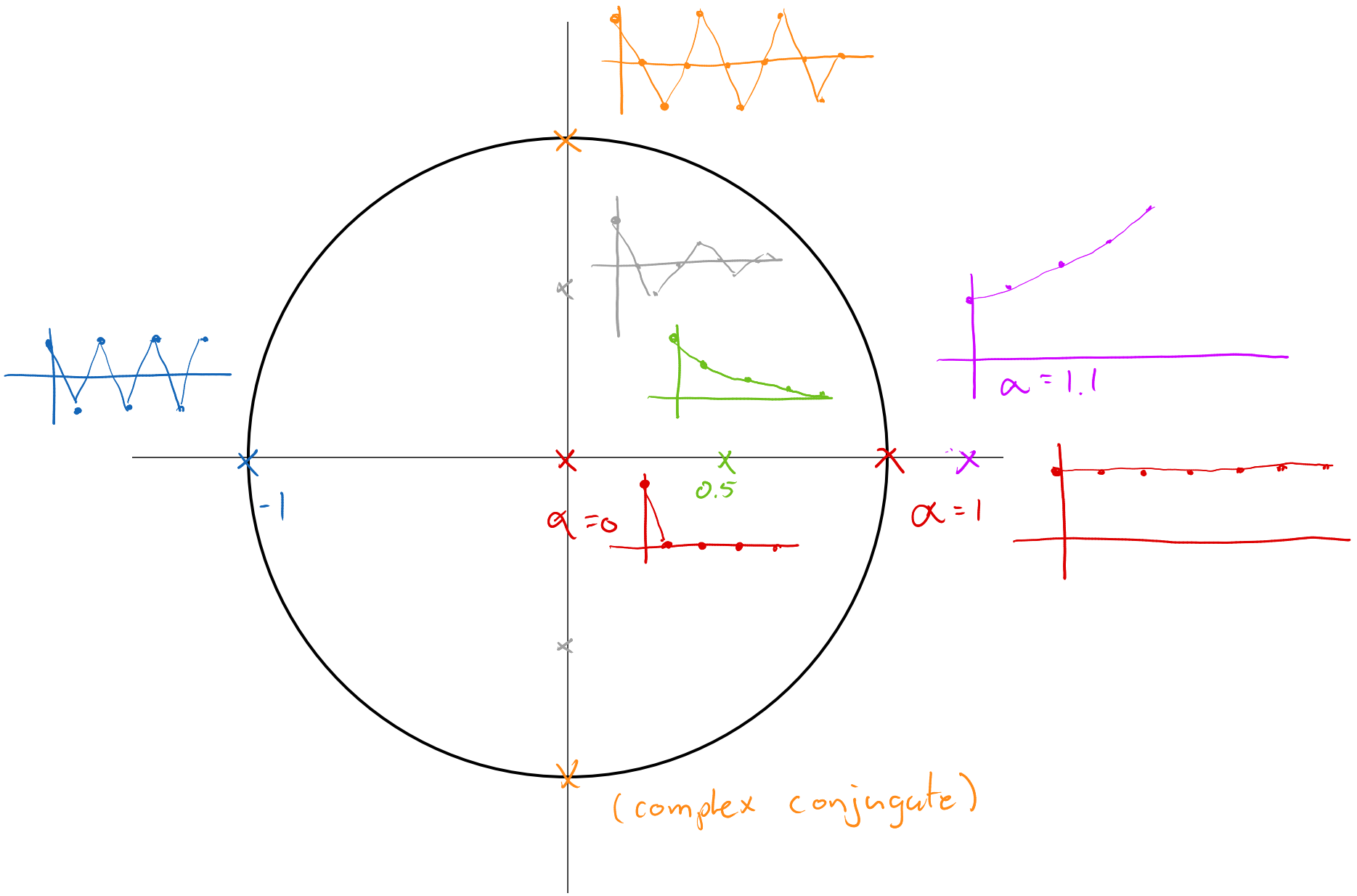
$$X_k = A\, x_{k-1} + B \cdot u(k)$$

eigenvalues ≤ 1

Z transform

$$y(k) = a\,y(k-1) + b\,u(k) \Rightarrow \frac{1}{1 - a z^{-1}} = \frac{z}{z - a}$$

