Outage probability:

$$P_o = Pr(\gamma < \gamma_o)$$

The average rate correctly received/decoded over many transmission blocks is

$$R_o = (1 - P_o)\log_2(1+\gamma_o)$$

ecture 9:

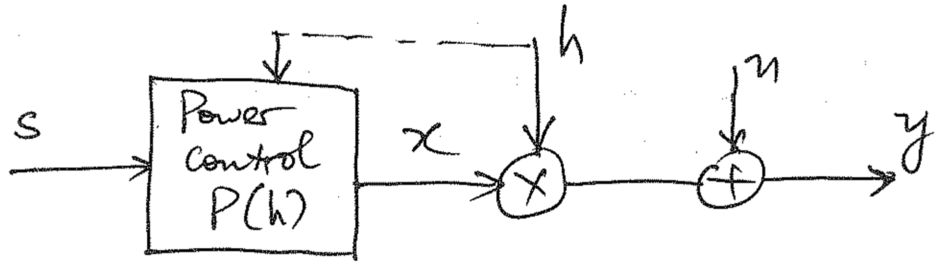
+) <u>Capacity with CSI at both Tx and Rx:</u>

With CSI at the transmitter, the transmission can now be adaptive to the channel fading.

The capacity in this case is obtained as a result of an optimization problem:

$$C = \max E_h \log_2\left(1 + |h|^2 \frac{P(h)}{\sigma^2}\right)$$

$$\text{s.t. } E_h[P(h)] \le \bar{P}$$



Here $\bar{P}$ is the average transmit power

The optimization can be written in terms of the "effective" SNR $\gamma = |h|^2 \cdot \frac{\bar{P}}{\sigma^2}$ as well

$$C = \max E_\gamma \log_2\left(1 + \gamma \cdot \frac{P(\gamma)}{\bar{P}}\right) \quad \text{s.t. } E_\gamma[P(\gamma)] \le \bar{P}.$$

or more explicitly

$$C = \max \int_0^{\infty} \log_2\left(1 + \gamma \frac{P(\gamma)}{\bar{P}}\right) p(\gamma)\, d\gamma$$

$$\text{s.t.} \quad \int_0^{\infty} P(\gamma) \cdot p(\gamma)\, d\gamma \le \bar{P} .$$

where $p(\gamma)$ is the distribution (probability density function) of γ.

Solving by the Lagrangian technique results in the optimal power allocation / adaptation as

$$\frac{P(\gamma)}{\bar{P}} = \begin{cases} 1/\gamma_0 - 1/\gamma & \text{if } \gamma \geq \gamma_0 \\ 0 & \gamma < \gamma_0 \end{cases}$$

for some cutoff value γ_0.

Then the capacity is

$$C = \int_{\gamma_0}^{\infty} \log_2\left(\frac{\gamma}{\gamma_0}\right) p(\gamma)\, d\gamma .$$

and the cutoff value is found from the power constraint of $E[P(\gamma)] = \bar{P}$ as

$$\int_{\gamma_0}^{\infty} \left(\frac{1}{\gamma_0} - \frac{1}{\gamma}\right) p(\gamma)\, d\gamma = 0$$

The value γ_0 depends only on the fading distribution and is usually found numerically.

This technique is water-filling over time (as γ is time varying).

+) Zero-outage capacity and channel inversion.

- A suboptimal but simple scheme is for the transmitter to adapt to CSI so that the received power is constant.

This adaptation is called channel inversion.

$$P(\gamma) = \frac{\gamma_r}{\gamma} \cdot \bar{P} \qquad \gamma_r\text{: constant received SNR}$$
$$\gamma = |h|^2 \cdot \frac{\bar{P}}{\sigma^2}$$

For $E[P(\gamma)] = \bar{P}$ then $\gamma_r = \dfrac{1}{E[\frac{1}{\gamma}]}$.

- Capacity with channel inversion is also called "zero-outage" capacity. It is equal to the capacity of an AWGN with SNR γ_r.

$$C = \log(1+\gamma_r) = \log\left(1 + \frac{1}{E[1/\gamma]}\right)$$

- The transmission strategy is simple using a fixed-rate encoder and a decoder designed for just AWGN channels. Fading impact is nulled and system design is simple.

This technique is common in spread spectrum systems with near-far problem (interference imbalance)

- Zero-outage capacity can exhibit a large data reduction compared to Shannon capacity.
For Rayleigh fading, zero-outage capacity is zero.

