Multicarrier modulation.

- In this topic we will study multicarrier modulation as a method of sending signals over a broad band (which is usually frequency selective).

- For broadband communications, since the transmission bandwidth is large, usually much larger than the channel coherence bandwidth, then the frequency selectivity of the channel has to be overcome in some way. In general there are three approaches:

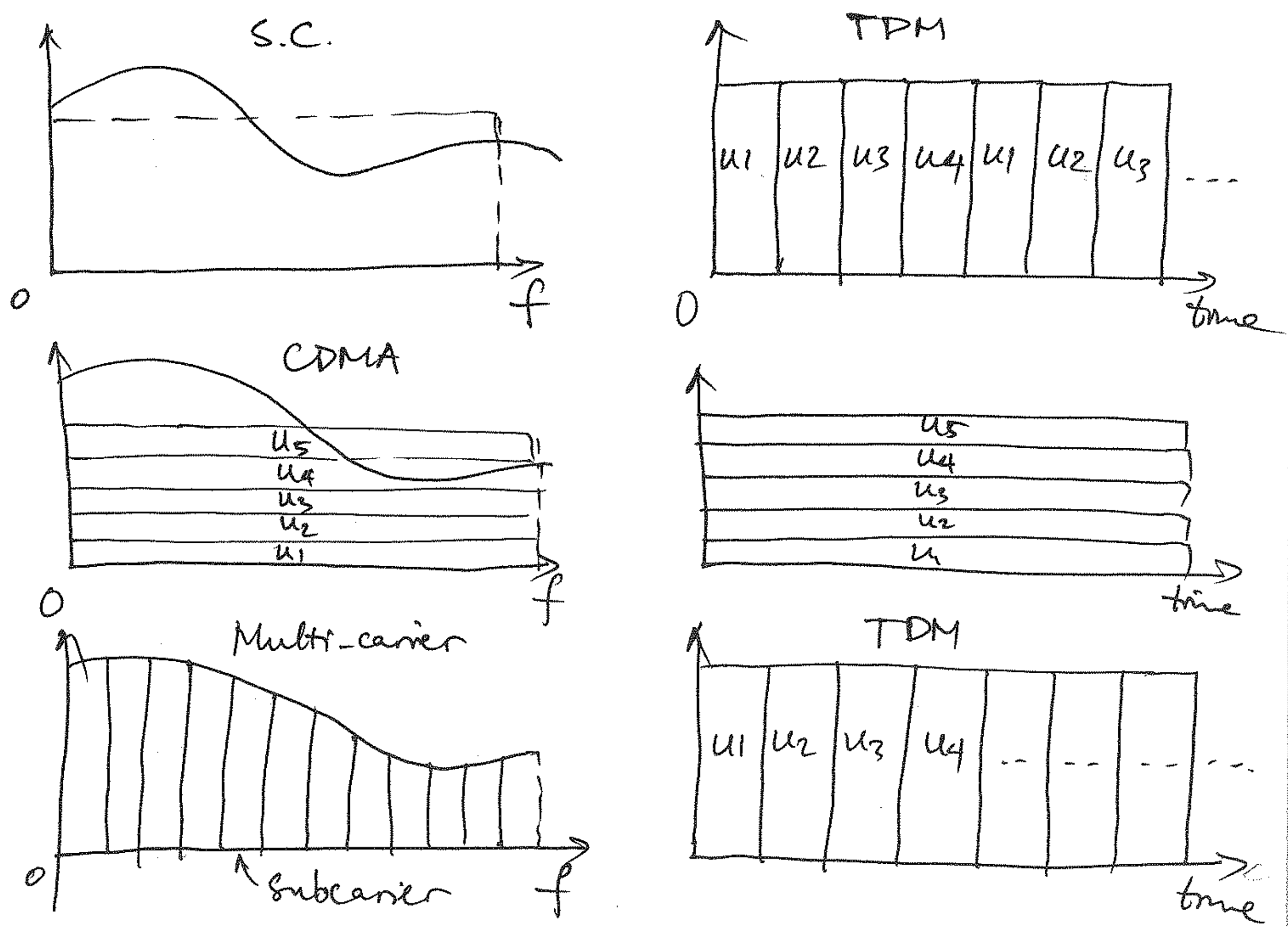
  - Single carrier equalization: Signals are sent over the whole transmission BW using a single carrier, and time equalization is employed at the receiver to deal with inter-symbol interference.
    This approach usually requires very long and accurate equalizers which adapt to the channel. For fading channels that change often, this approach requires continuous updates of the equalizer filter taps and therefore is not appealing in practice.

