

Lecture 3: Large-scale fading: path loss and shadowing.

+ Path loss: attenuation of signal power over distance
Shadowing: variation in the mean (average) signal power caused by large obstacles in the environment.

◦ Path loss and shadowing are models for large-scale propagation effects. They capture the change in the mean (average) received power over the distance between the transmitter and the receiver in a given environment.

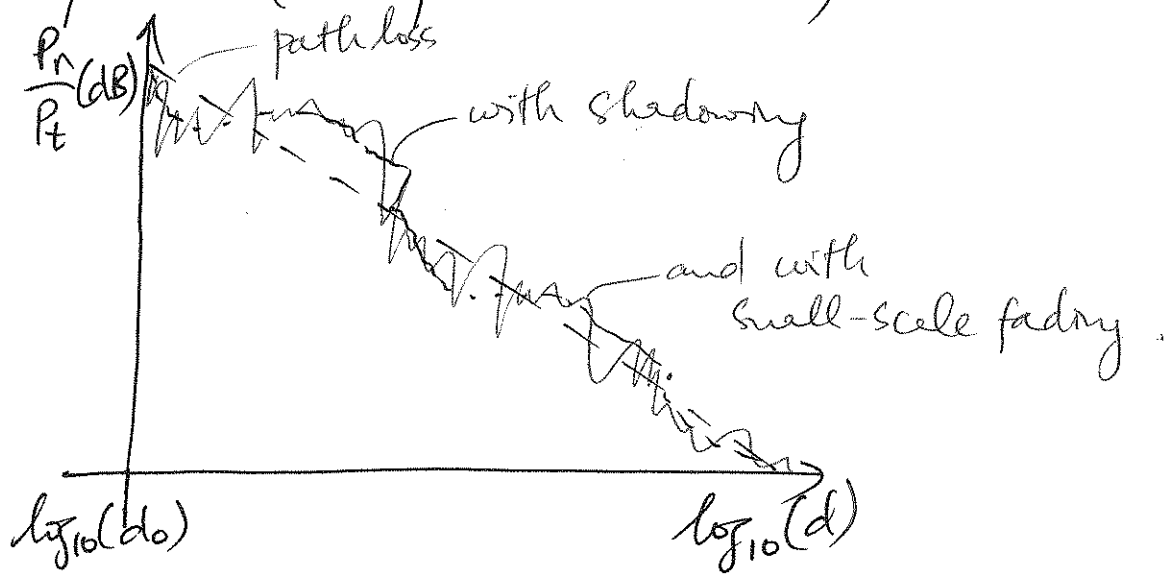
◦ Path loss usually occurs over a long distance (100-1000 m)
Shadowing occurs over distances proportional to the sizes of the obstructing objects (buildings, 10-100m outdoor and less indoor).

◦ The actual received power at a given Tx-Rx distance and fixed receiver location will fluctuate around the mean. This fluctuation is caused by the multipath effect because of reflection and scattering components, which can occur randomly and superpose constructively and destructively at the receiver.

◦ These multipath components cause variation in the received power on a small scale in the order of the signal wavelength. This effect is called small-scale fading and will be discussed in the next topic.

◦ Fading: received signal power decreases (usually in a random manner).

A rough picture (conceptual illustration)



1. Path loss: a simplified model

Recall the free-space propagation law:

$$P_{RX} = P_{TX} \cdot G_{TX} \cdot G_{RX} \cdot \left(\frac{\lambda}{4\pi d}\right)^2$$

Take $10 \log_{10}$ of both sides:

$$P_{RX}(\text{dB}) = P_{TX}(\text{dB}) + 10 \log_{10} \left(G_{TX} \cdot G_{RX} \cdot \frac{\lambda^2 d_0^2}{(4\pi)^2} \right) - 20 \log_{10} \left(\frac{d}{d_0} \right)$$

or

$$P_{RX}(\text{dB}) = P_{TX}(\text{dB}) + K(\text{dB}) - 10 \times 2 \log_{10} \left(\frac{d}{d_0} \right)$$

where K : a unitless constant

d_0 : a reference distance for antenna far field

For free-space, the path loss exponent is 2.

◦ For a practical environment, a log-distance pathloss model is given as:

$$P_{RX}(\text{dB}) = P_{TX}(\text{dB}) + K(\text{dB}) - 10\gamma \log\left(\frac{d}{d_0}\right)$$

where γ is the pathloss exponent in a specific environment.

◦ Usually K , d_0 and γ are obtained based on empirical measurements in that environment and then numerical fitting to the model.

The fitting can be done through minimizing the mean-squared-error (MSE) between the measured data and the fitted model.

◦ Another approach is to do curve fitting on the empirical data to obtain a more detailed model. Such an empirical model can be more accurate for a specific environment, but the model needs to be calibrated for each new environment.

◦ Some well-known empirical models for pathloss/outdoor propagation includes:

- Longley-Rice model: 40 MHz - 100 GHz range

- over irregular terrains
- usually assume a line of sight (LOS)
- use path geometry and terrain profile.