+) Truncated channel inversion: Drop the channel during a bad fade:

$$\frac{P(\gamma)}{\bar{P}} = \begin{cases} \dfrac{\gamma_r}{\gamma} & \text{if } \gamma > \gamma_0 \\ 0 & \gamma < \gamma_0 \end{cases}$$

where γ_0 is based on the outage probability : $P_0 = Pr(\gamma \leq \gamma_0)$

With the power constraint $E[P(\gamma)] = \bar{P}$ then

$$\gamma_r = \frac{1}{E_{\gamma_0}\left[\frac{1}{\gamma}\right]} \quad \text{where} \quad E_{\gamma_0}\left[\frac{1}{\gamma}\right] \triangleq \int_{\gamma_0}^{\infty} \frac{1}{\gamma} p(\gamma) d\gamma .$$

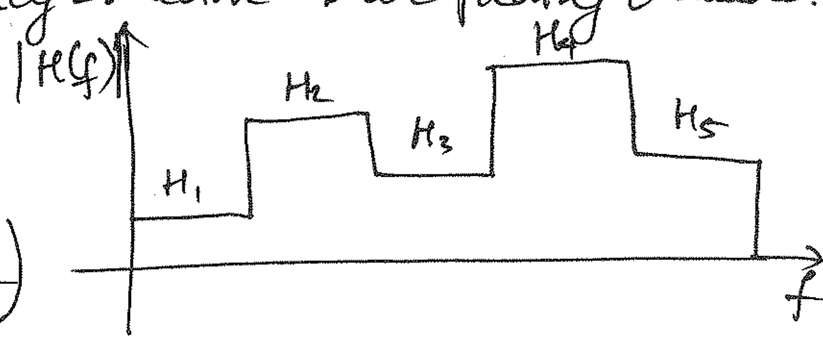
- The outage capacity for a given P_0 is

$$C(P_0) = \log\left(1 + \frac{1}{E_{\gamma_0}[1/\gamma]}\right) p(\gamma \geq \gamma_0)$$

Choosing the γ_0 to maximize the above expression leads to the maximum outage capacity (with truncated channel inversion). This maximization is usually done numerically.

+) <u>Capacity of frequency selective channels:</u>

- Consider a set of frequency-selective block fading channels. We can treat this as a set of parallel channel with shared power constraint



$$C = \max \sum_j \log\left(1 + \frac{|H_j|^2 P_j}{\sigma^2}\right)$$

$$\text{s.t.} \quad \sum_j P_j \leq P$$

Solving this problem via the Lagrangian technique leads to the water-filling power allocation.

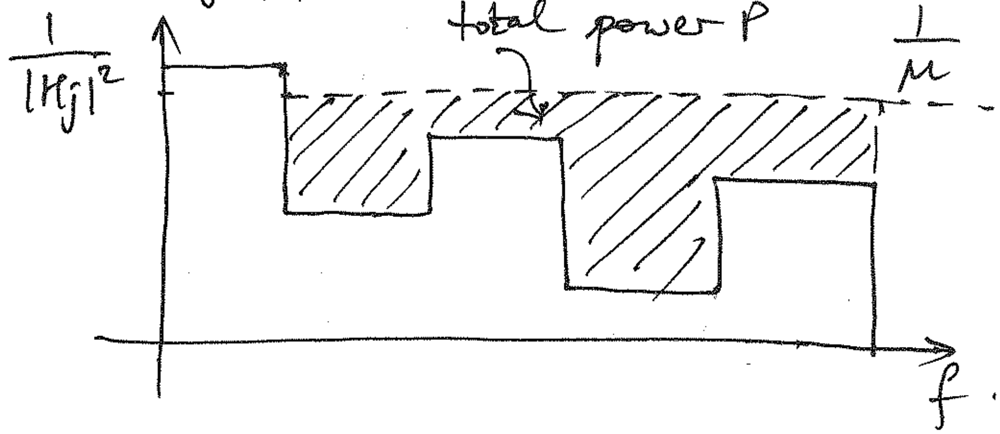
$$P_j = \begin{cases} -\frac{\sigma^2}{|H_j|^2} + \frac{\sigma^2}{\mu} & \text{if } |H_j|^2 \geq \mu \\ 0 & |H_j|^2 < \mu \end{cases}$$

where μ is the "water-level" such that

$$\sum_j P_j = P \quad \text{or} \quad \sum_j \left(\frac{1}{\mu} - \frac{1}{|H_j|^2}\right)^+ = \frac{P}{\sigma^2}.$$

The capacity is

$$C = \sum_{j:\,|H_j|^2 \geq \mu} \log_2\left(\frac{|H_j|^2}{\mu}\right).$$



- When the frequency response $H(f)$ is continuous, the capacity is given by

$$C = \max \int \log\left(1 + \frac{|H(f)|^2 P(f)}{N_0}\right) df$$

$$\text{s.t.} \quad \int P(f)\, df \leq P$$

- In practice, spectrum is divided into subchannels (subcarriers) in OFDM systems, and power allocation over all subcarriers is an important practical problem.

- For both time-varying and frequency selective channel, if CSI is known at Tx, the optimal strategy is to water-fill over both time and frequency. But this is rarely applicable in practice due to CSI issues.