  - Spread spectrum: Uses (quasi) orthogonal code sequences to spread symbol over the wideband. Multiple users transmit concurrently over the whole transmission band and are separated at a receiver based on the orthogonal codes. The system works well when there are a small number of users but suffers from multiple access interference when the number of users is large.
    This approach is first used in military application because of its ability to hide signals (below noise floor) and resist narrowband jamming. The approach is used in cellular systems in 3rd generation and some 2nd generation systems.

– Multicarrier modulation: Here the wide transmission band is divided into subcarriers, each is much smaller than the channel cohence bandwidth, and therefore exhibits frequency flat fading. The data is modulated onto these multiple subcarriers, different techniques of modulation exist which we will study. Usually no equalization is necessary.

Multicarrier modulation is used in all current WiFi (WLAN) systems and in 4G cellular, the specific modulation technique is OFDM.

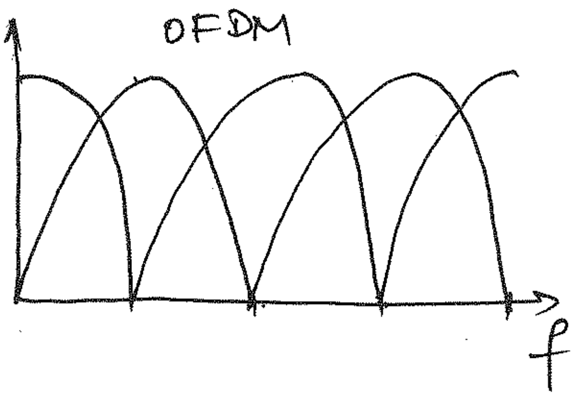
Multicarrier modulation is being considered for 5G cellular, although the specific technique is not yet settled. Several candidates are contending: OFDM, FBMC, UFMC.

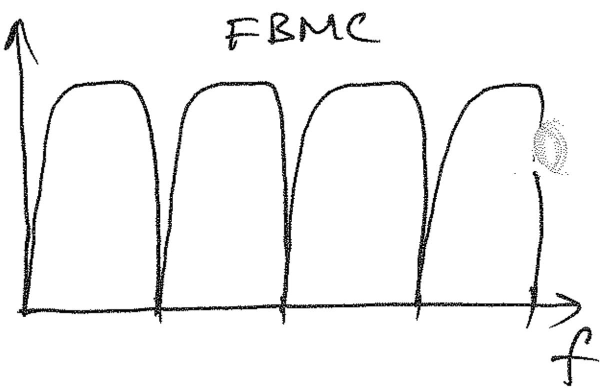


+) In multicarrier modulation, there are also two different approaches: overlapping subcarriers and non overlapping subcarriers!

∘ Overlapping subcarriers: this approach relies on orthogonal modulation waveforms to ensure no inter_carrier interference. This is the approach in OFDM, also called DMT (discrete multi-tone)

∘ Non-overlapping subcarriers: this approach relies on sharp filters to separate the subcarriers. This approach is generally called filtered multitone (FMT) or FBMC (filter bank multicarrier).



