channels with multiple antennas, and also on the capacity of some existing spatial diversity techniques.

2.1 Characteristics of multipath fading channel

A wireless channel is often characterized as a multipath fading channel with a complex and time-varying impulse response. The multipath arises from scattering, reflection, refraction or diffraction of the radiated energy off objects in the environment. Propagation path loss and fading due to moving or random objects in the environment between the transmitter and receiver causes the time variation in the channel. This time variation also depends on the speed of the moving objects, either the transmitter or receiver, where a mobile channel is more likely to experience greater time variation than an indoor fixed wireless channel. In this section, we will study multipath fading channel impulse response, power spectra and channel parameters. These will form the basis for establishing channel models in the next section.

2.1.1 Channel impulse response

A multipath fading channel is characterized by a complex time-varying channel impulse response $c(\tau; t)$ (Figure 2.1). When an ideal impulse is transmitted over a multipath fading channel, there will be two effects on the received signal. Firstly, since the signal may follow different paths with different lengths and attenuation factors, the received signal may appear as a train of pulses with different delays and magnitudes. The second effect is due to the time varying nature of the channel, which means the nature of the multipath is varying with time. Thus the number of pulses, the delay between them and their magnitude may change from time to time as the experiment of sending the impulse is repeated. This time variation, moreover, appears to be unpredictable to the user of the channel. Therefore a time-varying multipath fading channel is often characterized statistically.

First, we will examine the effect of the channel on a modulated transmitted signal that is represented generally as

$$s(t) = Re\big[s_l(t)e^{j2\pi f_c t}\big] \tag{2.1}$$

where f_c is the carrier frequency and $s_l(t)$ is the lowpass information carrying signal. Assuming that there are multiple propagation paths, each with a propagation delay

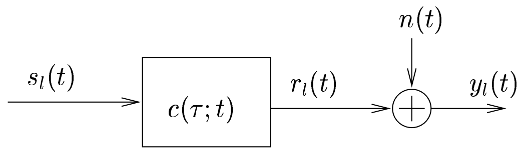

Figure 2.1: Multipath fading channel.

and an attenuation factor which are both time variant, the received bandpass signal without noise may be expressed as

$$r(t) = \sum_{k=1}^{K} \alpha_k(t) s\big(t - \tau_k(t)\big) \tag{2.2}$$

where K is the total number of paths, $\alpha_k(t)$ is the k^{th} path attenuation factor and $\tau_k(t)$ is k^{th} path propagation delay. Substituting $s(t)$ in (2.1) yields

$$r(t) = Re\Big\{\Big[\sum_{k=1}^{K} \alpha_k(t) e^{-j2\pi f_c \tau_k(t)} s_l\big(t - \tau_k(t)\big)\Big] e^{j2\pi f_c t}\Big\} .$$

It is apparent from this equation that the equivalent low pass received signal is

$$r_l(t) = \sum_{k=1}^{K} \alpha_k(t) e^{-j2\pi f_c \tau_k(t)} s_l\big(t - \tau_k(t)\big) .$$

Since the channel bandwidth is generally much smaller than the carrier bandwidth, the system can be effectively modeled by equivalent lowpass signals through equivalent lowpass channels [1]. Since $r_l(t)$ is the response of an equivalent lowpass channel to the equivalent lowpass signal $s_l(t)$, it follows that the equivalent lowpass channel can be described by the time-varying impulse response

$$c(\tau;t) = \sum_{k=1}^{K} \alpha_k(t) e^{-j2\pi f_c \tau_k(t)} \delta\big(t - \tau_k(t)\big) = \sum_{k=1}^{K} \alpha_k(t) e^{j\theta_k(t)} \delta\big(t - \tau_k(t)\big) \tag{2.3}$$

where $\theta_k(t) = -2\pi f_c \tau_k(t)$ is a time-varying phase sequence. This $c(\tau;t)$ represents the response of the channel at time t due to an impulse applied at time $t - \tau$. The channel is completely characterized by the number of multipath components K and the path variables: amplitude $a_k(t)$, delay $\tau_k(t)$ and phase $\theta_k(t)$. These parameters change unpredictably with time and are often described statistically. The received signal $r_l(t)$ therefore is also random, and when there are a large number of paths, the central limit theorem applies. This means $r_l(t)$ may be modeled as a complex-valued Gaussian random process. Thus the channel impulse response $c(\tau;t)$ is a complex-valued Gaussian random process in the t variable. The statistical models are described in more detail in the next section.

