Diversity Techniques

- We see that fading makes the received SNR fluctuate. This fluctuation in the SNR not only reduces the capacity (compared to an AWGN channel with the same average SNR), it also makes signal detection / decoding error prone.

- Diversity techniques combines independent fading signal paths in order to improve the error performance.

  The main idea is that independent signal paths have a low probability of experiencing deep fades simultaneously.

  Hence correctly combining these paths leads to a signal SNR with less variation, hence decoding has lower error probability.

- Diversity can be applied over the time, frequency or spatial dimensions.

  - Time diversity: transmit the same signal at different times separated by more than the channel coherence time

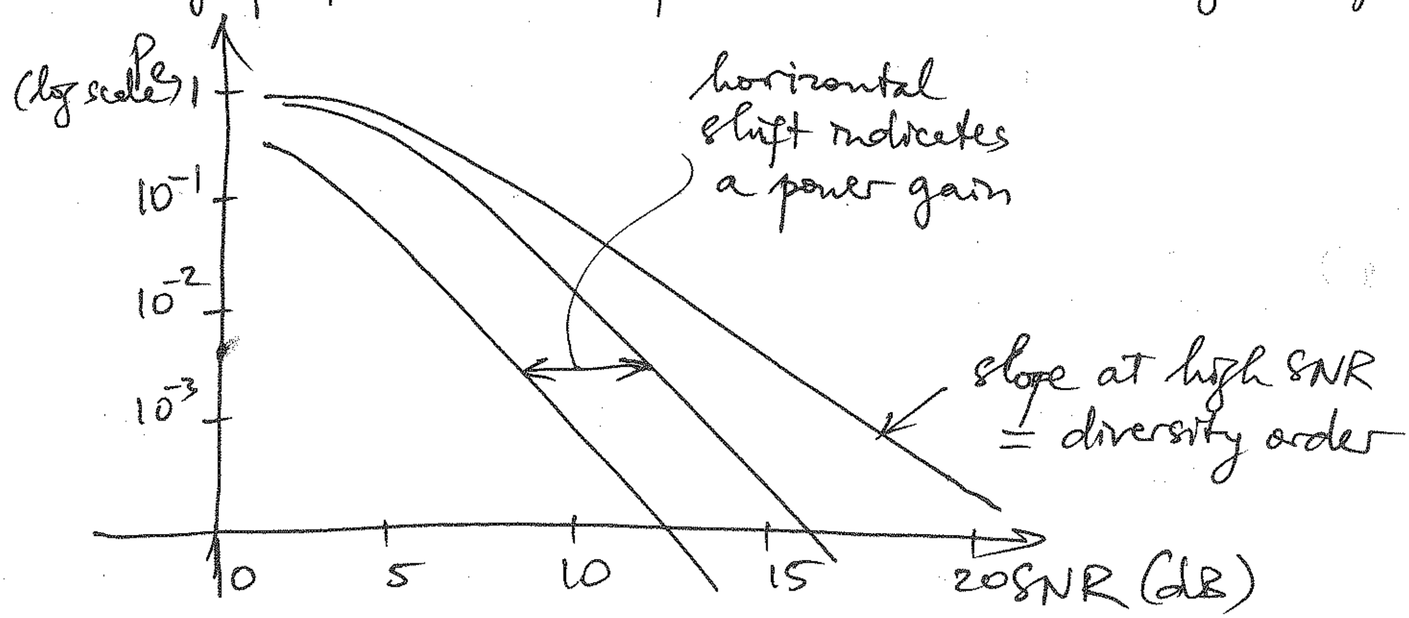
  - Frequency diversity: transmit the same narrowband signal over different carrier frequencies separated by more than the channel coherence bandwidth.

  - Spatial diversity: use an antenna array to transmit or receive signals. The separation among the antennas need to be at least half a wavelength (carrier wavelength) to obtain uncorrelated or independent fading at different antennas.

- Polarization diversity: transmit signals on two orthogonal polarizations. Only two diversity branches are available.

- Diversity performance includes a power gain (array gain) and a diversity gain (change in the slope of the error probability vs. SNR).

Diversity performance depends on the combining technique.