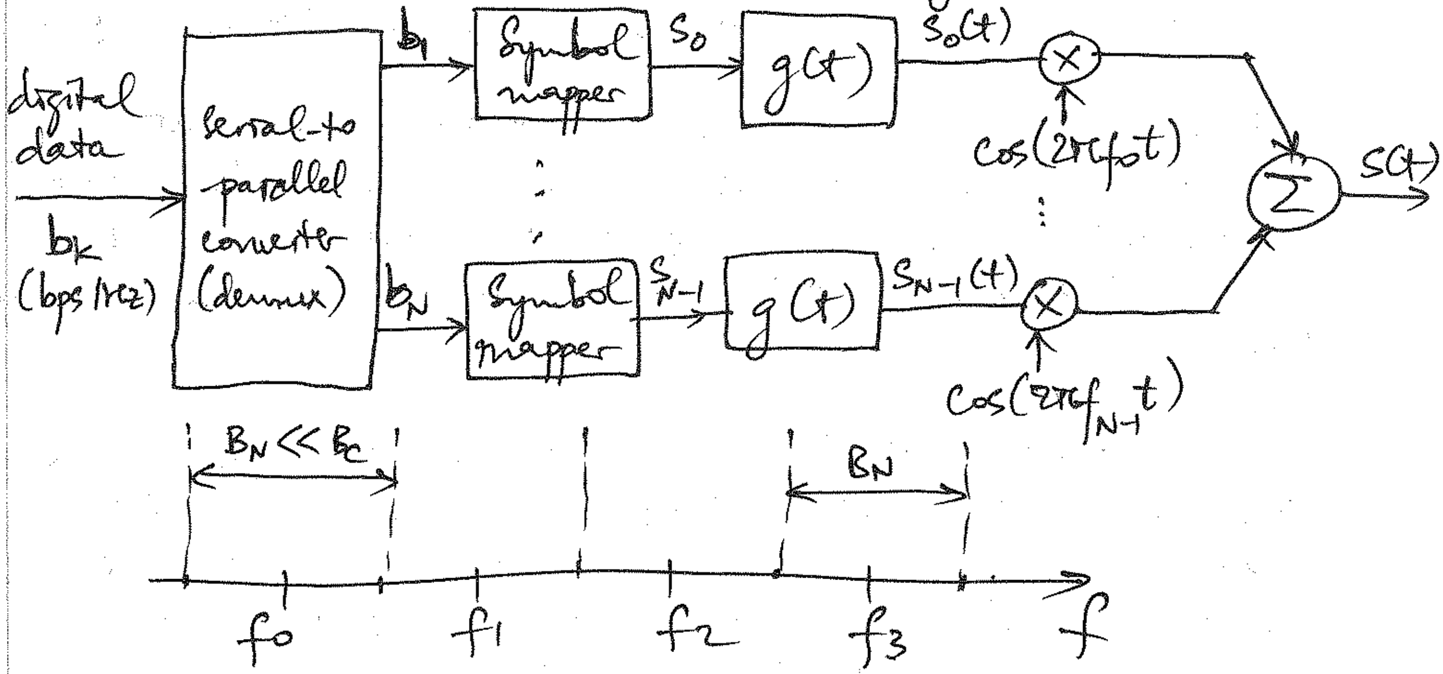


∘ There are pros and cons in each approach which we will study.

+) In both approaches, the wide transmission band is divided into N subcarriers, each with bandwidth $B_N = \frac{B}{N} \ll B_c$ the coherence bandwidth.

Each subcarrier is much smaller than the channel coherence BW and can be considered as having flat frequency. The level of the frequency response in each subcarrier, however, is random due to fading. (flat fading).

General multicarrier transmitter diagram:



+) <u>MC modulation with overlapping subcarriers:</u>

- Sub-carriers $\{\cos(2\pi f_i t + \phi_i)\}$, $f_i = \frac{i}{T_N} + f_0$ forms an orthogonal basis:

  T_N = Symbol time of modulated signals.
  ($T_N \gg T_c$ channel coherence time)
  $T_N \approx \frac{1}{B_N}$ usually.

- Denote $\varphi_k(t) = \cos(2\pi f_k t + \phi_k)$, $0 \le t \le T_N$

then

$$\frac{1}{T_N}\int_0^{T_N} \varphi_k(t)\varphi_j(t)\,dt$$

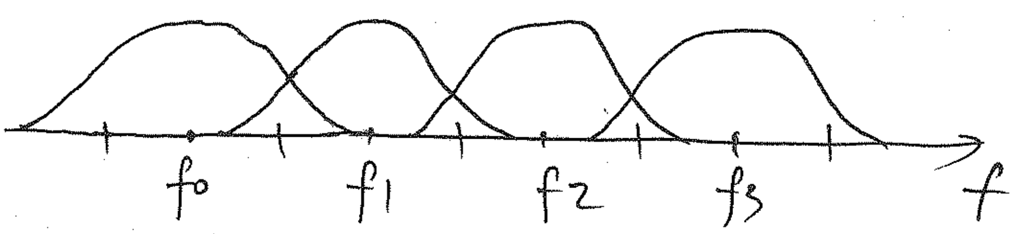
$$= \frac{1}{2T_N}\int_0^{T_N}\left(\cos\left[2\pi\frac{(k-j)t}{T_N} + \phi_k - \phi_j\right] + \cos\left[2\pi\left(\frac{(k+j)}{T_N} + 2f_0\right)t + \phi_j + \phi_k\right]\right)dt$$

$= 0$, usually $f_0 \gg \frac{1}{T_N}$ and is integer multiple of $\frac{1}{T_N}$!