Large dynamic changes in the transmitting medium are required for the $\alpha_k(t)$ to change sufficiently to cause a significant change in the received signal. On the other hand, $\theta_k(t)$ will change by 2π radians whenever $\tau_k(t)$ (or in effect the path length) is changed by $1/f_c$, which is a small amount due to large carrier bandwidth. Therefore $\theta_k(t)$ can change quite rapidly with relatively small motions of the medium. This time variation of the phases $\{\theta_k(t)\}$ is the primary cause of fading phenomena in a multipath channel. The randomly time-varying phases $\{\theta_k(t)\}$ associated with the vectors $\{\alpha_k(t)e^{j\theta_k(t)}\}$ at times result in the received vectors adding constructively or destructively. This causes amplitude variations in the received signal and is termed *signal fading*.

The lowpass received signal $y_l(t) = r_l(t) + n(t)$ therefore becomes

$$y_l(t) = c(\tau;t) * s_l(t) + n(t) = \int_{-\infty}^{\infty} c(\tau;t)s(t-\tau)d\tau + n(t) \tag{2.4}$$

where $n(t)$ is lowpass complex-valued additive Gaussian noise.

2.1.2 Channel power spectra and coherence parameters

To see the effect of signal characteristics on a channel model, we describe a number of correlation functions and power spectral density functions that define the characteristics of a fading multipath channel. We begin with an equivalent lowpass channel impulse response $c(\tau;t)$, which is characterized as a complex-valued random process and is assumed to be wide-sense stationary, (i.e. the impulse response has a constant mean and its autocorrelation function depends only on the time difference but not the absolute time). We define the autocorrelation function of $c(\tau;t)$ as:

$$\phi_c(\tau_1, \tau_2; \Delta t) = E\left[c^*(\tau_1;t)c(\tau_2;t+\Delta t)\right] .$$

In most radio transmission media, the attenuation and phase shift of the channel associated with different paths are uncorrelated, which is usually called *uncorrelated scattering* [1]. With this assumption, the channel response associated with path delay τ_1 and the channel response associated with path delay τ_2 are uncorrelated, and the above equation becomes:

$$E[c^*(\tau_1;t)c(\tau_2;t+\Delta t)] = \phi_c(\tau_1;\Delta t)\delta(\tau_1 - \tau_2) . \tag{2.5}$$

Similarly in the frequency domain, we have the time-varying transfer function

$C(f;t)$ as the Fourier transform of $c(\tau;t)$

$$C(f;t) = \int_{-\infty}^{\infty} c(\tau;t)e^{-j2\pi\Delta f\tau}d\tau \ . \tag{2.6}$$

Under the wide sense stationary assumption, define the autocorrelation function

$$\Phi_C(f_1, f_2; \Delta t) = E[C^*(f_1;t)C(f_2;t+\Delta t)] \ .$$

It can be shown that $\Phi_C(f_1, f_2; \Delta t)$ is related to $\phi_c(\tau_1, \tau_2; \Delta t)$ by the Fourier transform

$$\Phi_C(f_1, f_2; \Delta t) = \int_{-\infty}^{\infty} \phi_c(\tau_1; \Delta t)e^{-j2\pi\Delta f\tau_1}d\tau_1 \equiv \Phi_C(\Delta f; \Delta t) \tag{2.7}$$

where $\Delta f = f_2 - f_1$. Furthermore, the assumption of uncorrelated scattering implies that the autocorrelation function of $C(f;t)$ in frequency is a function of only the frequency difference $\Delta f = f_2 - f_1$. Therefore $\Phi_C(\Delta f; \Delta t)$ is called the *spaced-time spaced-frequency correlation function* of the channel.

