# Lecture 4: Small-scale fading: Statistical Multipath Channel Models

- As discussed earlier, the actual received signal/power at a receive antenna varies significantly over time, even at a given Tx - Rx distance and a given frequency band.

- This fluctuation in the received signal power is caused by reflection and scattering effects.

- Usually the number of scatterers is large, causing the signal to arrive at the receive antenna via multiple propagation paths.

- These multipaths can add constructively and destructively, causing the signal at the receiver to appear random.

- This randomness in received signal is called small-scale fading (caused mainly by scatterers in the vicinity of the receiver) and is usually characterized statistically.

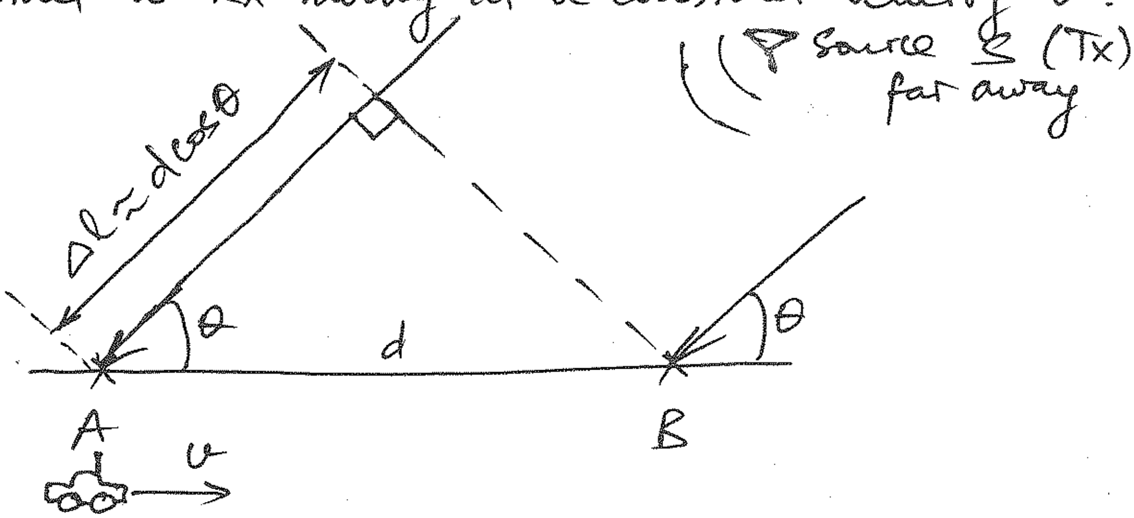
+) Small-scale fading effects include:
  - rapid changes in signal strength over a small distance or time interval.
  - random frequency expansion due to the Doppler effect
  - time dispersion (echos) caused by multipath propagation delays.

The received signal at any point in space may consist of a large number of plane waves (paths) with random amplitudes, phases, and angles of arrival.

+) Factors influencing small-scale fading:
- multipath propagation from reflection and scattering objects.
- speed of the Tx and/or Rx
- speed of surrounding objects
- the transmission bandwidth of the signal.

## 1) Doppler shift:

Consider a Rx moving at a constant velocity $v$.




Assume source S (Tx) is far away such that far field propagation holds between point A & B.

Suppose: $d$ = distance traveled between A & B.
$\theta$ = angle of arrival
$v$ = speed of moving object (Rx)
$\Delta t$ = time required for traveling from A→B.

Then there will be a change in the phase of the received signal due to mobility. The phase change is proportional to the travel distance measured in wavelengths:

$$\Delta\phi = \frac{2\pi \Delta l}{\lambda} = \frac{2\pi d \cos\theta}{\lambda} = \frac{2\pi}{\lambda} v . \Delta t . \cos\theta$$

This phase change will cause a change in the instantaneous frequency of the received signal. Specifically the frequency is shifted by an amount called the "Doppler" shift, given as

$$f_d = \frac{1}{2\pi}\frac{\Delta\phi}{\Delta t} = \frac{v}{\lambda}\cos\theta, \quad -\pi \le \theta \le \pi.$$

- If $|\theta| < \frac{\pi}{2}$ : Rx moving towards Tx $\rightarrow \cos\theta > 0$
  $f_d > 0$ or the frequency is increased.
  $|\theta| > \frac{\pi}{2}$ : Rx moving away from Tx $\rightarrow \cos\theta < 0$
  $f_d < 0$ or the frequency is decreased.