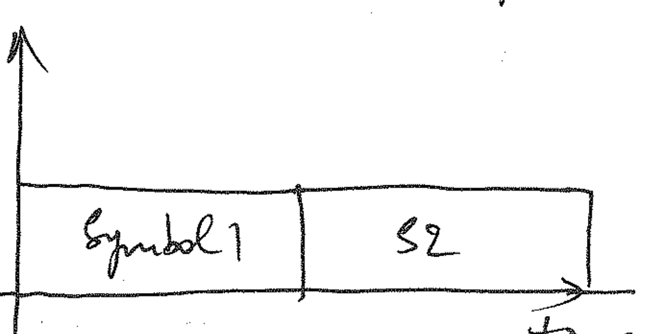
- Thus the subcarriers modulated by $\{\cos(2\pi f_k t + \varphi_t)\}$ are orthogonal regardless of individual phases φ_t.

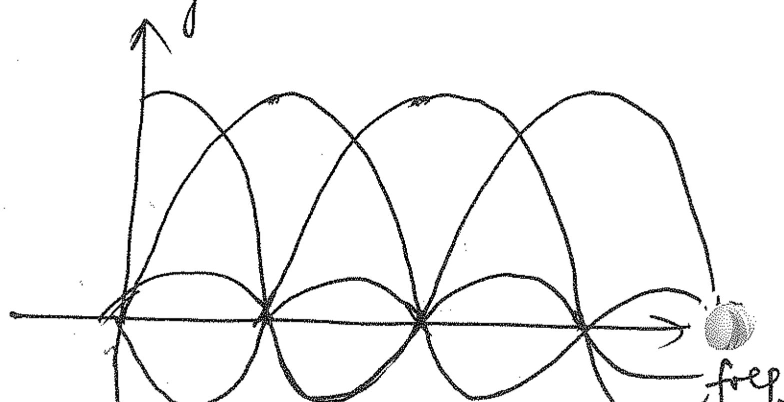
- This orthogonality would hold perfectly without any frequency overlapping if the time domain is infinite (T_N). But the signal in time is always of finite duration, dictated by a pulse shape $g(t)$. Hence the subcarriers will overlap in frequency.

- For example, if $g(t)$ is a raised cosine pulse, then there is a bandwidth expansion factor α so that each subcarrier's signal actually occupies a bandwidth of $(1+\alpha)B_N$, thus an overlapping band of αB_N between 2 adjacent subcarriers.



- For OFDM, the pulse shaping function is the square pulse so the frequency response is the sinc function. The subcarriers overlaps significantly.





- Luckily, OFDM can be implemented entirely in both discrete time and discrete frequency so the overlapping of subcarriers doesnot matter as long as they do not overlap at sampled frequencies.

+) <u>Orthogonal frequency division multiplexing - OFDM:</u>

- OFDM uses rectangular pulse shaping functions and produces sinc frequency responses that overlap between subcarriers.

- OFDM can be implemented as DFT and IDFT operations, via Fast Fourier Transform (FFT) algorithms.

We will not go into the details of FFT algorithms but will review the DFT and IDFT (transforms) to understand the architecture of an OFDM system.

- The discrete Fourier transform - DFT:

Consider an N-point discrete-time signal $x[n]$, $n=0 \ldots N-1$. Its N-point DFT is defined as

$$X[k] \triangleq \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} x[n] e^{-j2\pi \frac{nk}{N}} \quad , k=0 \ldots N-1$$

The time sequence can be recovered from the frequency sequence through the inverse DFT (IDFT):

$$x[n] \triangleq \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X[k] e^{j2\pi \frac{nk}{N}} \quad , n=1 \ldots N-1$$

DFT operation has a special circular convolution property.