Mapping out the Z-plane



Relating Z back to the S-plane

$$z = e^{-sT}$$

1) let $s = j\omega$ s-plane $j\omega$-axis is unit circle

$$z = e^{-j\omega T}$$

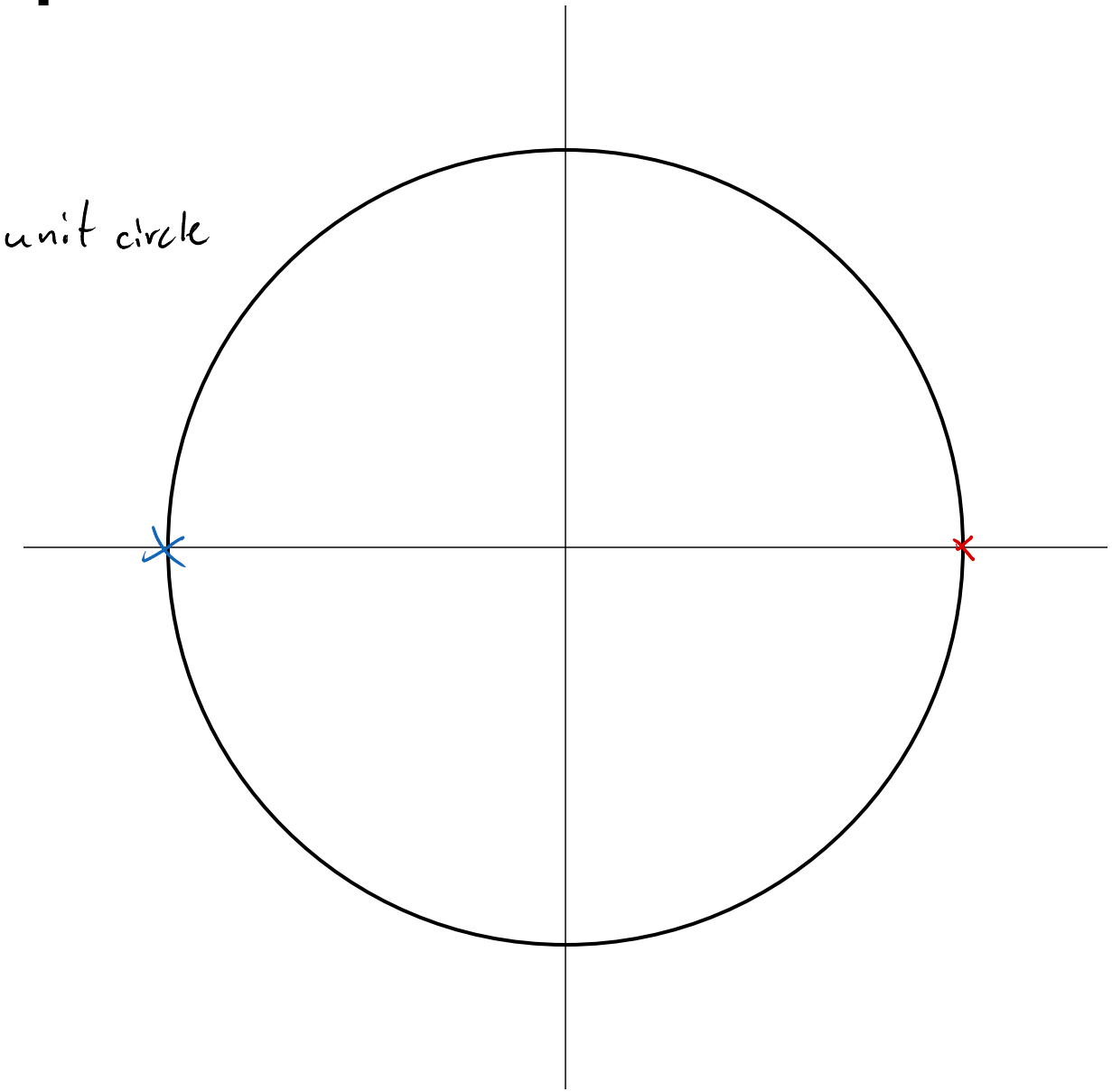
$$= \cos(-\omega T) + j\sin(-\omega t)$$

