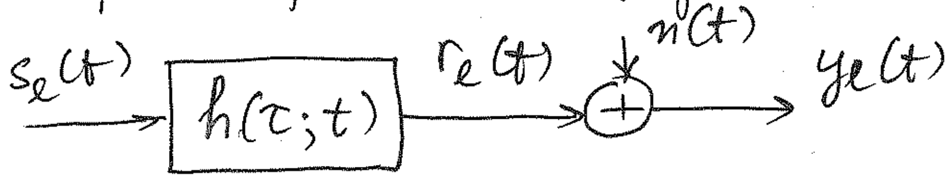
Lecture 5:

3/. Properties of a time-varying channel.



- $h(\tau;t)$: time-varying channel impulse response

Subscript "ℓ" is for low-pass equivalent signal, which is related to the modulated signal as:

$$x(t) = Re\left\{x_\ell(t)\, e^{j2\pi f_c t}\right\}$$

for any signal $x(t)$ (replaced by s, r or y respectively).

+) Channel correlation function and power spectra.

- Recall that for a random process, we study its auto-correlation function and its power spectrum density.

The auto-correlation function tells how fast the process (channel) de-correlates over time.

The power spectrum density is the Fourier transform of the auto-correlation function and tells how the average power of the process is distributed over frequency.

- We will examine the same measures for the wireless channel.

The only difference now is that since the channel is time-varying, all the functions become two-dimensional

- We assume that the wireless channel is wide-sense stationary, which means that its first and second order statistics are consistent over time (the statistics do not change over time, or that they are time-invariant).

+) Define the channel auto-correlation as:

$$R_h(\tau_2, \tau_1; \Delta t) = E[h^*(\tau_1; t)\, h(\tau_2; t+\Delta t)]$$

Uncorrelated scattering: usually the channel assocrated with path delay τ_1 is uncorrelated with the channel at path delay τ_2.

Thus the auto-correlation becomes

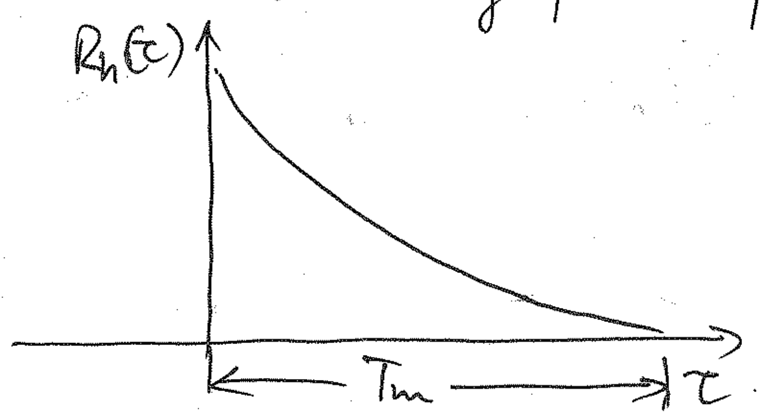
$$E[h^*(\tau_1; t)\, h(\tau_2; t+\Delta t)] = R_h(\tau_1; \Delta t)\, \delta(\tau_2 - \tau_1)$$

∘ Let $\Delta t = 0$ and denote $R_c(\tau) = R_c(\tau; 0)$, then this function is simply the average power output of the channel as a function of time delay τ.

$$R_h(\tau) = E[h^*(\tau; t)\, h(\tau; t)] = E[|h(\tau; t)|^2]$$

$\triangleq$ multipath intensity profile.

or delay power spectrum.



The range of τ over which $R_h(\tau)$ is essentially non-zero is called the "multipath spread" of the channel.

$R_h(\tau; \Delta t)$ is measured by sending very narrow pulses and cross-correlating the received signal with a delayed version of itself.

+) Now consider the frequency domain and define a time-varying transfer function $H(f;t)$ as:

$$H(f;t) = \int_{-\infty}^{\infty} h(\tau;t)\, e^{-j2\pi f_c \tau}\, d\tau$$

Under wide-sense stationarity, define the auto-correlation as

$$R_H(f_1, f_2; \Delta t) = E\left[H^*(f_1;t)\, H(f_2; t+\Delta t)\right]$$

Note the relationship:

$$h(\tau;t) \xrightarrow{FT} H(f;t)$$

$$R_h(\tau;\Delta t) \xrightarrow{FT} R_H(f_1, f_2; \Delta t) = R_H(\Delta f; \Delta t)$$

The function $R_H(\Delta f;\Delta t)$ is called the "spaced-frequency, spaced-time correlation function" of the channel.