- Circular convolution:

Given 2 N-point discrete-time signals $x[n]$ and $h[n]$, their circular convolution is defined as

$$y[n] = x[n] \circledast_N h[n] = \sum_{k=0}^{N-1} h[k]\, x[n-k]_N$$

where $[n-k]_N = (n-k) \bmod N$

$x[n-k]_N$ is the circular shift of sequence $x[n]$.

- Circular convolution property for DFT:

$$x[n] \circledast_N h[n] \xleftrightarrow{DFT} X[k] \cdot H[k]$$

$$x[n] \cdot h[n] \xleftrightarrow{DFT} X[k] \circledast_N H[k].$$

+) <u>Cyclic prefix</u>:

- Cyclic prefix is added to the input OFDM sequence to create an effective circular convolution operation through the linear channel.

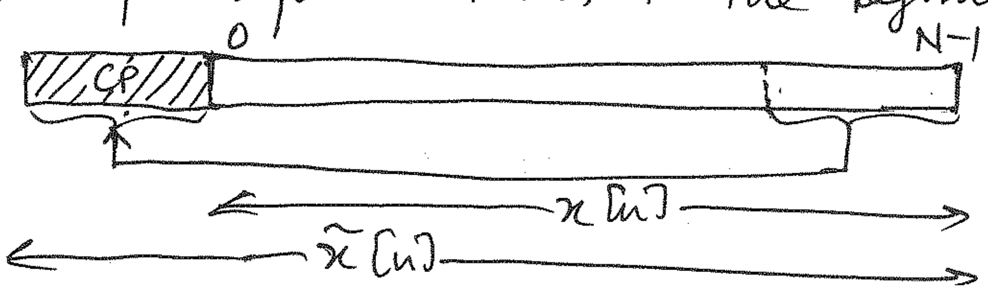
- Suppose that the input sequence is length N

$x[n]$, $n = 0 \ldots N-1$

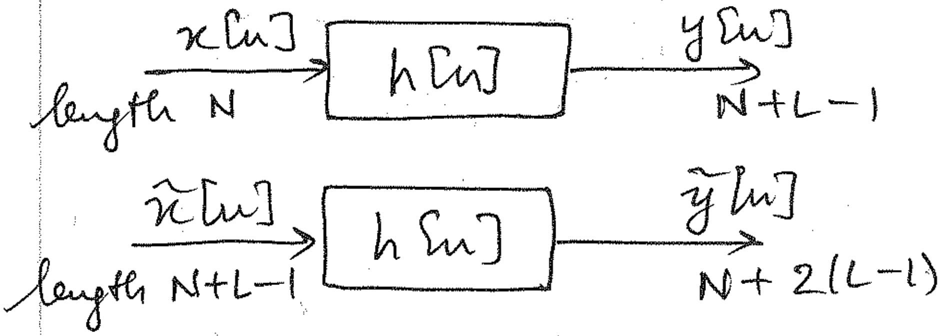
The channel with delay spread has length L.

$h[k]$, $k = 0 \ldots L-1$.

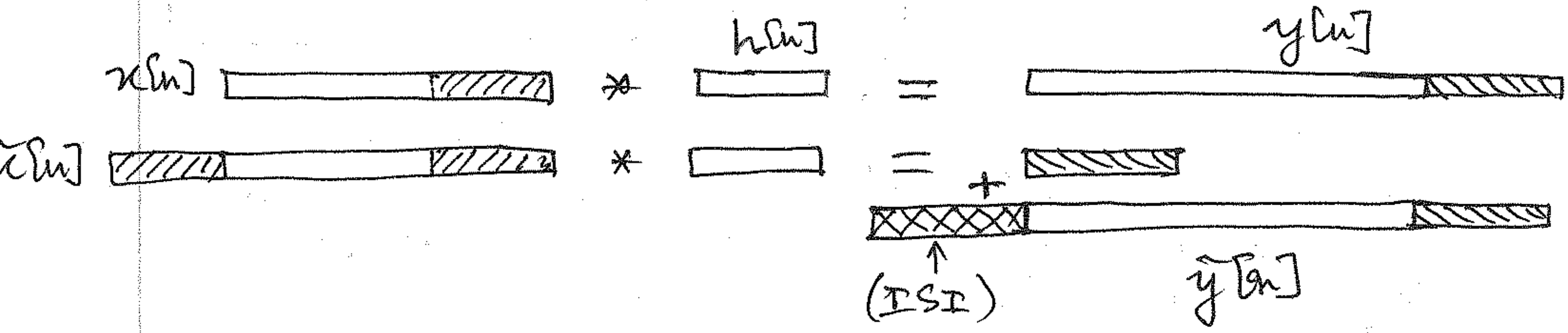
- The cyclic prefix is to append the last L-1 symbols of the input sequence $x[n]$ to the beginning of it.