2) Origin $s = 0 \Rightarrow z = 1$

3) Origin on z-plane is $s = -\infty$

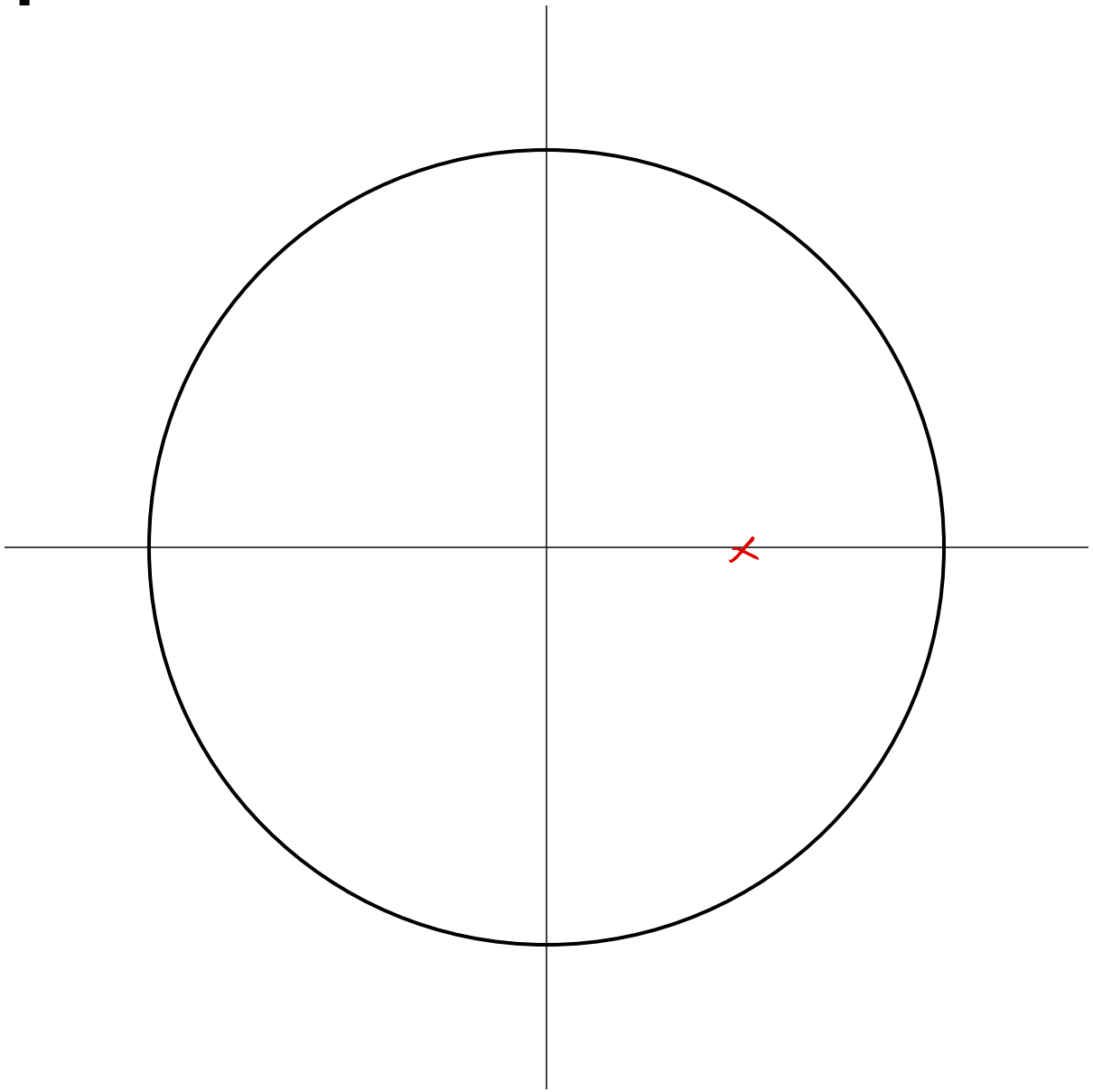
4) $-1 = \cos(-\pi) = \cos(-\omega T)$

$\omega T = \pi \Rightarrow \omega = \frac{\pi}{T}$



Relating Z back to the S-plane

Real pole at $z = 0.5$

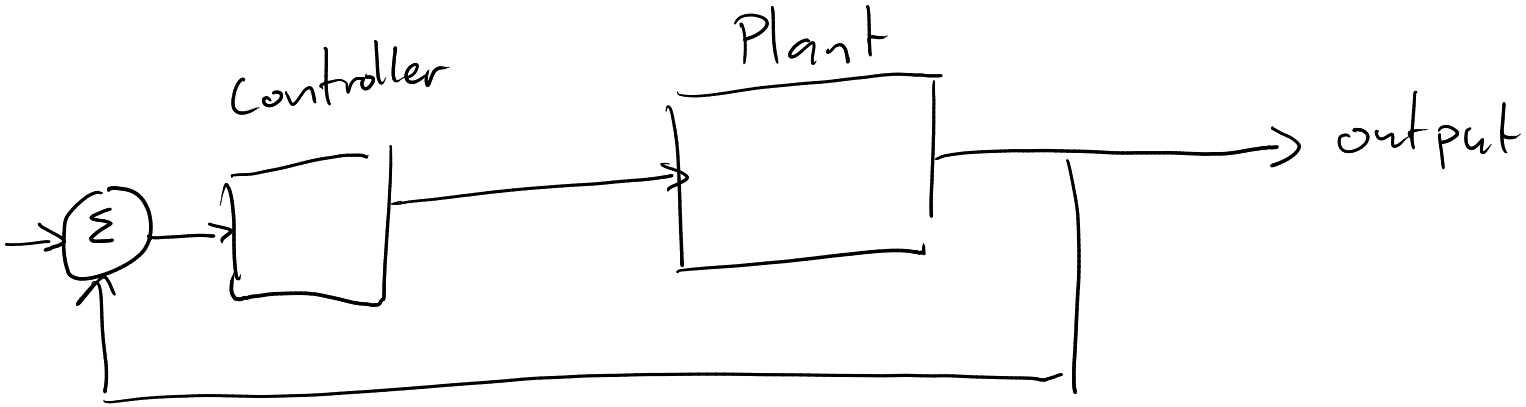


Experimenting with MATLAB

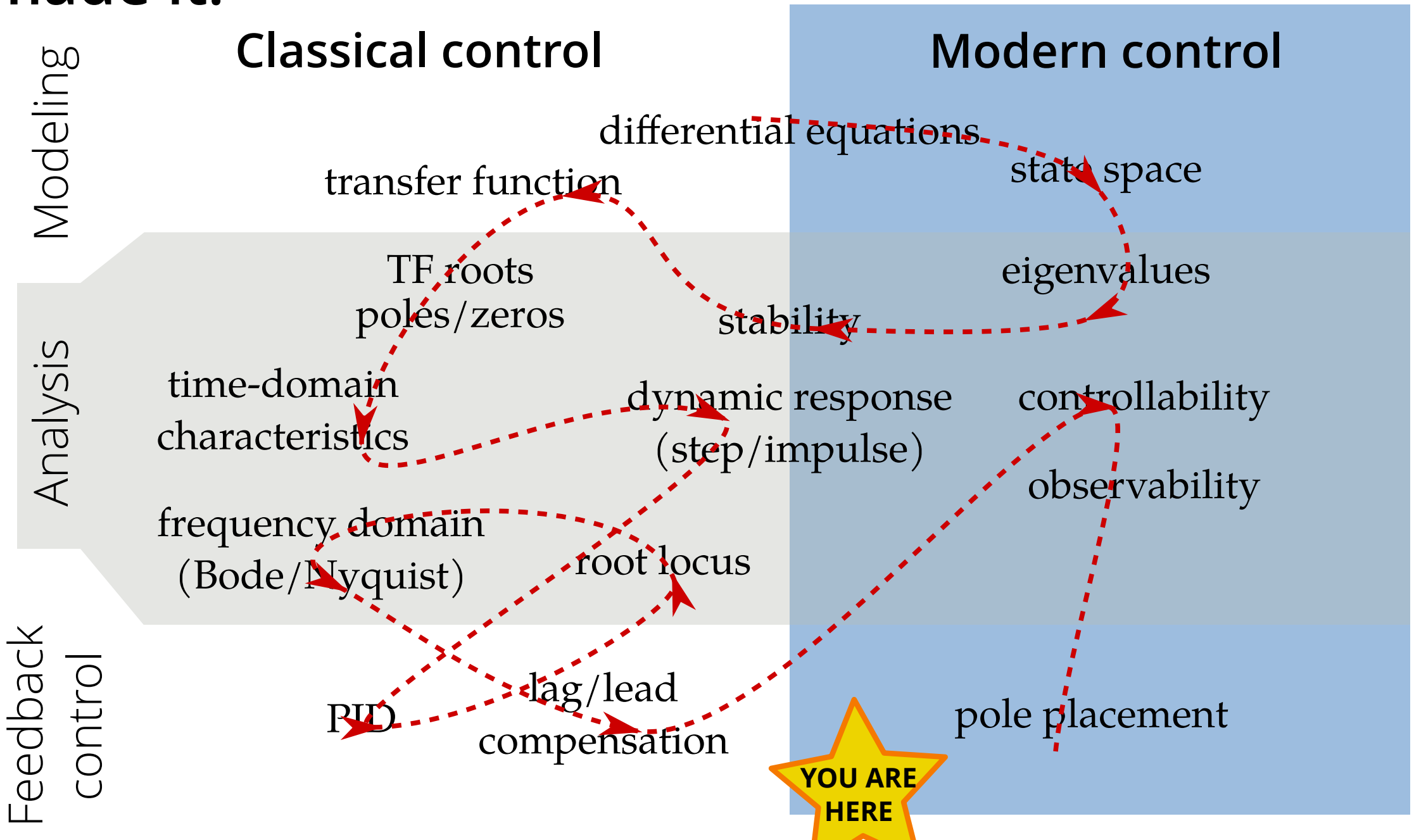
What happens as we lower the sample rate?

How fast do we need to sample for good response?

What does a root locus look like in discrete space?


Controller
Plant
output
ε

We made it!


Classical control
Modern control
Modeling
Analysis
Feedback control
differential equations
transfer function
state space
TF roots
poles/zeros
eigenvalues
stability
time-domain
characteristics
dynamic response
(step/impulse)
controllability
observability
frequency domain
(Bode/Nyquist)
root locus
PID
lag/lead
compensation
pole placement
YOU ARE HERE