$$S_H(\Delta f;\lambda) = \int_{-\infty}^{\infty} R_H(\Delta f;\Delta t)\, e^{-j2\pi\Delta t\lambda}\, d\Delta t$$

Now let $\Delta f = 0$ and define $S_H(\lambda) \triangleq S_H(0;\lambda)$, then this function gives the signal intensity as a function of the Doppler frequency λ.

- $S_H(\lambda)$ is called the "Doppler power spectrum" of the channel.

$$S_H(\lambda) = \int_{-\infty}^{\infty} R_H(0;\Delta t)\, e^{-j2\pi\lambda\Delta t}\, d\Delta t .$$

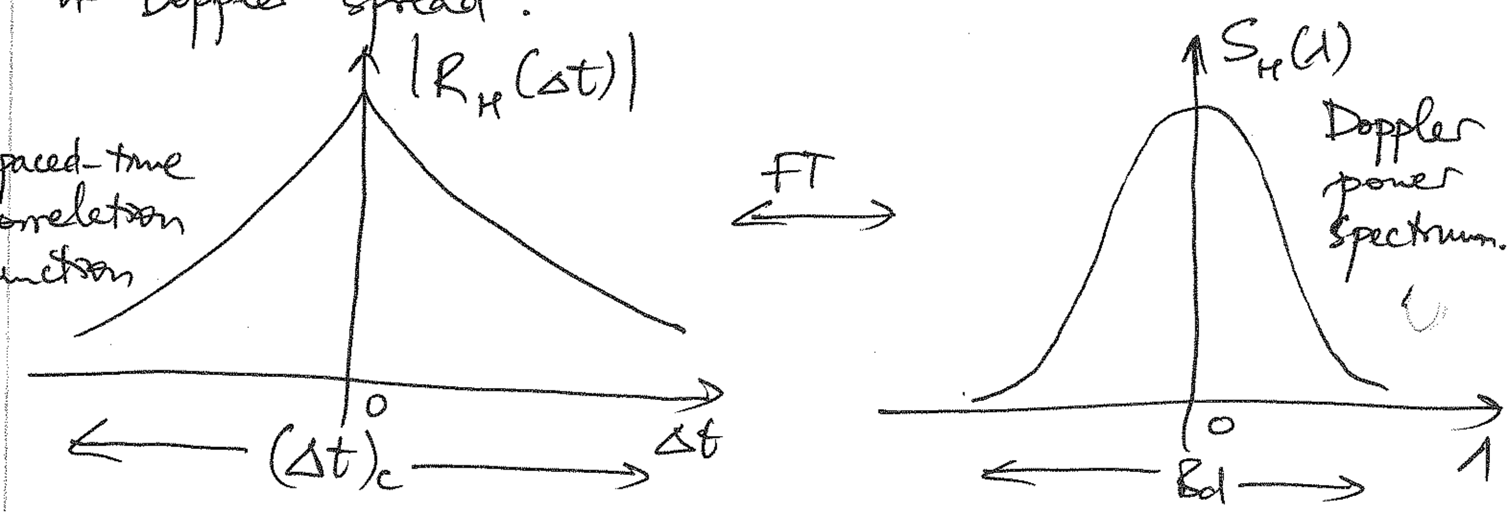
- The range of λ over which $S_H(\lambda)$ is essentially non_zero is called the "Doppler spread" of the channel.

The reciprocal of the Doppler spread is a measure of the coherence time of the channel.

$$(\Delta t)_c \simeq \frac{1}{B_d}$$

B_d : Doppler spread
$(\Delta t)_c$: coherence time.

- Note: If there is no time variation then $R_H(0;\Delta t) = 1$ and $S_H(\lambda) = \delta(\lambda)$. Then $B_d = 0$ and the channel is the same over time ($(\Delta t)_c = \infty$). There is no Doppler spread.



- The coherence time measures how fast the channel is changing with time. $(\Delta t)_c$ represents the maximum separation in time such that the signals are still strongly correlated.