$$\tilde{x}[n] = x[n]_N, \quad -(L-1) \le n \le N-1$$

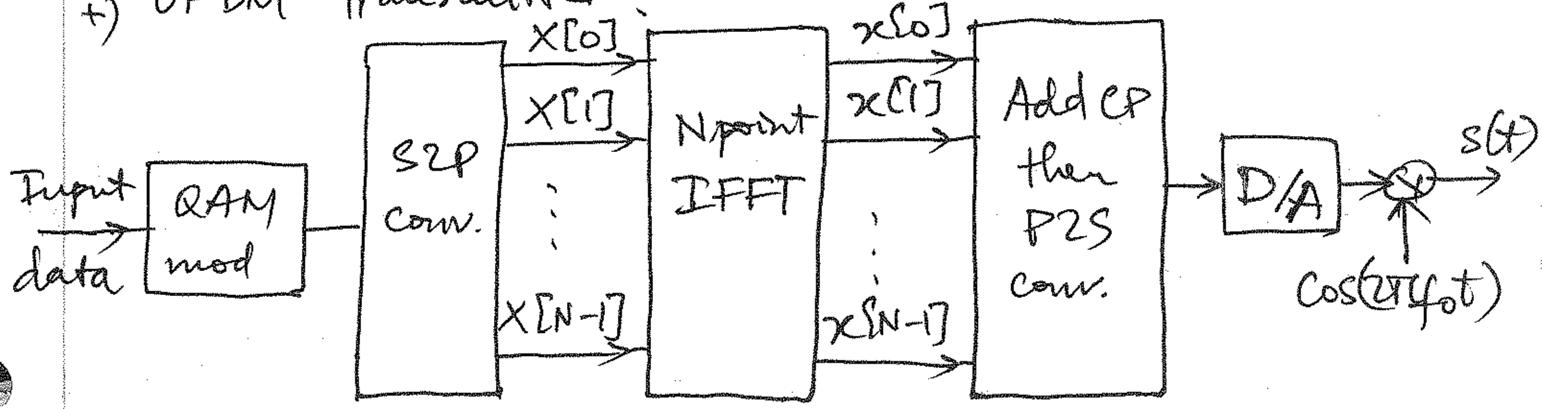


$\tilde{y}[n] = x[n] \circledast_N h[n]$ for $0 \le n \le N-1$.



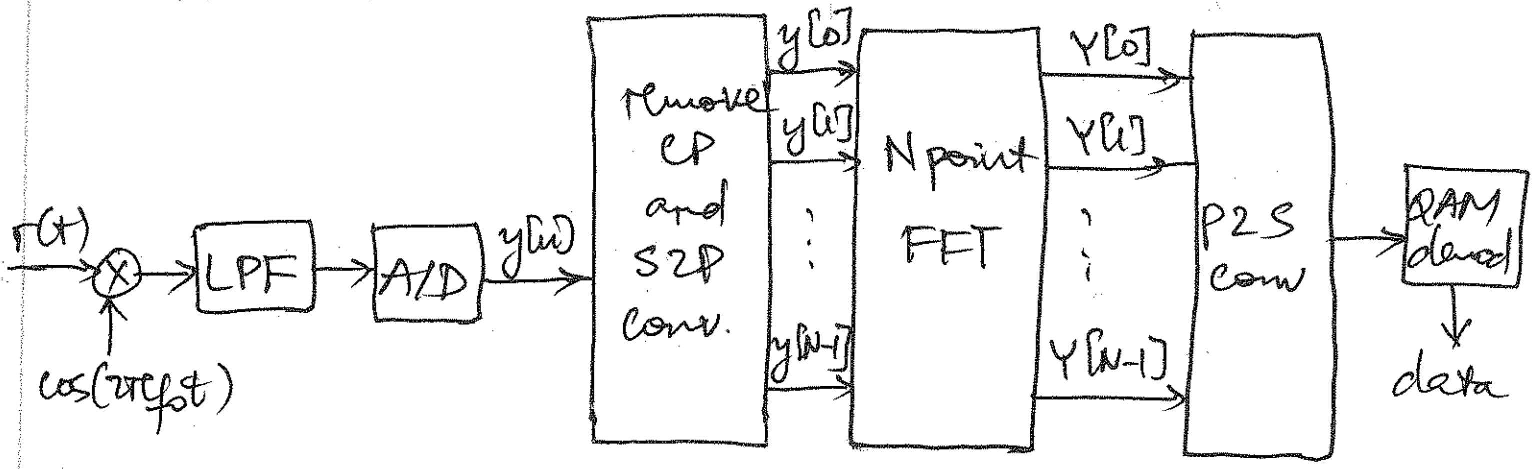
∘ The cyclic prefix allows for the implementation of OFDM via DFT (FFT) operations but also adds overheads to the transmission.
The CP sequence of $L-1$ symbols carry no new data and yet consumes transmit power.
The data rate is reduced by a ratio of $\frac{N}{N+L-1}$.

