- MRC combiner gives the best output SNR and hence the largest power (or array) gain.

- The diversity order of MRC receiver is also M.

- To compute the outage probability, we need to determine the distribution of the output SNR $\gamma_\Sigma$.

In general this distribution can be derived from the characteristic function or moment generating function of the channel power gain (fading distribution).

- If fading is Rayleigh, then $\gamma_\Sigma$ has a chi-squared distribution ($\chi^2$) with 2M degrees of freedom.

$$f_{\gamma_\Sigma}(\gamma) = \frac{\gamma^{M-1} e^{-\gamma/\bar{\gamma}}}{\bar{\gamma}^M (M-1)!}, \quad \gamma \geq 0$$

and the outage probability can be computed in closed forms

$$P_o = \Pr(\gamma_\Sigma < \gamma_0) = 1 - e^{-\gamma_0/\bar{\gamma}} \sum_{i=1}^{M} \frac{(\gamma_0/\bar{\gamma})^{k-1}}{(k-1)!}.$$

type II:

+) Equal gain combining:

As MRC requires the knowledge of the instantaneous channel gain on each branch, a simpler scheme is to only match the phase of the channel (co-phasing) then combines with equal weights.

Let $h_i = a_i e^{j\theta_i} \longrightarrow w_i = e^{-j\theta_i}$.

The output SNR is $\gamma_\Sigma = \frac{P}{\sigma^2}\left(\sum |h_i|\right)^2.$

Equal gain combining performance falls in between that of MRC and SC (selection combining)

Closed form distribution for $\gamma_Z$ doesnot exist under equal gain combining so error performance (outage) is usually obtained numerically, except for $M = 2$.

+) Receive diversity for frequency selective channels: The RAKE receiver.

∘ Consider a frequency selective channel caused by multipath propagation.

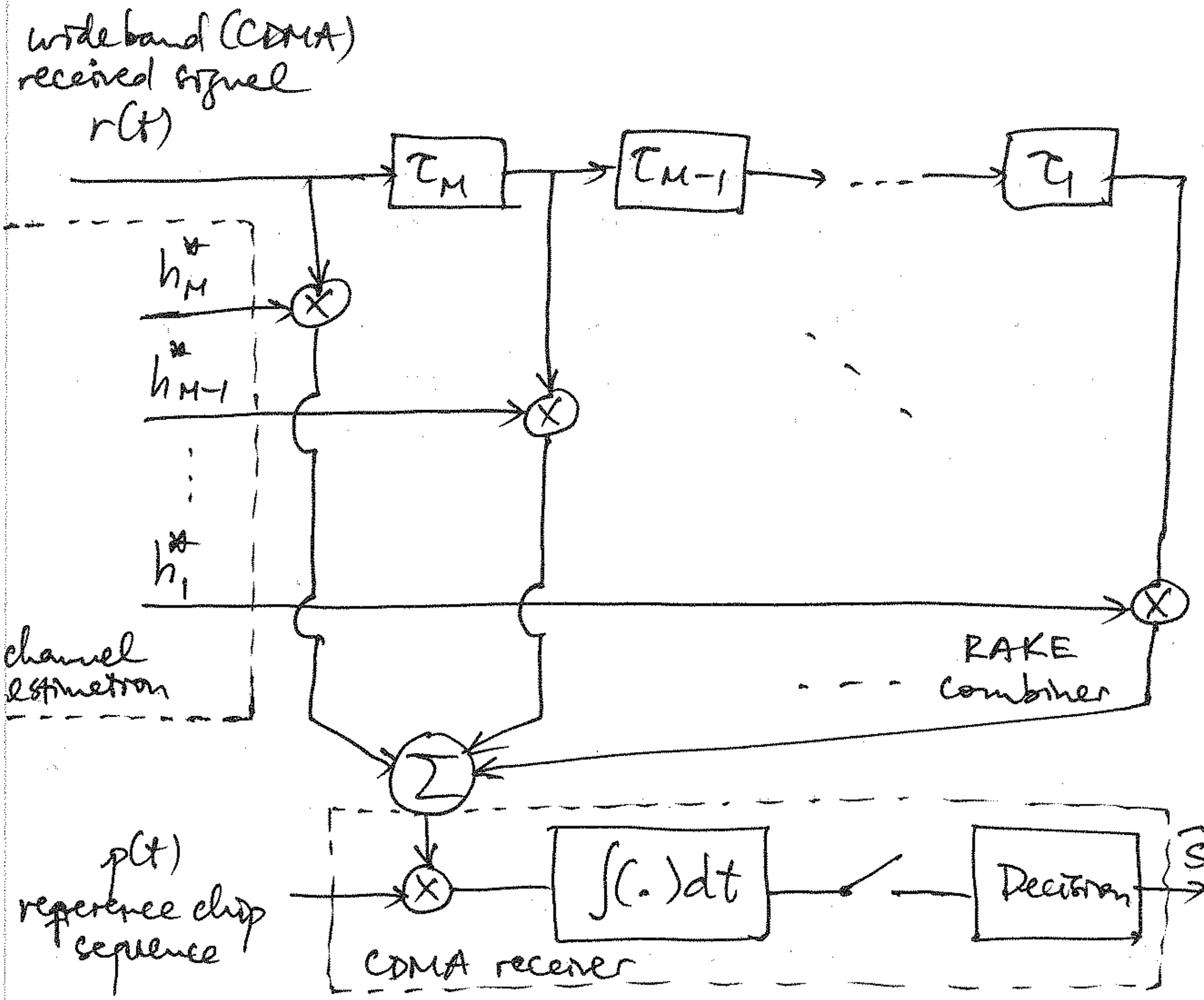
The resolvable paths are statistically independent and provides multiple replicas of the same transmitted signal.