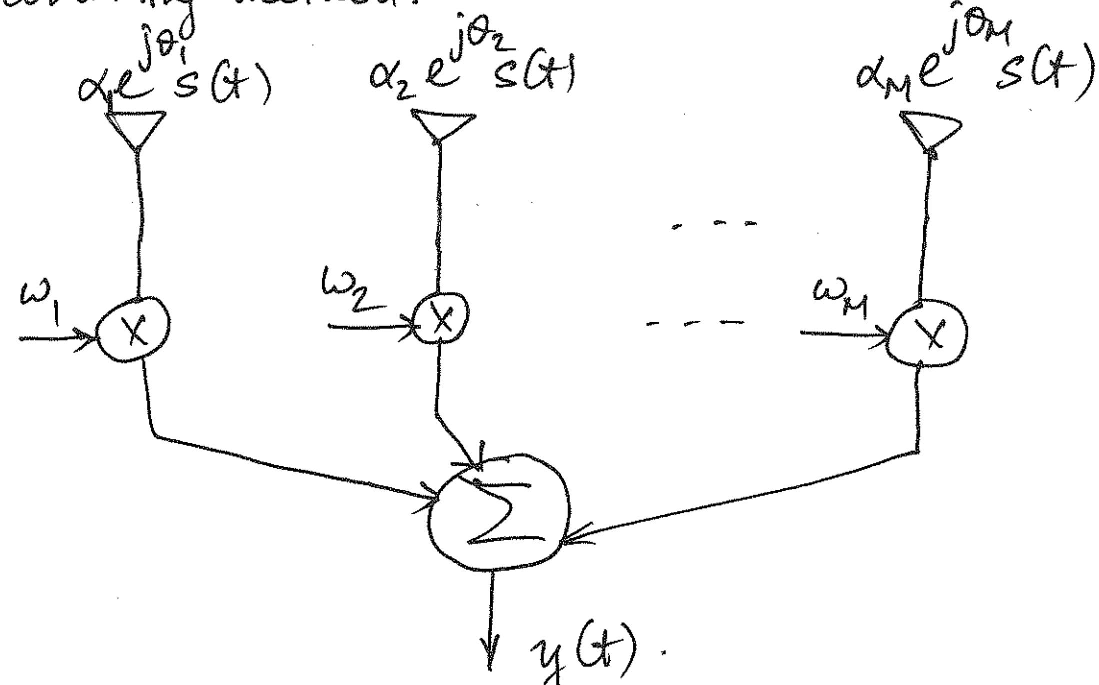
+) Receiver diversity combining models:

+ System model:

- Take multiple antennas as the example, but the model applies equally well to frequency and time diversity.

- The receiver receives M independent branches of the same transmit signal.

- The question is how to combine these M independent received signals, and what is the performance of such an combining method.



- The received signal can be written as a vector

$$r = h s + n$$

where

$$h = \begin{bmatrix} h_1 \\ h_2 \\ \vdots \\ h_M \end{bmatrix} = \begin{bmatrix} \alpha_1 e^{j\theta_1} \\ \alpha_2 e^{j\theta_2} \\ \vdots \\ \alpha_M e^{j\theta_M} \end{bmatrix} ; \quad r = \begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_M \end{bmatrix} ; \quad n = \begin{bmatrix} n_1 \\ n_2 \\ \vdots \\ n_M \end{bmatrix}$$

and s is a complex scalar representing the transmit symbol.

- The combining technique is usually linear: each branch is scaled by a complex weighting factor w_i and then sum. The combining weight vector can be written as

$$w = [w_1 \; w_2 \; \dots \; w_M]^T$$

The combiner output is a single value

$$y = w^T r = \sum_{i=1}^{M} w_i r_i = w^T h . s + w^T n$$

Different ways of designing the weight vector lead to different combining techniques.

- To measure performance, we examine the received SNR after combining:

Suppose s has average power P
noises $n_i \in CN(0, \sigma^2)$ and are independent at different branches.

$\gamma_i = \frac{P}{\sigma^2} |h_i|^2$ is the received SNR at branch i

The SNR after combining is

$$\gamma_\Sigma = \frac{P}{\sigma^2 (\sum |w_i|^2)} \cdot \left| \sum w_i h_i \right|^2 .$$

- This combined SNR is still a random quantity. We look at several attributes:

i) The average SNR:

$$\bar{\gamma}_\Sigma = \frac{P}{\sigma^2 (\sum |w_i|^2)} E\left[\left| \sum w_i h_i \right|^2 \right]$$

This $\bar{\gamma}_\Sigma$ gives the average SNR gain or array gain.

ii) The error performance at high SNR:

- Error performance depends directly on the distribution of γ_Σ, denoted as $f_{\gamma_\Sigma}(\gamma)$.

The outage probability for a given target SNR γ_0 is an indication of the symbol error probability and is equal to

$$P_0 = \Pr(\gamma_\Sigma < \gamma_0) = \int_0^{\gamma_0} f_{\gamma_\Sigma}(\gamma)\, d\gamma .$$

- Usually this error probability can be evaluated to be of the form

$$P_0 \cong c\,\bar{\gamma}^{-\beta}$$

where c is a constant

β is called the diversity order.