+) OFDM transmitter:



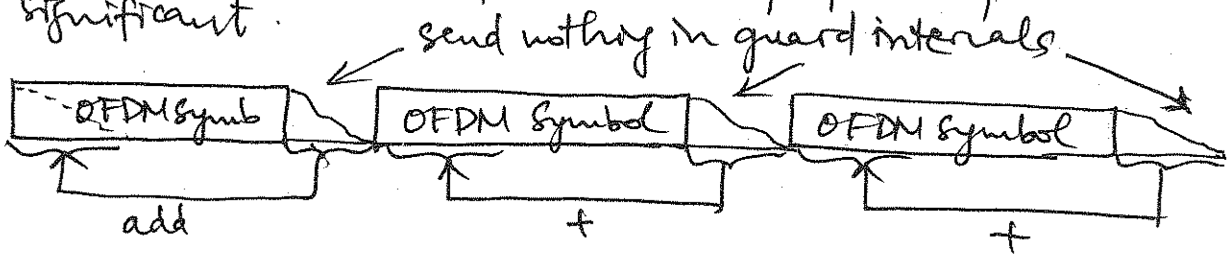
$X[k]$: block of size N input (QAM) symbols is referred to as an OFDM symbol.

OFDM Receiver:



- Instead of spending power to send the CP, one can also sends zero-symbol CP, hence consuming no transmit power on the CP, and just add the tail of each receive sequence back to the beginning to create the circular convolution effect.

This implementation adds noise from the tail as well but the difference in SNR of the two prefix implementation is not significant.



+) Mitigation of subcarrier fading:

- Even though each subcarrier is flat, it still experiences fading such that the attenuation (or received power) is random.

- Suppose that the channel gain in subcarrier k is h_k. The received power in subcarrier k is

$$\gamma_k = \frac{P_k}{\sigma_n^2} \cdot |h_k|^2 .$$

Since γ_k is random, it affects the performance (BER, outage). Several techniques are available to mitigate the effect of fading.

i) Coding with interleaving over time and frequency.

. Each codeword is spread over both time and frequency, thus the coded bits experience independent fading and hence improves reliability.

. This technique is simple, requiring no CSIT or complicated Tx/Rx processing. It works well when the whole transmission band is much larger than the channel coherence BW, thus spreading a codeword over subcarriers can bring freq. diversity.

ii) Frequency equalization:

. This technique is essentially channel inversion on each sub carrier to equalize the output SNR. But it enhances noise power in bad subcarriers. Other equalizers (MMSE) are possible.

iii) Precoding:

. This technique relies on CSIT to compensate for the channel fading. The simplest form is channel inversion. Other form of precoding are available depending on the type of CSIT and the design criterion.

iv) Power control:

. This technique also relies on CSIT (and can be thought of as a type of precoding) to allocate power on the subcarriers according to a design criterion.

. For example, to maximize transmission rate, the power allocation will be waterfalling across subcarriers. This technique requires accurate instantaneous CSIT.