∘ A RAKE receiver ~~is~~ combines these paths in an optimal way to achieve diversity.

RAKE receiver is based on the assumption that different signals superposed over the multiple paths (i.e. sharing the spectrum) are orthogonal! In other words, different signals that need to be detected by the RAKE receiver are orthogonal.

∘ RAKE receiver is used in CDMA system provided that the chip rate is fast enough so that the large delay between the multipath components make the multipaths resolvable and received signals appear uncorrelated.

∘ Performance of the RAKE receiver is similar to MRC if all path gains are known and wideband signals are orthogonal.




- Rake receiver works well for outdoor CDMA systems as the delay spread is large. It doesn't work well for indoors due to the small delay spread which make the multipaths unresolvable (the channel indoor is almost flat).

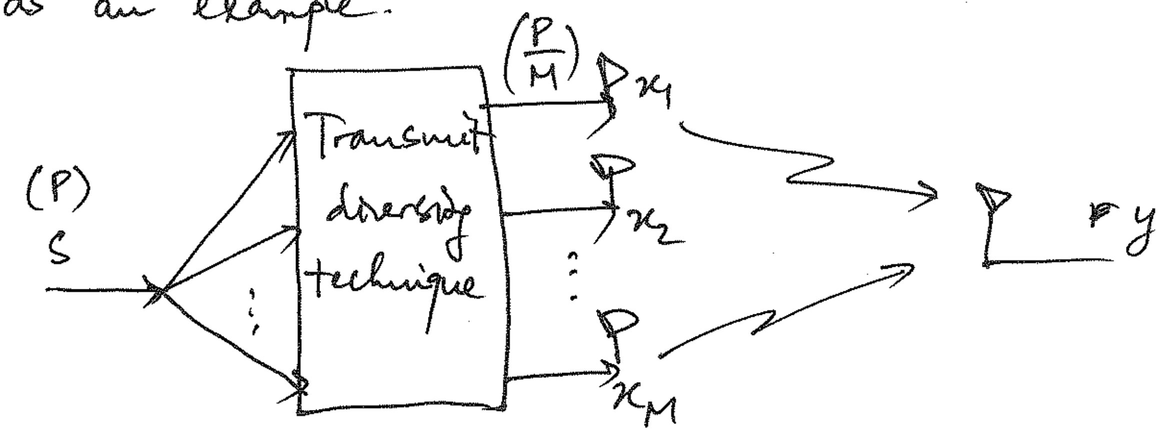
## B.) Transmitter diversity techniques.

- For receiver diversity techniques, the channel coefficients can be estimated and hence are usually known to the diversity combiner. This CSI knowledge leads to both power gain and diversity gain.

- For transmit diversity, we need to consider ~~tw~~ two cases: when the channel is unknown, and when it is known at Tx.

- Here we will look at the case of unknown CSI at Tx, and leave the other case until later when we look at MIMO capacity and beamforming.

- For transmit diversity, again it can be performed over time, frequency or spatial domains. We will take spatial diversity as an example.




- For fair comparision, the transmit power is divided equally among all diversity branches. Signal transmitted in each branch has an average power of $\frac{P}{M}$.

+) <u>Basic principle</u>: If we send the same signal from all antennas (i.e. in all diversity branches), we will not obtain any diversity gain. The performance is then exactly the same as using only a single antenna.

To see why, consider the received signal

$$y = \sum_{i=1}^{M} h_i \cdot x + n$$

$x$: transmit signal w/power $P/M$
$h_i$: channel gain
$n$: Gaussian noise.

- We can re-write the received signal as

$$y = \left(\frac{1}{\sqrt{M}}\sum_{i=1}^{M} h_i\right).\sqrt{M}x + n$$

$$= \tilde{h}.s + n$$

where $s$ is the same as the transmit signal in a single antenna system (no diversity) with power $P$.

- Consider the equivalent channel:

$$\tilde{h} = \frac{1}{\sqrt{M}}\sum_{i=1}^{M} h_i$$

Assuming that all branches $h_i$ are independent fading with the same fading distribution $CN(0,\sigma^2)$.

Then the distribution of $\tilde{h}$ is exactly the same as that of any $h_i$: $\tilde{h} \sim CN(0,\sigma^2)$.

The "combined" channel has the same statistic as a single fading link and no gain is realized (no diversity nor power gain).