This is the effect you hear on a siren, the pitch increases as the siren moves towards you and then decreases as it moves away from you.

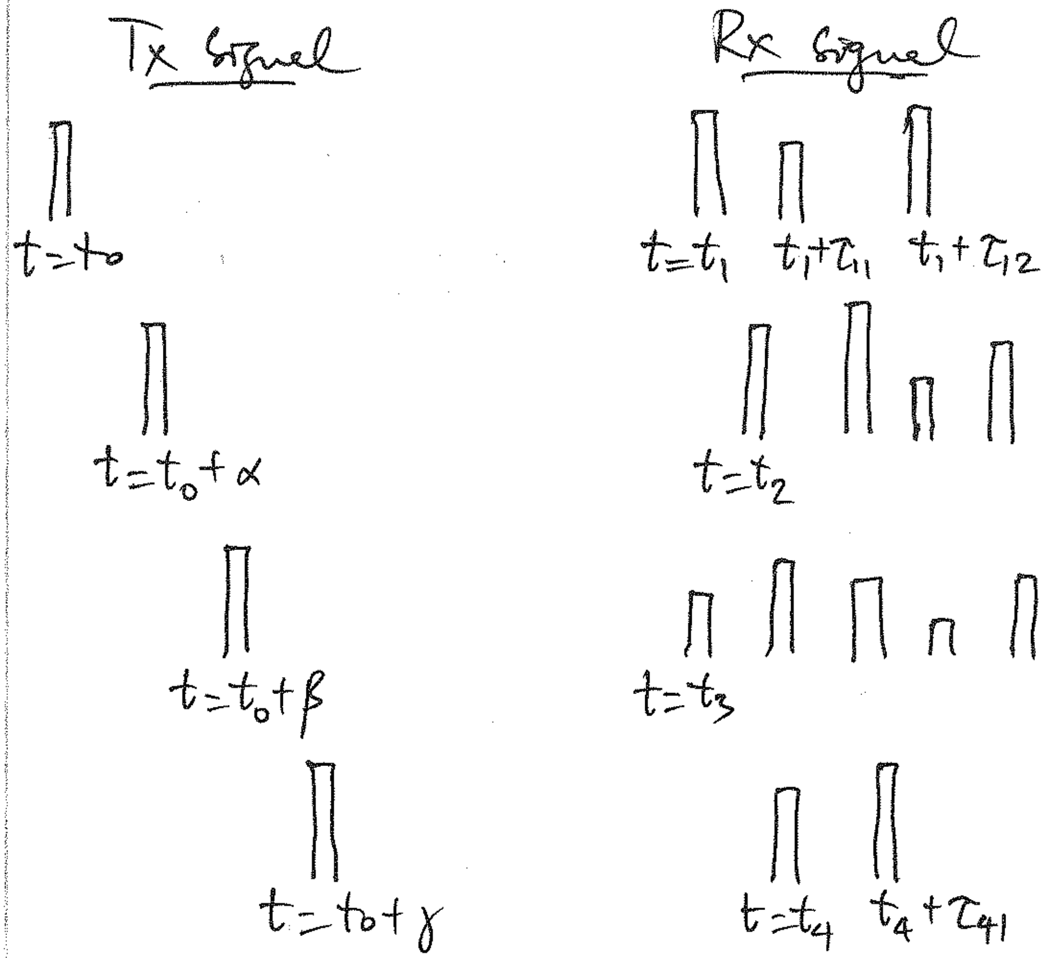
- For moving Tx/Rx, the Doppler effect creates an expansion in the frequency of the received signal that is called the Doppler spread.

If the transmitted signal were a pure sinusoidal signal (single tone), it will be received spread over a range of frequencies.

## 2/. Characterization of fading multipath channels:

- We will now characterized the channel by its canonical form: the impulse response.

- Suppose that we transmit extremely short pulses, ideally impulses, over a multipath fading channel.