Delay power spectrum and coherence bandwidth

First we will study the effect of path delays on the channel characteristics. If we let $\Delta t = 0$ and $\tau_1 = \tau_2 = \tau$ in (2.5), the resulting autocorrelation function $\phi_c(\tau;0) \equiv \phi_c(\tau)$ is simply the average power output of the channel as a function of the time delay τ. Thus the function $\phi_c(\tau)$ is termed *multipath intensity profile* or the *delay power spectrum* of the channel. The range of values of τ over which $\phi_c(\tau)$ is essentially non-zero is termed the *multipath spread* of the channel and is denoted by T_m. With $\Delta t = 0$ in (2.7), the transform relationship is simply

$$\Phi_C(\Delta f) = \int_{-\infty}^{\infty} \phi_c(\tau)e^{-j2\pi\Delta f\tau}d\tau \ . \tag{2.8}$$

Since $\Phi_C(\Delta_f)$ is an autocorrelation function in the frequency variable, it provides us with a measure of the frequency cohesion of the channel. As a result of (2.8), the reciprocal of the multipath spread is a measure of the *coherence bandwidth of the channel*

$$(\Delta f)_c \approx \frac{1}{T_m} \tag{2.9}$$

where $(\Delta f)_c$ denotes the coherence bandwidth. This means that two sinusoids (single tone frequency signal) with frequency separation greater than $(\Delta f)_c$ are affected differently by the channel. Thus when the signal bandwidth is larger than $(\Delta f)_c$, different frequency components of the signal will experience different amounts of fading and the signal is severely distorted by the channel. In this case the channel is said to be *frequency selective.* On the other hand, if the signal bandwidth is small compared to $(\Delta f)_c$, the channel appears to be flat in the frequency spectrum and is termed a *frequency nonselective* or *flat channel.*

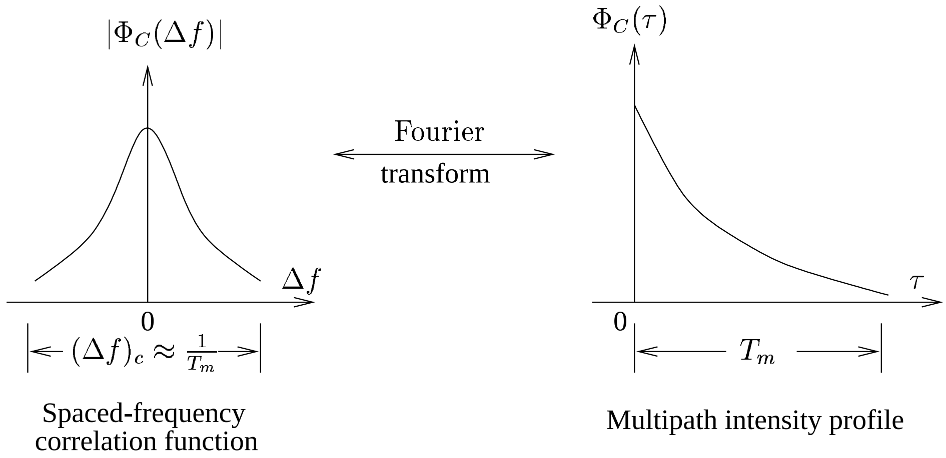

Figure 2.2: Relationship between delay spread and bandwidth coherence of a channel.

