Homework 2
Due: February 28, 2018

1. Consider a transmitter which radiates a sinusoidal carrier frequency of 1850 MHz. For a vehicle moving 60 mph, compute the received carrier frequency if the mobile is moving
   (a) directly toward the transmitter,
   (b) directly away from the transmitter, and
   (c) in a direction which is perpendicular to the direction of arrival of the transmitted signal.

2. Assume a mobile traveling at a velocity of 10 m/s receives two multipath components at a carrier frequency of 1000 MHz. The first component is assumed to arrive at $\tau = 0$ with an initial phase of 0^o and a power of -70 dBm, and the second component which is 3 dB weaker than the first component is assumed to arrive at $\tau = 1$ μs, also with an initial phase of 0^o. Suppose the mobile moves directly toward the direction of arrival of the first component and directly away from the direction of arrival of the second component. Plot the amplitude of the channel response within the time interval from 0 s to 0.5 s. Compute the narrowband instantaneous power at time intervals of 0.1 s from 0 s to 0.5 s. Compute the average narrowband power received over this observation interval. Assume the amplitudes of the two multipath components do not fade over the local area.

3. The goal of this problem is to develop a Rayleigh fading simulator for a mobile communications channel using the method of filtering Gaussian processes that is based on the in-phase and quadrature PSDs as described below. In this problem you must do the following:
   (a) Develop simulation code to generate a signal with Rayleigh fading amplitude overtime. Your sample rate should be at least 1000 samples per second, the average received envelope should be 1, and your simulation should be parameterized by the Doppler frequency f_D. Matlab is the easiest way to generate this simulation, but any code is fine.
   (b) Write a description of your simulation that clearly explains how your code generates the fading envelope; use a block diagram and any necessary equations.
   (c) Turn in your well-commented code.
   (d) Provide plots of received amplitude (dB) versus time for $f_D = 1$, 10, and 100 hertz over 2 seconds.

   A common method for simulating the envelope of a narrowband fading process is to pass two independent white Gaussian noise sources with PSD $N_0/2$ through low pass filters with a frequency response $H(f)$ that satisfies

$$S_{r_I} = S_{r_Q} = \frac{N_0}{2}|H(f)|^2$$

   The filter outputs then correspond to the in-phase and quadrature components of the narrow-band fading process with PSDs $S_{r_I}(f)$ and $S_{r_Q}(f)$.

   Under uniform scattering, the power spectral densities (PSDs) of the in-phase and quadrature received signal parts $r_I(t)$ and $r_Q(t)$ denoted by $S_{r_I}(f)$ and $S_{r_Q}(f)$, respectively are

$$S_{r_I}(f) = S_{r_Q}(f) = \mathcal{F}[A_{r_I}(\tau)] = \begin{cases} \dfrac{2P_r}{\pi f_D} \dfrac{1}{\sqrt{1-(f/f_D)^2}} & |f| \le f_D, \\ 0 & \text{else.} \end{cases}$$

4. Assume a Rayleigh fading channel with average signal power $2\sigma^2 = -80$ dBm. What is the power outage probability of this channel relative to the threshold $P_0 = -95$ dBm? How about $P_0 = -90$ dBm?

5. Show that the magnitude (envelope) of the sum of two independent identically distributed complex (quadrature) Gaussian sources is Rayleigh distributed. Assume that the Gaussian sources are zero mean and have unit variance. Specifically prove, for X and Y independent zero-mean Gaussian random variables with variance σ^2, that the distribution of $Z = \sqrt{X^2 + Y^2}$ is Rayleigh distributed and that the distribution of Z^2 is exponentially distributed.

6. A local spatial average of a power delay profile measured at 900 MHz is shown in Figure 1.

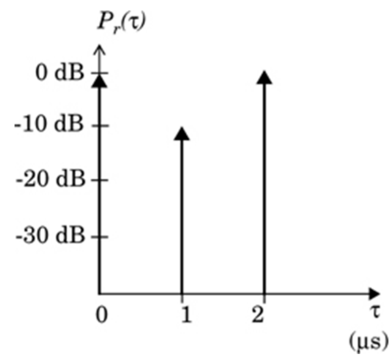

Figure 1: Power delay profile.

(a) Determine the rms delay spread and mean excess delay for the channel.
(b) Determine the maximum excess delay (20 dB).
(c) If the channel is to be used with a modulation that requires an equalizer whenever the symbol duration T is less than 10 σ_τ, determine the maximum RF symbol rate that can be supported without requiring an equalizer.
(d) If a mobile traveling at 30 km/hr receives a signal through the channel, determine the time over which the channel appears stationary (or at least highly correlated).

The following definitions and equations might be useful:

- The mean excess delay is the first moment of the power delay profile and is defined to be

$$\bar{\tau} = \frac{\sum_k a_k^2 \tau_k}{\sum_k a_k^2} = \frac{\sum_k P(\tau_k)\tau_k}{\sum_k P(\tau_k)}$$

- The rms delay spread is the square root of the second central moment of the power delay profile and is defined to be

$$\sigma_\tau = \sqrt{\overline{\tau^2} - (\bar{\tau})^2} \qquad \text{where} \qquad \overline{\tau^2} = \frac{\sum_k a_k^2 \tau_k^2}{\sum_k a_k^2} = \frac{\sum_k P(\tau_k)\tau_k^2}{\sum_k P(\tau_k)}$$

- o The maximum excess delay (X dB) of the power delay profile is defined to be the time delay during which multipath energy falls to X dB below the maximum.

7. If a particular modulation provides suitable BER performance whenever $\sigma_\tau/T_s \leq 0.1$ determine the smallest symbol period T_s (and thus the greatest symbol rate) that may be sent through RF channels shown in Figure 2, without using an equalizer.

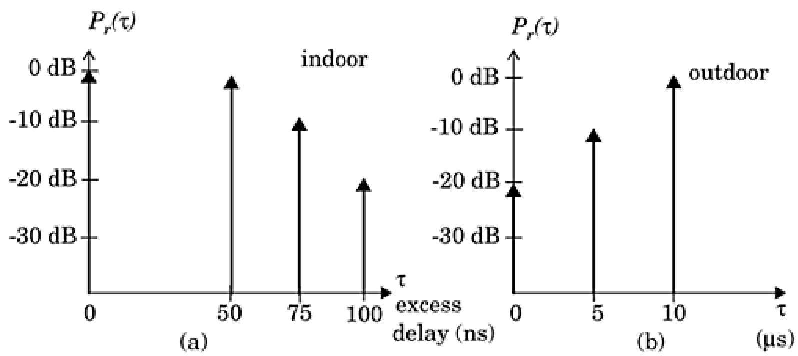

Figure 2: Two channel responses.