Lecture 7: Statistical models for fading channels

In this lecture we will study the common statistical models for wireless fading channels.

+) Quasi-stationary channel model:

Recall the multipath channel impulse response

$$h(\tau;t) = \sum_n \alpha_n(t)\, e^{j\theta_n(t)}\, \delta(t-\tau_n(t)).$$

When the channel is varying relatively slowly in time such that the channel response remains relatively time-invariant for at least a signaling interval (a frame), then the channel can be estimated for that interval using some training sequences (pilot symbols) or blind estimation, then re-estimated for the next interval.

In this case (which applies to most practical system), the channel is modeled as quasi-stationary. During each signaling interval, the channel is assumed to be time-invariant. Then the channel changes in the next signaling interval.

With this quasi-stationary model, the channel during each signaling interval reduces to

$$h(\tau;t) = \sum_k \underbrace{\alpha_k e^{j\theta_k}}\, \delta(t-\tau_k)$$

$$= \sum_k a_k \quad \delta(t-\tau_k)$$

where $a_k = \alpha_k e^{j\theta_k}$ is the complex-value response of path k.

Note here the path response $a_k = \alpha_k e^{j\theta_k}$ is no longer time-varying, but is modeled statistically and is usually assumed to change independently from frame to frame.

The complex path gain a_k with amplitude α_k and phase θ_k, and path delay τ_k are all random variables.

+) <u>Rayleigh fading model:</u>

- Consider a narrowband channel where the multipaths are not distinct or resolvable over a signaling interval. Then a single resolvable path will be used to represent all multipaths:

$$a_k = \alpha_k e^{j\theta_k} = \sum_{i=1}^{p} \alpha_{ki} \cdot e^{j\theta_{ki}}$$

- Over a small area and provided there is no line of sight, the path amplitudes are approximately equal

$$a_k = \alpha \sum_{i=1}^{p} e^{j\theta_i}$$

$$= \alpha \sum \cos\theta_i + j\alpha \sum \sin\theta_i$$

- For large number of multipath, applying the Central Limit Theorem, each real and imaginary part of the path gain becomes Gaussian.

For phase θ_i independent and uniformly distributed in $[0, 2\pi]$, it can be shown that the real and imaginary parts are uncorrelated and have the same variance. Thus model the channel response as

$$a_k = a_{kr} + j a_{ki}$$

where $a_{kr}, a_{ki} \in N(0, \frac{\sigma^2}{2})$ and are independent.

This model is called the circularly complex Gaussian channel.

- The path amplitude $\alpha_k = \sqrt{a_{kr}^2 + a_{ki}^2}$ is Rayleigh distributed. The path phase is uniform.

The Rayleigh fading model is widely used due to its tractability and it can be a good fit in environments with many scatterers and no direct path (LOS) between the Tx and Rx.

+) <u>Rician fading model:</u>

- When there exists a dominant path together with a low level of scattered multipath components, then the channel response can have a non-zero mean.

$$a_k = a_{kr} + j a_{ki}$$

where

$$a_{kr} \in N(\mu_r, \frac{\sigma^2}{2}), \quad a_{ki} \in N(\mu_i, \frac{\sigma^2}{2})$$

- Physically, when a dominant or LOS path exists, the received signal can be written as a sum of 2 parts:

$$r = \underbrace{u e^{j\beta}}_{\text{Rayleigh fading component}} + \underbrace{v e^{j\gamma}}_{\text{strong (dominant) path component}} = \alpha e^{j\theta}$$

The strong path component usually is changing with uniform phase. It then can be shown that the amplitude and phase of r are independent, where the amplitude is Rician distributed and the phase is uniform.

Rician pdf of channel response's amplitude:

$$p_\alpha(\alpha|\upsilon) = \frac{\alpha}{\sigma^2} \exp\left(-\frac{\alpha^2+\upsilon^2}{2\sigma^2}\right) I_0\left(\frac{\alpha\upsilon}{\sigma^2}\right) \cdot u(\alpha)$$

where $I_0(.)$ is the modified Bessel function of the first kind and order zero.

- Other more complicated models exist, including Nakagami, log normal, Weibull and Suzuki distributions.

Nakagami model allows random path amplitudes instead of having all path amplitudes equal as in the Rayleigh fading model. It includes Rayleigh fading as a special case, and can approximate Rician fading with high accuracy.

Nakagami shows the best fit in urban environments.

+) <u>Discrete-time channel model:</u>

Suppose that the signal bandwidth is W and the received waveform is sampled at the symbol rate $T = \frac{1}{W}$.

We can then use the equivalent discrete-time channel impulse response as

$$h[n;k] = h(nT;kT)$$

$h[n;k]$: n indicates time-varying aspect
k indicates the channel sample at discrete delay k.

The time-varying frequency response of this channel can be written as

$$H(\omega;n) = \sum_k h[n;k] e^{-j\omega kT}$$

For the transmit signal $s[n]$, the received signal (in the absence of noise) is:

$$r[n] = h[n;k] * s[n] = \sum_k h[n;k] s[n-k]$$

When the channel is slow fading, or as in the quasi-stationary model, $h[n;k]$ doesnot depend on n and the output becomes:

$$r[n] = \sum_k h[k] s[n-k] = h[n] * s[n].$$

When the channel is frequency non-selective, then $h[n;k] = h[n].\delta[k]$ and the output becomes a simple multiplication in time:

$$r[n] = h[n].\, s[n]$$

When the channel is both frequency nonselective (flat fading) and slow fading, the channel reduces to a scaling factor

$$r[n] = h.\, s[n]$$

where h is a complex random quantity (scalar).

+) <u>Channel state information (CSI) acquisition:</u>

- The measurement or estimation of the channel response $h[n;k]$ in a communication system is called channel acquisition (or CSI acquisition).

- CSI acquisition for wireless comm. usually has to be done in real time given the time-varying nature.

- Receivers are usually able to obtain CSI of fading channel from the received signals, either by training sequences (pilots) or via blind estimation.

This channel estimation at the Rx is usually done for every frame. The estimation is rellatively accurate (enough to recover the transmitted data), so receiver CSI is usually assumed to be perfect.

- Transmitter CSI is a different matter. It requires explicit feedback from receivers, or relies on the reciproxity priciple. Usually transmitter CSI is imperfect or in the form of partial CSI.

+) Forward and reverse channels:

- Principle of reciprocity: The channel is identical on the forward and reverse link as long as the channel is measured at the same frequency and at the same time instant.

- Reciprocity generally only applies to TDD (time division duplexing) systems, provided that the time separation between the forward and reverse links is very small compared to the channel coherence time.

- Reciprocity usually doesnot hold in FDD systems since reverse and forward links are usually separated enough in frequency to make the channels completely different even statistically.

+) <u>Channel model with multiple antennas.</u>

- Consider a channel with M transmit and N receive antennas. The transmitter has a total power constraint of P regardless of the number of transmit antennas.

- Assuming antenna separation of at least half a wavelength, then the fading between each pair of Tx-Rx antennas is independent of other fading links.

- For a slow flat fading channel, the MIMO input-output relation can be written as

$$y = H.x + n$$

where x is the transmitted vector, $x \in C^M$ (complex value)
y " received " , $y \in C^N$
n " noise vector , $n \in C^N$
H is the channel matrix , $H \in C^{N \times M}$

H is a random complex matrix of channel coefficients

For Rayleigh and Rician fading, the entries of H (h_{ij}) are circularly complex Gaussian random variable (the real and imaginary parts have the same variance and are independent)
For Rayleigh fading : $h_{ij} \sim CN(0, \sigma^2)$.