$\bar{\gamma}$ is average SNR per branch.

The higher diversity order β, the faster the error rate falls off with increasing (average) SNR.

+) Selection combining:

- This method only selects the output with the highest instantaneous SNR.

- Simple to implement, requires minimum hardware.

- Requires switching among different branches

- The combining weight vector contains only one entry of 1 and the rest are zero. The location of this 1 varies with each received set of signals.

$$\gamma_\Sigma = \max_i \{\gamma_i\} = \max_i \left\{ \frac{P}{\sigma^2} |h_i|^2 \right\}.$$

- Suppose error occurs when the received SNR is below γ_0, which is the same as the cummulative distribution functions (cdf).

For selection Combining, all M branches must have received SNR below γ_0 for an error to occur.

- The cdf of γ_Σ is

$$F_{\gamma_\Sigma}(\gamma) = \Pr(\gamma_\Sigma < \gamma) = \Pr\left(\max_i \{\gamma_i\} < \gamma\right)$$

$$= \prod_i \Pr(\gamma_i < \gamma)$$ because of uncorrelated branches.

$$= \left[\Pr(\gamma_i < \gamma)\right]^M$$ if all γ_i are identically distributed.

<u>Example</u>: For Rayleigh fading:

$$f_\gamma(\gamma_i) = \frac{1}{\bar{\gamma}_i} e^{-\gamma_i / \bar{\gamma}_i}$$

Then for a single branch (no combining)

$$P_o(\gamma_0) = 1 - e^{-\gamma_0/\bar{\gamma}_i}$$

With selection combining, the outage probability becomes:

$$P_o(\gamma_0) = \prod_{i=1}^{M} \left(1 - e^{-\frac{\gamma_0}{\bar{\gamma}_i}}\right)$$

$$= \left(1 - e^{-\frac{\gamma_0}{\bar{\gamma}}}\right)^M$$ if all $\bar{\gamma}_i = \bar{\gamma}$.

+) An even simpler scheme is threshold combining: find the first branch that has SNR above a given threshold γ_T, then switch to another branch when SNR of the current branch falls below γ_T.

This method is also called switch and stay combining. It has the same outage probability as selection combining provided the threshold is optimized.

+) <u>Maximal Ratio Combining</u>: (MRC).

- The goal of MRC is to maximize the output SNR γ_Σ.

$$\gamma_\Sigma = \frac{P}{\sigma^2} \cdot \frac{\left|\sum_i w_i h_i\right|^2}{\sum_i |w_i|^2}$$

By Cauchy-Schwartz inequality:

$$\left|\sum_i w_i h_i\right|^2 \leq \left(\sum |w_i|^2\right)\left(\sum |h_i|^2\right)$$

"=" iff $w_i = c \cdot h_i^*$ for some scaling factor c.

- Thus the optimal combining weight vector can be chosen as

$$w = h^* \quad (* \text{ stands for complex conjugate})$$

Then the resulting output SNR is:

$$\gamma_\Sigma = \frac{P}{\sigma^2}\left(\sum_i |h_i|^2\right) = \sum_i \gamma_i .$$

→ output SNR increases linearly with the number of combining branches.

- MRC combiner gives the best output SNR and hence the largest power (or array) gain.

- The diversity order of MRC receiver is also M.

- To compute the outage probability, we need to determine the distribution of the output SNR γ_Σ.

In general this distribution can be derived from the characteristic function or moment generating function of the channel power gain (fading distribution).

- If fading is Rayleigh, then γ_Σ has a chi-squared distribution (χ^2) with 2M degrees of freedom.

$$f_{\gamma_\Sigma}(\gamma) = \frac{\gamma^{M-1} e^{-\gamma/\bar{\gamma}}}{\bar{\gamma}^M (M-1)!} \quad , \gamma \geq 0$$

and the outage probability can be computed in closed forms

$$P_o = \Pr(\gamma_\Sigma < \gamma_0) = 1 - e^{-\gamma_0/\bar{\gamma}} \sum_{i=1}^{M} \frac{(\gamma_0/\bar{\gamma})^{k-1}}{(k-1)!}.$$

+) Equal gain combining:

As MRC requires the knowledge of the instantaneous channel gains on each branch, a simpler scheme is to only match the phase of the channel (co-phasing) then combines with equal weights.

Let $h_i = \alpha_i e^{j\theta_i} \longrightarrow w_i = e^{-j\theta_i}$.

The output SNR is $\gamma_\Sigma = \frac{P}{\sigma^2}\left(\sum |h_i|\right)^2$.