- Hence to achieve diversity from the transmitter, different signals must be sent from different antennas, even if these signals represent the same transmit symbols. (Note: the case is different if we know the channel at the transmitter.)

- Many transmit diversity techniques were proposed, including time shifting, frequency shifting, phase shifting, or introducing an LTI filter in front of each branch. These techniques work in that they achieve the diversity order equal to the number of diversity branch. However, they often transform a frequency flat channel into a frequency selective system and hence complicate the receiver by requiring equalization.

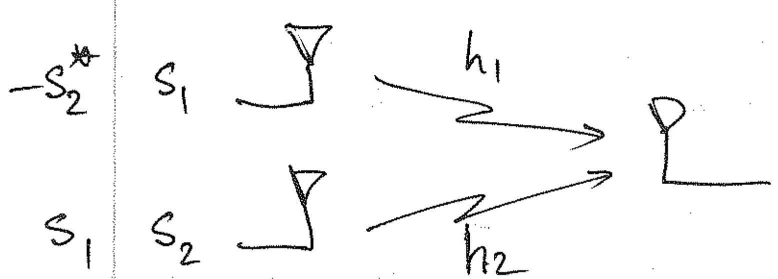
+) <u>Space - time code</u>: The Alamouti scheme.

- Space - time code is a breakthrough idea for achieving transmit diversity (assuming no CSI at the transmitter) without a complicated receiver (no equalization).

- The idea is to send different symbols from different antennas then repeat these symbols in some way over the next few symbol intervals.

- The simplest and groundbreaking work is the Alamouti code, designed for 2 transmit antennas.

- Alamouti code is sent over 2 symbol periods. An underlying assumption is the channel stays the same during these 2 symbol periods.




The received signals during the two symbol period are

$$\begin{cases} y_1 = h_1 s_1 + h_2 s_2 + n_1 \\ y_2 = -h_1 s_2^* + h_2 s_1^* + n_2 \end{cases}$$

where $n_1, n_2 \in \mathcal{CN}(0, \sigma^2)$.

Express the received signals during the 2-symbol periods in a vector form as

$$y = \begin{bmatrix} y_1 \\ y_2^* \end{bmatrix} = \begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2^* \end{bmatrix}$$

or $y = Hs + n$

where $y = \begin{bmatrix} y_1 \\ y_2^* \end{bmatrix}$, $s = \begin{bmatrix} s_1 \\ s_2 \end{bmatrix}$, $H = \begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix}$, $n = \begin{bmatrix} n_1 \\ n_2^* \end{bmatrix}$

- The receiver has vector $y$. It is also assumed to know the channel coefficients $h_i$, hence the matrix $H$.

The receiver processes the received signal as follows. It creates a vector

$$z = H^* y = \begin{bmatrix} |h_1|^2 + |h_2|^2 & 0 \\ 0 & |h_1|^2 + |h_2|^2 \end{bmatrix} s + H^* n$$

Then the two output signals $z_1$ and $z_2$ have no intersymbol interference between $s_1$ and $s_2$.

- Define the new noise as $\tilde{n} = H^* n = [\tilde{n}_1 \ \tilde{n}_2]^T$. Then $\tilde{n}_1$ and $\tilde{n}_2$ are independent Gaussian noise with zero mean and variance $(|h_1|^2 + |h_2|^2).\sigma^2$.

The two signals $z_1, z_2$ can be processed separately to decode the symbol $s_1$ and $s_2$ respectively.

Assuming each transmit antenna sends at average power $\frac{P}{2}$, then the output SNR for decoding each symbol $s_1, s_2$ is

$$\gamma = \frac{(|h_1|^2 + |h_2|^2). P}{2\sigma^2}$$

- The Alamouti code achieves a diversity order of 2 (maximum for 2 transmit antennas).
Compared to MRC, the output SNR is half so the

power gain of Alamouti code is less than of MRC. This lower power gain is because of the lack of channel knowledge at the transmitter in Alamouti.

- The Alamouti code design leads to the generalization into orthogonal space-time code, and consequently a plethora of space-time code & designs with the goal of keeping the (maximum) diversity order while trying to increase the effective symbol rate per symbol interval, or the code rate.

- Space-time codes, including the Alamouti code, is part of many communication standards.

- Space-time code is usually expressed in the following form:

$$C = \begin{bmatrix} s_{11} & s_{12} & \cdots & s_{1T} \\ s_{21} & s_{22} & \cdots & s_{2T} \\ \vdots & & & \\ s_{N1} & s_{N2} & \cdots & s_{NT} \end{bmatrix}$$

$N$: # of transmit antennas
$T$: # of symbol intervals (assuming the channel does not change over $T$).

For a single received antenna, the received vector over $T$ symbols is

$$y = hC + n \quad \text{where} \quad h = [h_1 \ h_2 \ \cdots \ h_N], \quad n = [n_1 \ n_2 \ \cdots \ n_T]$$

A design criterion for space-time code is to maximize the minimum codeword distance, defined as

$$d_{ij} = \|(C_i - C_j)^* (C_i - C_j)\|_F$$

There are other criteria for designing STBC but we will not go into the details here.