- Okumura model: 150 MHz - 1920 MHz

- for urban areas
- includes base station heights 30 - 1000 m.
- wholly based on empirical data, little analytical
- good accuracy for urban, widely used in Japan

- Hata model: 150 MHz - 1500 MHz.
 - provide analytical extension to the graphical Okumura model
 - empirical formula for urban areas
 - correction terms for other situations.
- COST 231 Extension to Hata: extends to 2 GHz.
 - by the European Cooperative for Scientific & Technical research
 - 1500 MHz - 2000 MHz, d : 1 km - 20 km.

◦ There are also indoor propagation models that consider losses through partition, walls, floor concretes, hall ways, windows...

2. Shadowing:

- Shadowing accounts for the fact that even at the same Tx-Rx distance, the surrounding environments may be vastly different at two different locations.
- This difference causes different values in the average received power of the measured signal: the shadowing effect.
- Shadowing is usually random and is model as a random variable. The most common model is log-normal shadowing.
- With shadowing, the received power can be written as

$$P_{RX}(\text{dB}) = P_{TX}(\text{dB}) + K(\text{dB}) - 10\gamma \log\left(\frac{d}{d_0}\right) + \varphi_{\text{dB}}$$
 where φ_{dB} is a zero-mean Gaussian random variable.
- Note that φ_{dB} is measured in dB, which is the $10\log_{10}$

of the linear component. Hence the name "log-normal".

$\Psi_{dB} \sim N(0, \sigma_{\Psi_{dB}}^2)$, $\sigma_{\Psi_{dB}}^2$: variance of shadowing.

3. Outage and cell coverage under large scale fading:

+) Outage occurs when the received power falls below a certain level P_{min} needed to have acceptable performance.

$$P_o(d) = \Pr(P_r(d) \leq P_{min})$$

$$= 1 - Q\left(\frac{P_{min} - P_{avg}}{\sigma_{\Psi_{dB}}}\right)$$

where

$$P_{avg} = P_{Tx}(dB) + K(dB) - 10\gamma \log_{10}\left(\frac{d}{d_0}\right).$$

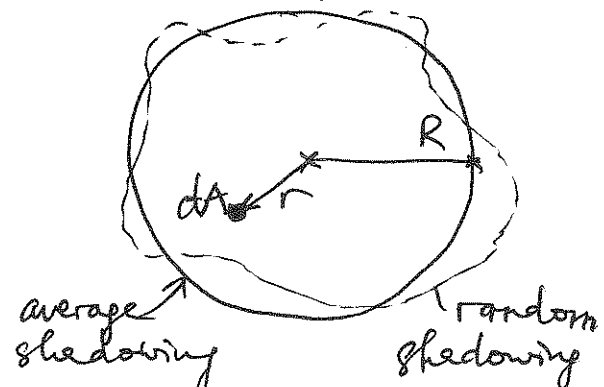
$Q(z) = \frac{1}{2} \operatorname{erfc}\left(\frac{z}{\sqrt{2}}\right)$ is the Q-function

$$\triangleq \Pr(X > z) = \int_z^{\infty} \frac{1}{\sqrt{2\pi}} e^{-y^2/2} dy$$

(probability that a standard Gaussian R.V is $> z$).

+) Cell coverage refers to the percentage of a cell within which the received power is at least P_{min} .

Let $P_{Rx}(r)$ be the received power at a radius r from the base station.



For an incremental area dA , let

$$P_A = \Pr(P_{RX}(r) > P_{min}) \text{ in } dA = 1 - P_0(r)$$

Then coverage area can be computed as

$$C = \frac{1}{\pi R^2} \int_{\text{cell area}} P_A dA = \frac{1}{\pi R^2} \int_0^{2\pi} \int_0^R P_A r dr d\theta$$

After some manipulation, we get a closed-form equation:

$$C = Q(a) + \exp\left(\frac{2-2ab}{b^2}\right) Q\left(\frac{2-ab}{b}\right)$$

where $a = \frac{P_{min} - \bar{P}_{RX}(R)}{\sigma_{4dB}}$, $b = \frac{10\gamma \log_{10}(e)}{\sigma_{4dB}}$

$\bar{P}_{RX}(R)$ is the average received power at the cell boundary

If $P_{min} = \bar{P}_{RX}(R)$ then the coverage area simplifies to

$$C = \frac{1}{2} + \exp\left(\frac{2}{b^2}\right) Q\left(\frac{2}{b}\right)$$

Example: Cell radius $R = 600\text{m}$; Path loss $\gamma = 3.71$

Base station power $P_{TX} = 100\text{mW} = 20\text{dBm}$

Then average received power at the cell edge is

$$\bar{P}_{RX}(R) = P_t + K - 10\gamma \log_{10}(600) = -114.6\text{dBm}$$

(31.54 dB)

If $P_{min} = -110\text{dBm}$, then coverage area is $C = .59$.

$P_{min} = -120\text{dBm} \rightarrow C = 0.988$.