- The received signal may appear as a train of pulses with:
  - varying number of pulses
  - varying strength of individual pulses
  - varying spacing among the pulses

These variations appear unpredictable. Thus multipath time-varying channel is often characterized statistically.

- Suppose that we transmit a signal that is represented in general as

$$s(t) = \mathrm{Re}\left\{ s_\ell(t)\, e^{j2\pi f_c t} \right\} \qquad (1)$$

where $f_c$: carrier frequency
$s_\ell(t)$: baseband (lowpass) equivalent signal (the part that actually contains information)

Assume multipath propagation, each path has a certain propagation delay and an attenuation factor.

- The received signal can then be expressed as

$$r(t) = \sum_n \alpha_n(t)\, s[t - \tau_n(t)] \qquad (2)$$

where:

$$n = \text{path number (index)}$$
$$\alpha_n(t) = \text{attenuation factor for the } n^{th} \text{ path}$$
$$\tau_n(t) = \text{propagation delay} \quad n$$

- Putting (1) and (2) together yields

$$r(t) = Re\left\{ \underbrace{\left( \sum_n \alpha_n(t)\, s_\ell[t - \tau_n(t)]\, e^{-j2\pi f_c \tau_n(t)} \right)}_{\text{equivalen low-pass (baseband) received signal}} e^{j2\pi f_c t} \right\}$$

- Thus the equivalent baseband channel has the following impulse response:

$$h(\tau; t) = \sum_n \alpha_n(t)\, e^{-j2\pi f_c \tau_n(t)} \cdot \delta(t - \tau_n(t)) \qquad (3)$$

- For channels with many scatterers and multipath, we can also express the receved signal on a continuum of multipath components. We re-write (2) as

$$r(t) = \int \alpha(\tau; t)\, s(t - \tau)\, d\tau \qquad (4)$$

where

$\alpha(\tau; t)$ : attenuation of the signal components at delay $\tau$.

Then the equivalent baseband impulse response becomes

$$h(\tau;t) = \alpha(\tau;t)\, e^{-j2\pi f_c \tau} \qquad (5)$$

and we can express the received signal via standard convolution:

$$r(t) = Re\left\{\left[\underbrace{h(\tau;t) * s_\ell(t)}_{\text{baseband equivalent received signal}}\right] \cdot e^{j2\pi f_c t}\right\}$$

where

$$h(\tau;t) * s_\ell(t) = \int_{-\infty}^{\infty} h(\tau;t)\, s_\ell(t-\tau)\, d\tau .$$

- The impulse response of a fading channel $h(\tau;t)$ as given in (3) and (5) is linear and time-varying (in contrast to LTI systems which are time-invariant).

To understand the effect of a linear time-varying channel, lets perform a simple thought experiment.

Suppose we send a constant signal, $s_\ell(t) = 1 \ \forall t$. Then the baseband equivalent received signal is

$$r_\ell(t) = \sum_n \alpha_n(t)\, e^{-j2\pi f_c \tau_n(t)} = \sum_n \alpha_n(t)\, e^{j\theta_n(t)}$$

where $\alpha_n(t)$ : path amplitude (of the $n^{th}$ path)
$\theta_n(t)$ : path phase

Both path amplitude and phase depend on the dynamics in the channel (or propagation environment/medium).

- Changes in $\alpha_n(t)$ usually requires large dynamic changes in the propagation medium.

- Changes in $\theta_n(t)$ however can be very rapid: $\theta_n(t)$ will change by $2\pi$ whenever the path delay $\tau_n$ changes by $\frac{1}{f_c}$, which is a very small number esp. at large carrier frequencies.

  $\theta_n(t)$ will change by $2\pi$ rad with relatively small motion in the medium.

  Path delays $\tau_n(t)$ also changes in a unpredictable (random) manner.

- Thus the channel impulse response $h(\tau;t)$ can be viewed as a complex-valued random process.

+) Signal fading: Multipath propagation results in signal fading, which is primarily the results of the time variation in the phases $\theta_n(t)$.
Sometimes these different paths add destructively, leading the very weak received signals.
Sometimes they can add constructively, leading to a strong received signal.
This amplitude variation in the received signal is termed "signal fading".

- We will study the statistical models of fading channels. But first we will examine the properties of a linear time-varying channel, without paying attention to the fact that it is random.