It is measured by transmitting a pair of sinusoids separated by Δf and cross-correlating the two (separately) received signals with a relative delay Δt.

∘ Again let $\Delta t = 0$ and define $R_H(\Delta f) \triangleq R_H(\Delta f; 0)$ then

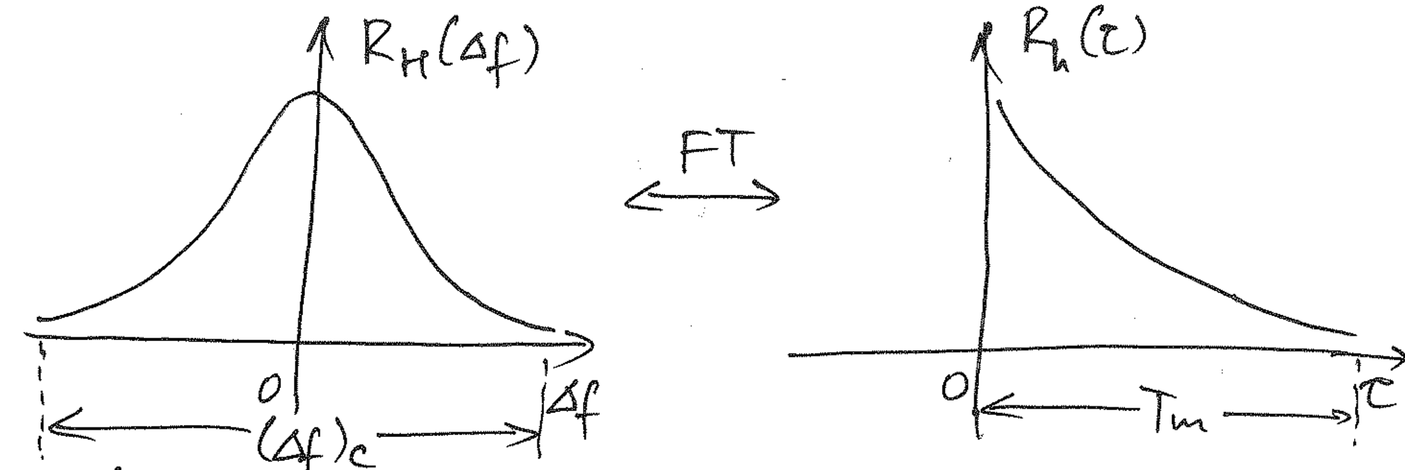
$$R_H(\Delta f) = \mathcal{F}\left[R_h(\tau)\right] = \int_{-\infty}^{\infty} R_h(\tau)\, e^{-j2\pi\Delta f \tau}\, d\tau$$

$R_H(\Delta f)$ is an auto-correlation in the frequency variable; it provides a measure of the frequency coherence of the channel.

$$(\Delta f)_c \approx \frac{1}{T_m}$$

T_m : delay spread

$(\Delta f)_c$: coherence bandwidth of the channel.



spaced-frequency correlation func. multipath intensity profile

- Meaning of cohence BW $(\Delta f)_c$: Two signals (sinusoids) separating by larger than $(\Delta f)_c$ will be affected differently by the channel.

If the signal BW is larger than $(\Delta f)_c$, different frequency components of the signal will experience different amounts of fading and the signal is severely distorted by the channel. The channel is said to be "frequency selective".

If the signal BW is much smaller than $(\Delta f)_c$ → the channel appears to be flat in spectrum and is called a "frequency non-selective" or "frequency flat" channel.

Lecture 6:

+) Doppler power spectrum and coherence time:

Now we will examine the time-varying aspect, which is measured in parameter Δt and is related to the Doppler effect.

Consider the spaced-frequency spaced-time function $R_H(\Delta f; \Delta t)$. Now take the FT with respect to Δt: