# Homework 3
Due: March 28

1. Consider a flat-fading channel of bandwidth 20 MHz where for a fixed transmit power $\bar{S}$, the received SNR is one of six values: $\gamma_1 = 20$ dB, $\gamma_2 = 15$ dB, $\gamma_3 = 10$ dB, $\gamma_4 = 5$ dB, and $\gamma_5 = 0$ dB and $\gamma_6 = -5$ dB. The probability associated with each state is $p_1 = p_6 = .1$, $p_2 = p_4 = .15$, $p_3 = p_5 = .25$. Assume only the receiver has CSI.

   (a) Find the capacity of this channel.

   (b) Plot the outage capacity versus outage for outage probability in the range $0 \leq p_{out} < 1$.

2. Consider a frequency-selective fading channel with total bandwidth 12 MHz and coherence bandwidth $B_c = 4$ MHz. Divide the total bandwidth into 3 subchannels of bandwidth B_c , and assume that each subchannel is a Rayleigh flat-fading channel with independent fading on each subchannel. Assume the subchannels have average gains $\mathbf{E}[|H_1(t)|^2] = 1$, $\mathbf{E}[|H_2(t)|^2] = .5$, and $\mathbf{E}[|H_3(t)|^2] = .125$. Let the total transmit power be 30 mW, and a receiver noise spectral density of $.001\mu$W per Hertz.

   (a) Find the optimal frequency domain water-filling power adaptation for this channel and the corresponding capacity, assuming only the receiver knows the instantaneous channel and the transmitter knows the average channel gain on each subchannel.

   (b) Compare the capacity of parts (a) with that obtained by allocating an equal average power of 10 mW to each subchannel without water-filling in either frequency or time dimension.

3. Simulate and plot the capacity of a Rayleigh fading channel vs. the average SNR for the following cases:

   (a) CSI is known at the receiver only,

   (b) AWGN channel capacity with the same average SNR as the Rayleigh channel.

   For your simulation, use the average transmit SNR, $\gamma = \frac{P}{\sigma_n^2}$, in the range $0 - 30$ dB in steps of 5 dB and generate 10,000 channel realizations for each value of SNR. To generate a Rayleigh fading channel vector of length N, use the following Matlab code:

   $$h = \frac{1}{\sqrt{2}} * (randn(1, N) + i * randn(1, N));$$

4. Use the outage probability expressions derived in class to plot the outage probability vs. SNR for Selection Combining, Maximum Ratio Combining, and Equal Gain Combining diversity techniques. Pick a fixed value for the threshold SNR, γ_0. These expressions are also available in the book by Goldsmith. Plot the outage probability for the number of diversity branches $M = 2, 4, 8,$, and SNR in the range $0 - 20$ dB in steps of 5 dB. Comment on the diversity and array gains as M increases.

5. Use simulation to plot the bit error probability vs. SNR for Selection Combining, Maximum Ratio Combining, and Equal Gain Combining diversity techniques using QPSK modulation. The simulation system is shown in the figure below, assuming a block fading channel model with block length N as a parameter of choice. You should generate enough random input bits b_k and the corresponding channel realizations to obtain a smooth error probability plot. Perform the simulation for the number of diversity branches $M = 1, 2, 4$ and 8, and for SNR in the range $0 - 20$dB in steps of 5dB. Compare with the outage plots in Problem 4 and comment.
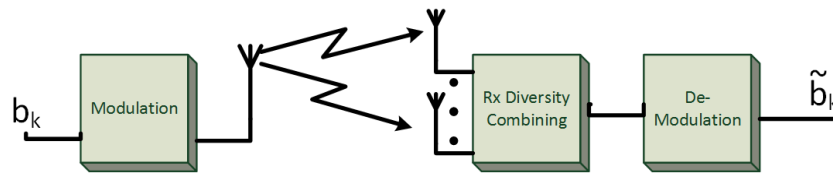
6. Perform simulation for the system using Alamouti code shown in the figure below, where b_k is a random input bit stream. Plot the bit error rate of the Alamouti code for SNR in the range $0-25$dB in steps of 5dB using QPSK modulation. Repeat with 16-QAM and superpose the error curves on the same plot. Assume a Rayleigh block fading channel. Your simulations should generate enough input bits and channel samples to obtain smooth error curves.
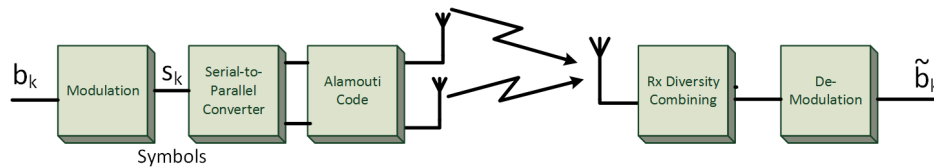
7. Problem 10-6 in Goldsmith, *Wireless Communications*.