A slowly varying channel has large coherence time and is called a "slow fading" channel. A slow fading channel has small Doppler spread.

A fast fading channel is the opposite: with short coherence time and large Doppler spread.

+) <u>The scattering function</u>:

- Define one more Fourier transform $S(\tau;\lambda)$ as

$$S(\tau;\lambda) = \mathcal{F}\left(R_h(\tau;\Delta t)\right) = \int_{-\infty}^{\infty} R_h(\tau;\Delta t) e^{-j2\pi\lambda\Delta t} d\Delta t$$

then we can also show that

$$S(\tau;\lambda) = \mathcal{F}^{-1}\left(S_H(\Delta f;\lambda)\right) = \int_{-\infty}^{\infty} S_H(\Delta f;\lambda) e^{j2\pi\tau\Delta f} d\Delta f$$

- The function $S(\tau;\lambda)$ is called the "scattering function" of the channel.

It provides the average power output of the channel as a function of the time delay τ and Doppler frequency λ.

- See the figure for relationships among all functions in the handout. (Figure 2.4).

+) Examples of wireless fading models.

∘ Multipath spread: This depends critically on the type of terrain.

For urban and suburban areas, $T_m \cong 1-10\,\mu s$

For rural mountainous areas, $T_m \cong 10-30\,\mu s$.

For mmWave (28 GHz): $T_m \cong 30-40\,ns$, $60-80\,ns$

→ Coherent BW is much larger at higher frequencies (mmWave) which allows a larger carrier BW.

∘ Uniform scattering model: (narrowband channel)

Recall the received signal as a result of multipath:

$$r_\ell(t) = \sum_n \alpha_n(t)\, e^{j\phi_n(t)}\, s_\ell(t - \tau_n(t))$$

For narrowband channel, the signal BW is much smaller than the channel coherence time ($B_T \ll B_c \cong \frac{1}{T_m}$) then most (all) path delays will be approximately the same from the signal point of view.

Thus for narrow band channel:

$$r_\ell(t) = \sum_n \alpha_n(t)\, e^{j\phi_n(t)}\, s_\ell(t-\tau).$$

For unmodulated signal, $s_\ell(t) = 1 \;\; \forall t$, then

$$r_\ell(t) = \sum_n \alpha_n(t) e^{j\phi_n(t)}$$ (see Lecture 4)

Recall from Lecture 4: For channel with many scatterers such that the received signal can be expressed as a continuum of many multipath components, the channel impulse response becomes

$$h(\tau;t) = \alpha(\tau;t)\, e^{-j2\pi f_c \tau(t)}$$

For a narrowband channel, since all multipath components have approximately the same delay, the channel can be written as

$$h(\tau;t) = \alpha(t)\, e^{-j2\pi f_c \tau(t)}$$

where the path delay $\tau(t)$ is time-varying, as is path amplitude $\alpha(t)$.

Now examine the channel auto-correlation:

$$R_h(\quad \Delta t) = E\left[h^*(\tau;t)\, h(\tau;t+\Delta t)\right]$$

$$= E\left[\alpha(t)\,\alpha(t+\Delta t)\, e^{+j2\pi f_c[\tau(t)-\tau(t+\Delta t)]}\right]$$

$$= E\left[\alpha(t)\,\alpha(t+\Delta t)\, e^{j\Delta\phi}\right]$$

where $\Delta\phi$ is the change in Phase over Δt.

Since path amplitude changes slowly, assume that $\alpha(t+\Delta t) \simeq \alpha(t)$ and $E[\alpha(t)\,\alpha(t+\Delta t)] = P_r$, the average received power.

Uniform scattering assumption: Assume that the angle of arrival is uniformly distributed because of dense scatterers.

Recall from the Doppler shift (lecture 4):

$$\Delta\phi = \frac{2\pi}{\lambda} v.\Delta t.\cos\theta$$

where θ is the angle of arrival.