Doppler power spectrum and coherence time

We will focus now on the time variation of the channel as measured by the parameter Δt in $\Phi_C(\Delta f; \Delta t)$. Define the Fourier transform of $\Phi_C(\Delta f; \Delta t)$ with respect to variable Δt to be

$$S_C(\Delta f; \lambda) = \int_{-\infty}^{\infty} \Phi_C(\Delta f; \Delta t) e^{-j2\pi\lambda\Delta t} d\Delta t \,. \tag{2.10}$$

With $\Delta f = 0$ and denoting $S_C(0; \lambda) \equiv S_C(\lambda)$, the above relation becomes

$$S_C(\lambda) = \int_{-\infty}^{\infty} \Phi_C(\Delta t) e^{-j2\pi\lambda\Delta t} d\Delta t \,. \tag{2.11}$$

The range of values of λ over which $S_C(\lambda)$ is essentially non-zero is called the *Doppler spread* B_d of the channel. It represents the spectral broadening of the transmitted signal due to the time variation in the channel. From (2.11), we observe that if the channel is time-invariant, that is $\Phi_C(\Delta t) = 1$, then $S_C(\lambda) \equiv \delta(\lambda)$, which means there is no spectral broadening observed in transmitting a pure tone. The function

$S_C(\lambda)$ is a power spectrum that gives the signal intensity as a function of the Doppler frequency λ and hence it is called the *Doppler power spectrum* of the channel.

Since $S_C(\lambda)$ is related to $\Phi_C(\Delta t)$ by the Fourier transform, the reciprocal of B_d is a measure of the coherence time of the channel. That is

$$(\Delta t)_c \approx \frac{1}{B_d} \tag{2.12}$$

where $(\Delta t)_c$ denotes the *coherence time* of the channel. Coherence time represents the time separation during which the channel impulse response at two time instances are still strongly correlated, and therefore it measures how fast a channel changes in time. A slowly varying channel will have a large coherence time, or equivalently, a small Doppler spread, and vice versa.

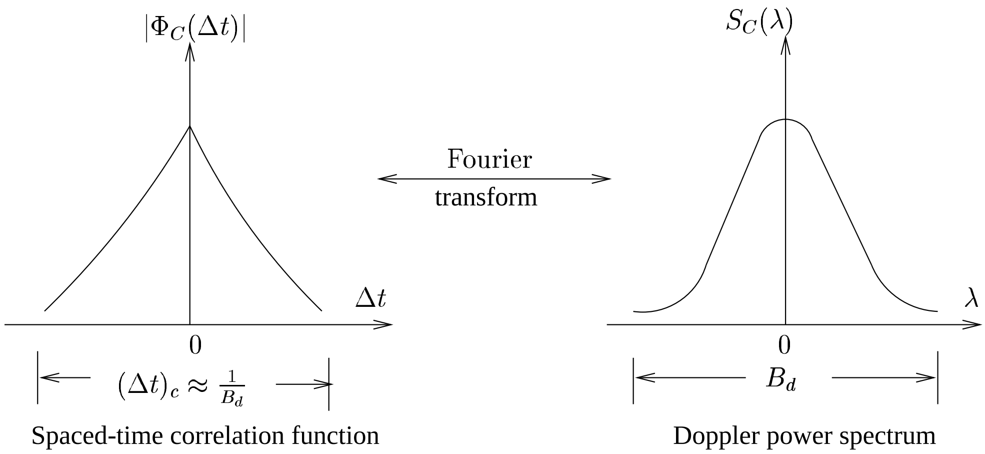

Figure 2.3: Relationship between Doppler spread and time coherence of a channel.

The scattering function

We have shown a Fourier transform relationship between $\phi_c(\tau;\Delta t)$, $\Phi_C(\Delta f;\Delta t)$ and a Fourier transform relationship between $\Phi_C(\Delta f;\Delta t)$ and $S_C(\Delta f;\lambda)$. Now, define a new function $S(\tau;\lambda)$ as

$$S(\tau;\lambda) = \int_{-\infty}^{\infty} \phi_c(\tau;\Delta t)e^{-j2\pi\lambda\Delta t}d\Delta t\ .$$

Then from(2.7) and (2.10), the following Fourier transform relation is apparent

$$S(\tau;\lambda) = \int_{-\infty}^{\infty} S_C(\Delta f;\lambda)e^{j2\pi\lambda\Delta f}d\Delta f\ .$$

