Lecture 8: Large-scale fading: path loss and shadowing.

Path loss: attenuation of signal power over distance

Shadowing: variation in the mean (average) signal power caused by large obstacles in the environment.

Path loss and shadowing are models for large-scale propagation effects. They capture the change in the mean (average) received power over the distance between the transmitter and the receiver in an indoor or outdoor environment.

Path loss usually occurs over a long distance (100-1000 m). Shadowing occurs over distances proportional to the sizes of the obstructing objects (buildings, 10-100 m outdoor and less indoor).

The actual received power at a given Tx-Rx distance and fixed receiver location will fluctuate around the mean. This fluctuation is caused by the multipath effect because of reflection and scattering components, which can occur randomly and superpose constructively and destructively at the receiver.

These multipath components cause variation in the received power on a small scale in the order of the signal wavelength. This effect is called small-scale fading and will be discussed in the next topic.

Fading: received signal power decreases (usually in a random manner).
1. Pathloss: a simplified model

Recall the free-space propagation law:

\[ P_{RX} = P_{TX} \cdot G_{TX} \cdot G_{RX} \cdot \left( \frac{\lambda}{4\pi d} \right)^2 \]

Take \( \log_{10} \) of both sides:

\[ P_{RX} (dB) = P_{TX} (dB) + 10 \log_{10} \left( G_{TX} \cdot G_{RX} \cdot \frac{\lambda^2}{4\pi d^2} \right) \]
\[ - 20 \log_{10} \left( \frac{d}{d_0} \right) \]

or

\[ P_{RX} (dB) = P_{TX} (dB) + K (dB) - 10 \times 2 \log_{10} \left( \frac{d}{d_0} \right) \]

where
- \( K \): a unitless constant
- \( d_0 \): a reference distance for antenna far field

For free-space, the pathloss exponent is 2.
For a practical environment, a log-distance path-loss model is given as:

$$P_{RX} (dB) = P_{TX} (dB) + K (dB) - 10 \gamma \log\left(\frac{d}{d_0}\right)$$

where $\gamma$ is the path-loss exponent in a specific environment.

Usually, $K$, $d_0$ and $\gamma$ are obtained based on empirical measurements in that environment and then numerical fitting to the model.

The fitting can be done through minimizing the mean-squared-error (MSE) between the measured data and the fitted model.

Another approach is to do curve fitting on the empirical data to obtain a more detailed model. Such an empirical model can be more accurate for a specific environment, but the model needs to be calibrated for each new environment.

Some well-known empirical models for path-loss (outdoor propagation) includes:

- Longley - Rice model: 40 MHz - 100 GHz range
  - over irregular terrains
  - usually assume a line of sight (LOS)
  - use path geometry and terrain profile

- Okumura model: 150 MHz - 1920 MHz
  - for urban areas
  - includes base station heights 30 - 1000 m
  - wholly based on empirical data, little analytical
  - good accuracy for urban, widely used in Japan
- Hata model: 150 MHz – 1500 MHz
  - provide analytical extension to the graphical Okumura model
  - empirical formula for urban areas
  - correction terms for other situations.

- COST 231 Extension to Hata: extends to 2 GHz
  - by the European Cooperative for Scientific & Technical Research
  - 1500 MHz – 2800 MHz, d: 1 km – 20 km

There are also indoor propagation models that consider losses through partition, walls, floor, concrete, hallways, windows...

2. Shadowing:

- Shadowing accounts for the fact that even at the same Tx-Rx distance, the surrounding environments may be vastly different at two different locations.
- This difference causes different values in the average received power of the measured signal: the shadowing effect.
- Shadowing is usually random and is modeled as a random variable. The most common model is log-normal shadowing.
- With shadowing, the received power can be written as

  \[ P_{RX} (dB) = P_{TX} (dB) + K (dB) - 10 \gamma \log \left( \frac{d}{d_0} \right) + \eta dB \]

  where \( \eta dB \) is a zero-mean Gaussian random variable.

  Note that \( \eta dB \) is measured in dB, which is the logarithm...
of the linear component. Hence the name "log-normal."

\[ Y_{dB} \sim N(0, \sigma_{Y_{dB}}^2) \]  
\[ \sigma_{Y_{dB}}^2 : \text{variance of shadowing.} \]

3. Outage and cell coverage under large scale fading:

Outage occurs when the received power falls below a certain level, \( P_{min} \), needed to have acceptable performance.

\[ P_o = \Pr (P_r(d) \leq P_{min}) \]

\[ = 1 - Q\left( \frac{P_{min} - P_{avg}}{\sigma_{Y_{dB}}} \right) \]

where

\[ P_{avg} = P_{tx}(dB) + K(db) - 10\log_{10}\left( \frac{d}{d_0} \right) \]

\[ Q(z) = \frac{1}{2} \text{erfc}\left( \frac{z}{\sqrt{2}} \right) \]

\[ = \Pr (X > z) = \int_z^\infty e^{-\frac{y^2}{2}} dy \]

(Probability that a standard Gaussian R.V. is > z).

Cell coverage refers to the percentage of a cell within which the received power is at least \( P_{min} \).

Let \( P_{rx}(r) \) be the received power at a radius \( r \) from the base station.
For an incremental area \( dA \), let
\[
P_a = Pr(Prx(r) > P_{\text{min}}) \text{ in } dA = 1 - P_0(r)
\]
Then coverage area can be computed as
\[
C = \frac{1}{\pi R^2} \int \int \frac{P_a}{dA} = \frac{1}{\pi R^2} \int_0^{2\pi} \int_0^R P_a \ r \ dr \ d\theta
\]
After some manipulation, we get a closed-form expression:
\[
C = Q(a) + \exp\left(\frac{2 - 2ab}{b^2}\right) Q\left(\frac{2-a}{b}\right)
\]
where \( a = \frac{P_{\text{min}} - Prx(R)}{64\pi R^2} \quad b = \frac{10g \log_{10}(e)}{64\pi R^2} \)

- \( Prx(R) \) is the average received power at the cell boundary.

If \( P_{\text{min}} = Prx(R) \) then the coverage area simplifies
\[
C = \frac{1}{2} + \exp\left(\frac{2}{b^2}\right) Q\left(\frac{2}{b}\right)
\]

Example: Cell radius \( R = 600 \text{m} \); Path loss \( g = 3.71 \)
Base station power \( Prx = 100 \text{mW} = 20 \text{dBm} \)
Then average received power at the cell edge is
\[
Prx(R) = Pt + K - 10g \log_{10}(600) = -114.6 \text{ dBm}.
\]
\[
(31.5 \text{ dB})
\]
If \( P_{\text{min}} = -110 \text{ dBm} \), then coverage area is \( C = .59 \).
If \( P_{\text{min}} = -120 \text{ dBm} \), then coverage area is \( C = 0.988 \).