

EE109 – Convex Optimization

Spring 2019

Course description

This course focuses on convex optimization theory and algorithms. Topics include convex sets, convex functions and convex optimization problems; duality theory and optimality conditions; algorithms for solving convex problems including descend, Newton and interior point methods. Examples of application are taken from communications, signal processing and other fields. Students will do a project as part of the course credit.

Prerequisite: vector calculus, linear algebra. Students should be comfortable with mathematical formulation and analysis. Prior Matlab or other programming experience is useful.

Objectives

We aim to provide a balance between theory and algorithms, so that students can recognize and formulate a convex problem, understand and analyze the optimality conditions, and at the same time are able to design an algorithm and implement codes to find the optimal solutions. The objectives of this course are for students to be able to

- Recognize and formulate convex optimization problems that arise in applications
- Analyze a convex problem using convexity theory and duality theory
- Understand how to solve convex problems using numerical techniques and obtain some practice in solving them.

Grading

Homework 20%, Midterm 20%, Final 35%, Project 25%.

- The course will have biweekly homework in the first three quarters of the semester, and a project in the last quarter.
- Course project should be carried out individually. If you intend to perform the project in a group of two, special permission from the instructor is needed – a group project is only allowed if the project has a big enough scope, and the group must have no more than 2 students. For the project, each student needs to formulate an optimization problem, analyze the optimal solutions or conditions for optimality, and implement a numerical algorithm to solve the problem. The problem can be students' own research (*advisor's consent is required*), or can be a reproduction of existing results in other papers; if a reproduction, a new algorithmic aspect or new analysis must be developed. The project involves a proposal in mid-semester and a class presentation at the end of semester. More information will be given in the project description.
- The final is a 24 or 48 hour take-home exam. The midterm exam format is to be determined. (The weight breakdown is approximate; we reserve the right to change it later if needed.)

Textbook

Boyd and Vandenberghe, *Convex Optimization*, Cambridge University Press, 2004.

References (these are optional)

- Ben-Tal and Nemirovski, *Lectures on Modern Convex Optimization: Analysis, Algorithms, and Engineering Applications*, MPS-SIAM Series on Optimization, 2001.
- Nesterov, *Introductory Lectures on Convex Optimization: A Basic Course*, Springer, 2003.
- David G. Luenberger, *Optimization by Vector Space Methods*, Wiley, 1997.
- R. Tyrell Rockafellar, *Convex Analysis*, Princeton University Press., 1996

Website

<http://www.ece.tufts.edu/ee/109>

The website contains information on homework, project and exams. Students should check the website regularly for updates.

Syllabus Outline (*tentative*)

	<u>No. of Lectures</u>	<u>Reading</u>
1. Introduction to optimization <ul style="list-style-type: none"> • Role of optimization, convexity • Examples of application (communications, signal processing) 	1	Ch1, App A
2. Review of linear algebra and mathematics background	2	App A, B, C
3. Convex set and convex function <ul style="list-style-type: none"> • Convex set, convex functions • Operations that preserve convexity (both sets and functions) • Conjugate function, conjugate sets • Separating hyper-plane theorem 	4	Ch 2, 3
4. Convex optimization problems <ul style="list-style-type: none"> • Optimization problem definition and examples • Linear programming • Quadratic programming • Geometric programming • Semi-definite programming 	2	Ch 4
5. Duality <ol style="list-style-type: none"> a. Lagrangian dual function (conjugate function) b. Lagrange dual problem <ol style="list-style-type: none"> i. Properties, weak and strong duality ii. Interpretation of dual variables, duality (geometric, saddle point, economics) c. Optimality conditions <ol style="list-style-type: none"> i. KKT, necessity and sufficiency ii. Sub-gradients for non-smooth functions d. Examples: <ol style="list-style-type: none"> i. Water-filling and reverse water-filling ii. Multiple-access sum capacity (scalar or MIMO version) iii. Compress sensing (using sub-gradient) 	4	Ch 5, HO
6. Methods and algorithms <ol style="list-style-type: none"> a. Unconstrained <ol style="list-style-type: none"> i. Gradient descent, steepest descent ii. Newton method b. With equality constraints <ol style="list-style-type: none"> i. Newton methods with equality constraints ii. ADMM method iii. Sub-gradient method c. With inequality constraints <ol style="list-style-type: none"> i. Barrier interior point method ii. Primal-dual interior point methods 	11	Ch 9,10,11
7. Advanced topics (if time) <ol style="list-style-type: none"> a. First-order methods for large-scale optimization <ol style="list-style-type: none"> i. First-order gradient descent ii. Application in machine learning b. Schur convexity 	2	HO