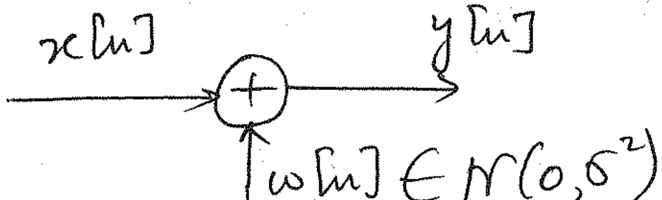
Lecture 8:

Capacity of wireless channels.

+) AWGN channel capacity

- Consider first a real additive white Gaussian noise channel (AWGN) represented as:

$$\underbrace{y[n]}_{\substack{\text{output}\\(\text{real})}} = \underbrace{x[n]}_{\substack{\text{input}\\(\text{real signal})}} + \underbrace{w[n]}_{\text{noise } \in \mathcal{N}(0,\sigma^2)}$$



- The transmit signal has an average power constraint of P, that is

$$E[x^2] \leq P.$$

- The capacity of this channel is the maximum transmission rate that, in theory, the information can be recovered at the receiver in the presence of noise with vanishing error probability.

Transmission at rates higher than the capacity will result in erroneous decoded information at the receiver no matter what code is used.

- For the AWGN channel, the capacity is

$$C = \frac{1}{2}\log_2\left(1 + \frac{P}{\sigma^2}\right) \quad \text{bps}$$

+) Band limited channel capacity:

- Now consider an AWGN channel with bandwidth W (Hz) and additive Gaussian noise with power spectral density $N_0/2$ (watts/Hz)

Assume a transmit power constraint of P (watts).

- Sample the signals at rate $\frac{1}{2W}$. Each sample can be represented by an AWGN discrete-time channel as

$$y_k[n] = x_k[n] + w_k[n] \quad , \quad k = 1 \ldots 2W.$$

- The noise power per "sampled" channel is $\frac{N_0}{2}$.
The average power constraint per "sampled" signal is $P/2W$.
Thus the capacity per "sampled" channel is

$$C_k = \frac{1}{2} \log_2\left(1 + \frac{P}{N_0 W}\right) \text{ bits per real channel dimension}$$

- Since there are $2W$ samples, the capacity of the band-limited channel is

$$C = W \log_2\left(1 + \frac{P}{N_0 W}\right) \text{ bps.}$$

Note that $\frac{P}{N_0 W} = SNR$ is the received signal to noise ratio.
The quantity below is called the spectral efficiency:

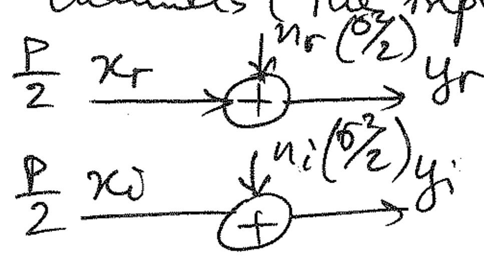
$$C = \frac{1}{2} \log_2(1 + SNR) \text{ bps/Hz.}$$

for a real channel.

+) <u>Capacity with complex signaling:</u>

When the transmitted and received signals are complex (two-dimensional) as in wireless communication, the factor $\frac{1}{2}$ in the front disappears.

This is because the communications can be viewed as occurring on two independent channels (the inphase and quadrature components)



$$y = x + n$$

where $x = x_r + j x_i$

$y = y_r + j\, y_i$

$n = n_r + j n_i$, n_r, $n_i \in \mathcal{N}(0, \frac{\sigma^2}{2})$, independent

Then on the real (inphase) dimension

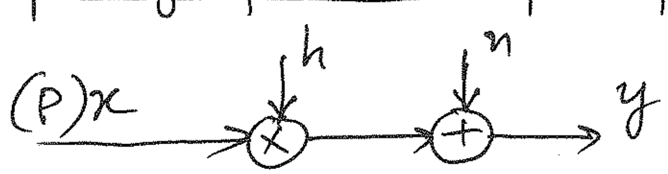
$y_r = x_r + n_r$ where $SNR_r = \dfrac{P/2}{\sigma^2/2} = \dfrac{P}{\sigma^2} = SNR$

Similarly $y_i = x_i + n_i$, $SNR_i = SNR_r = SNR$.

The spectral efficiency of a complex channel becomes

$$C = 2 \times \frac{1}{2} \log(1 + SNR) = \log(1 + SNR).$$

+) <u>Capacity of wireless flat fading channels:</u>



A flat-fading channel can be modeled as a random scaling factor.

$$y = h.x + n$$

where h: complex channel response coefficient
n: complex Gaussian noise, $n \in \mathcal{CN}(0, \sigma^2)$

- Since now there is a channel h, the capacity depends on whether the channel is known at the Rx, Tx or both.

- Since the channel is random, there are also different definitions of capacity.

+) Capacity with channel information at receiver:

This is the most common case for wireless communications since the channel is usually known at the receiver but not at the transmitter.

- Ergodic capacity: This refers to the "average" rate that we can transmit information over the channel, provided that each transmission block is long enough to experience all fading states.

The transmitter sends signal at a fixed rate and is not able to adapt to the changes in the channel as it doesn't have CSI (channel state / side information).

The receiver waits until the end of each transmission block to decode the information.

Ergodic capacity:

$$C = E_h \log_2\left(1 + |h|^2 \cdot \frac{P}{\sigma^2}\right) = E_\gamma \log_2(1+\gamma)$$

where $\gamma = |h|^2 \frac{P}{\sigma^2}$ is the (received) SNR.

The average SNR is $\bar{\gamma} = E|h|^2 \cdot \frac{P^2}{\sigma^2}$.

- By Jensen's inequality:

$$E_\gamma \log(1+\gamma) \leq \log(1 + E[\gamma]) = \log(1+\bar{\gamma})$$

Thus the ergodic capacity of a fading channel is less than the capacity of an AWGN channel at the same average SNR.

*) <u>Capacity with outage</u> (or outage capacity)

For slow fading channel where the instantaneous SNR γ is constant over a number of transmissions (or over a transmission burst) then changes to a new value based on the fading distribution in the next block.

Then at a constant transmission rate, there is a possibility that the channel in that transmission block does not support that rate and the received signal cannot be decoded. Then an outage occurs.

Usually at a fixed transmission rate, there is a corresponding probability of outage.

Suppose the transmission rate is $R = \log(1+\gamma_0)$. Then outage occurs when the instantaneous SNR falls below γ_0.