With the assumption that θ is uniformly distributed between $[0, 2\pi]$, then the autocorrelation function becomes:

$$R_h(\Delta t) = E\left[\alpha(t)\alpha(t+\Delta t)\, e^{j\frac{2\pi}{\lambda} v.\Delta t.\cos\theta}\right]$$

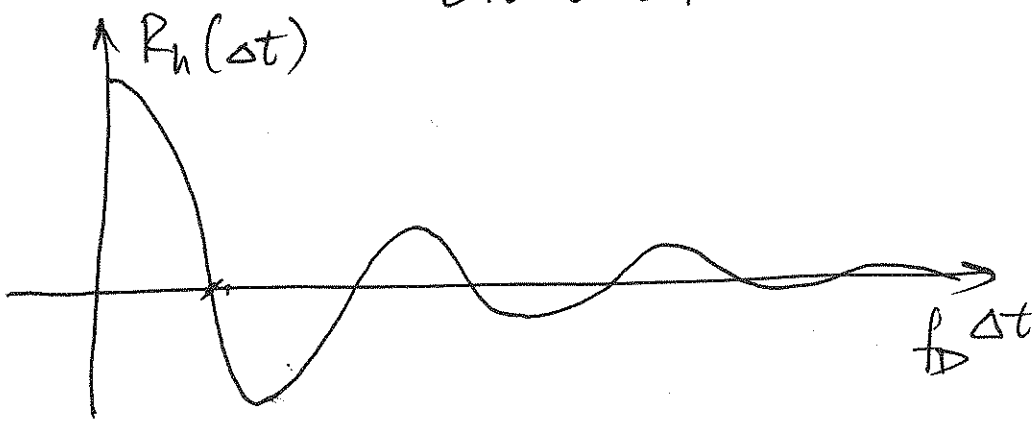
$$= P_r . \int_0^{2\pi} e^{j\frac{2\pi}{\lambda} v\Delta t.\cos\theta} . \frac{1}{2\pi} d\theta$$

$$= P_r . J_0(2\pi f_D \Delta t)$$

where: $f_D = \frac{v}{\lambda} = \frac{v.f_c}{c}$ ($c = 10^8 \times 3$ m/s is the speed of light).

is the maximum Doppler spread

$J_0(\cdot)$ is the first kind Bessel function of zero order.



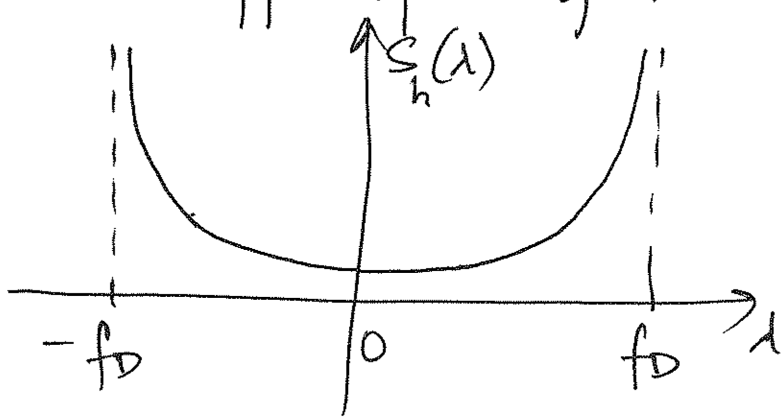
First zero of $R_h(\Delta t)$ occurs at approximately $f_D\Delta t \simeq 0.5$ which corresponds to a traveling distance of

$v\Delta t \simeq 0.5\lambda$, half a wavelength.

The signal becomes uncorrelated over distance and then becomes somewhat correlated again.

The 0.5λ distance is usually used as a rule of thumb for placing antennas in a MIMO system to get uncorrelated signals at different antennas.

- Under the uniform scattering assumption then the channel's Doppler power spectrum becomes an U shape.



- The uniform scattering assumption holds for microwave channels (below 6 GHz) but doesn't seem to hold for mmWave channels because the number of multipath is fewer.