Furthermore, $S(\tau;\lambda)$ and $\Phi_C(\Delta f;\Delta t)$ are related by the double Fourier transform

$$S(\tau;\lambda) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \Phi_C(\Delta f;\Delta t) e^{-j2\pi\lambda\Delta t} e^{-j2\pi\lambda\Delta f} d\Delta f d\Delta t \, .$$

The function $S(\tau;\lambda)$ is called the *scattering function* of the channel. It provides a measurement of the average power of the channel as a function of time delay τ and the Doppler frequency λ.

The relationship between the channel correlation and power spectrum functions and the coherence parameters can be summarized in figure 2.4.

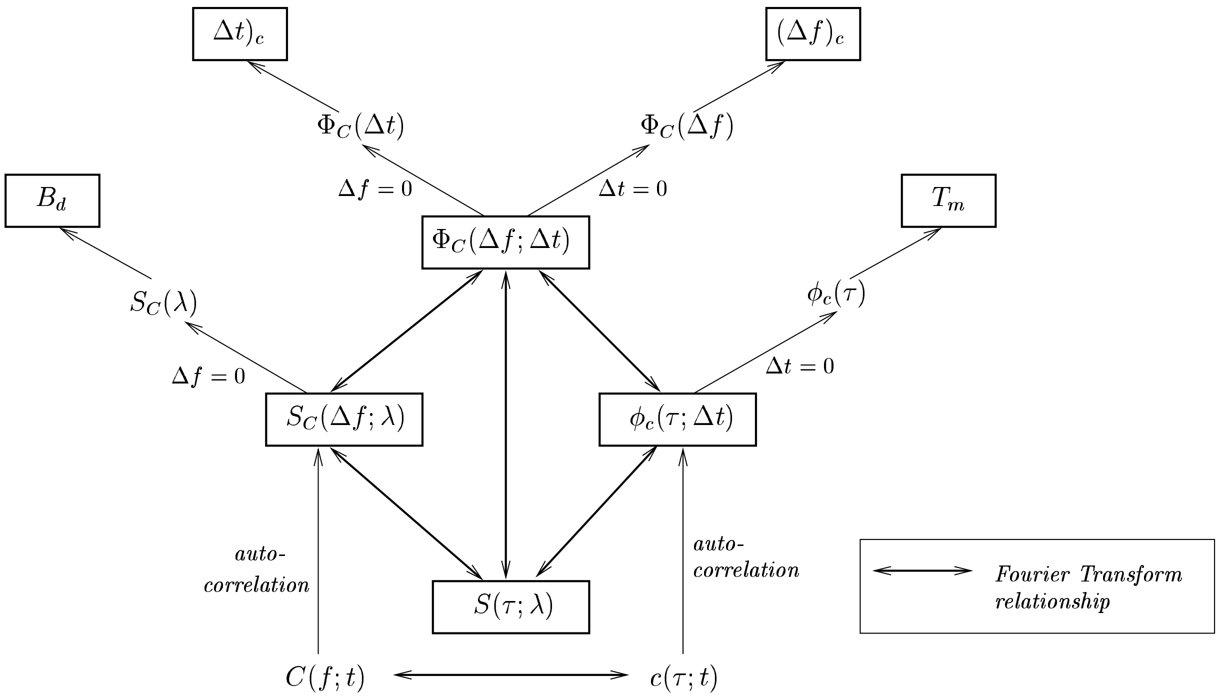

Figure 2.4: Multipath fading channel functions and parameters.

2.2 Wireless channel models

Having established the power spectrum functions and coherence parameters in the previous section, we now proceed to building the mathematical models for a wireless channel that can be used to study and analyze the performance of a particular system. First we will study the effect that the transmitted signal has on establishing a channel model. Since the channel is often characterized by its statistical properties, we will then present the common distributions used to model fading path amplitude, phase and delay time with some insights into the theoretical explanation and empirical justification of