Then the input-output relationship over T blocks can be written as

$$Y = HX + N$$

- For maximum-likelihood detection (ML), the receiver looks for the codeblock that matches closest given the received signal:

$$\hat{X} = \underset{X \in \mathcal{X}^{M_t \times T}}{\arg\min} \|Y - HX\|_F^2$$

$\mathcal{X}$ denotes the set of all codeblocks.

- The pair-wise error probability for mistaking a transmit code matrix X with another matrix $\hat{X}$ depends on the distance between these two matrices:

$$\mathbb{P}(\hat{X} \to X) = Q\left(\sqrt{\frac{\|H(X-\hat{X})\|_F^2}{2\sigma_n^2}}\right)$$

Denote $E = X - \hat{X}$ as the error matrix and apply the Chernoff bound to get

$$\mathbb{P}(\hat{X} \to X) \leq \exp\left(-\frac{\|HE\|_F^2}{4\sigma_n^2}\right)$$

Assuming the channel H contains i.i.d. entries that are zero-mean unit-variance Gaussian (i.e. Rayleigh fading), then taking expectation over the channel on both sides and after some malnipulations we get

$$\overline{\mathbb{P}}(\hat{X} \to X) \leq \left(\frac{1}{\det\left(I + \frac{1}{4\sigma_n^2} \cdot EE^*\right)}\right)^{M_r}$$

Define $\Delta = \frac{1}{P} E E^*$ as the product of the codeword pair difference, normalized by the transmit power (so that the distances are only property of the code design).

At high SNR $\left(\rho = \frac{P}{\sigma_n^2}\right)$ then the error bound can be simplified to

$$\mathbb{P}(X \to \hat{X}) \le \det(\Delta)^{-M_r} \cdot \left(\frac{\rho}{4}\right)^{-M_r}$$

This bound lead to the rank and determinant criteria for STBC design:

+) Rank criterion: For the STBC to achieve the full diversity order of $M_r \times M_t$, the rank of Δ over all codeblock pair difference matrices must be M_t (full rank).

+) Determinant criterion: To optimize the coding gain (power or array gain) then the minimum determinant of Δ (over all codeblock pair differences) should be maximized.

- The above criteria are the basic ones for STBC design. There is a rich literature on STBC, including for example orthogonal STBC, linear dispersion and threaded STBC which provide higher multiplexing gain (spatial code rate)... But we will not go into the details in this course.

- Examples of some orthogonal STBC (OSTBC):

OSTBC are those that have special structure: Codeword matrices are unitary, error matrices are unitary.

For OSTBC: $\Delta = I$.

- OSTBC decouples symbols at the receiver and greatly simplify receiver processing and decoding complexity!

- This advantage is often at the sacrifice of reduced rates.

- The Alamouti code is an OSTBC, coderate = 1 (symbol per time slot).

Other examples:

$$X = \begin{bmatrix} x_1 & -x_2 & -x_3 & -x_4 \\ x_2 & x_1 & x_4 & -x_3 \\ x_3 & -x_4 & x_1 & x_2 \\ x_4 & x_3 & -x_2 & x_1 \end{bmatrix}$$

real constellation
coderate = 1

$$X = \begin{bmatrix} x_1 & -x_2^* & \frac{x_3^*}{\sqrt{2}} & \frac{x_3^*}{\sqrt{2}} \\ x_2 & x_1^* & x_3^*/\sqrt{2} & -x_3^*/\sqrt{2} \\ \frac{x_3}{\sqrt{2}} & \frac{x_3}{\sqrt{2}} & a & b \end{bmatrix}$$

where $a = \frac{1}{2}(x_1 - x_1^* + x_2 - x_2^*)$

$b = \frac{1}{2}(x_2 + x_2^* + x_1 - x_1^*)$

complex constellation
code rate = 3/4.

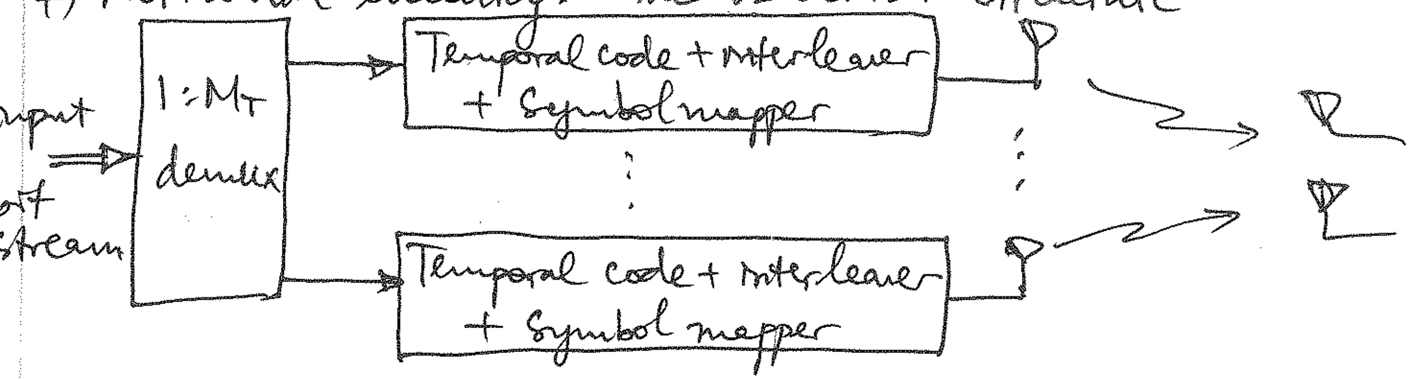
- For complex symbols at $M_t > 4$, we can construct OSTBC codes with rate $\frac{1}{2}$, but don't know if rate $> \frac{1}{2}$ possible.

- For real symbols it is always possible to design an OSTBC with rate 1.

- STBC with rate higher than $1 \rightarrow$ will need more complicated receiver / decoding.

6/. Spatial multiplexing architectures:

+) Horizontal encoding: the V_BLAST structure

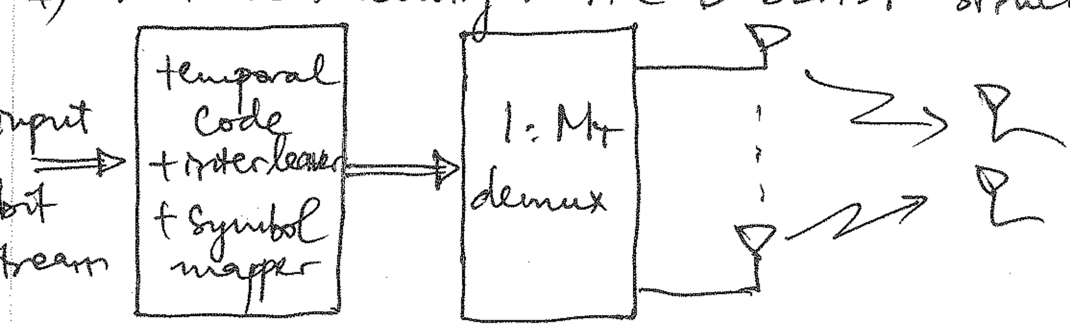


. In horizontal encoding, separate data streams undergo independent temporal coding (channel code), interleaving and symbol mapping.

. The multiplexing gain is M_t.

The diversity gain is at most M_r since any given symbol is transmitted from only one antenna.

+) Vertical encoding: the D-BLAST structure



. Here data stream undergoes temporal coding, interleaving and symbol mapping before being demultiplexed into M_t streams to be transmitted over the antennas.

. The multiplexing gain is M_t.
The diversity gain can potentially reach $M_r M_t$ if each