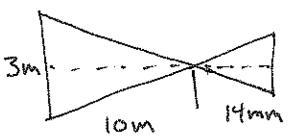
1)

We need to find s_i

$$\frac{1}{f} = \frac{1}{s_o} + \frac{1}{s_i}$$

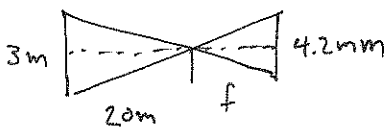
$$s_i = \frac{1}{\frac{1}{f} - \frac{1}{s_o}} = 14.06\,mm \approx 14\,mm$$

Now, using pinhole optics:



$$\frac{h}{14mm} = \frac{3m}{10m} \qquad h = 14mm \cdot \frac{3m}{10m} = \boxed{4.2\,mm}$$

2) Assume $f \approx s_i$ still.



$$\frac{f}{4.2mm} = \frac{20m}{3m} \Rightarrow f = 4.2mm \cdot \frac{20m}{3m} = 28mm$$

(which is what we expect)

3)

$$DOF \approx \frac{2NCU^2}{f^2} \qquad (U = s_o)$$

so doubling U and f has no effect!
(in practice, it's hard to get the same aperture when you double the focal length)

Background objects:

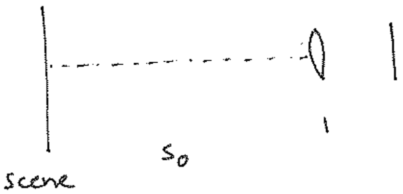
Assume a 1-m object 50 m away. (60m in second case)

in (1), object is $14mm \cdot \frac{1m}{50m} = 0.28mm$

in (2) object is $28mm \, \frac{1m}{60m} = 0.46\,mm$

Which is part of why a portrait lens has a moderate focal length

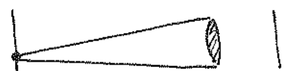
4) Start by finding the flux going through the lens.



A patch of the scene has luminance $2\times10^4 \frac{cd}{m^2} = 2\times10^4 \frac{lm}{sr\cdot m^2}$

Letting the patch shrink to a point, we can find the solid angle that will reach the lens.

$\Omega = \frac{A}{r^2}$



$f\# = 2.8 = \frac{f}{d}$

lens area $= \pi\left(\frac{d}{2}\right)^2$

$d = \frac{f}{f\#} = \frac{3.7\,mm}{2.8} = 1.32\,mm$

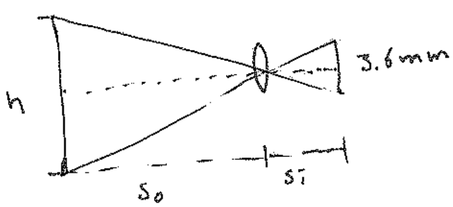
area $= \frac{\pi}{4}\cdot(1.32\,mm)^2 = \cancel{1.04}\,mm^2 = \overset{1.37}{\cancel{1.04}} \times10^{-6}\,m^2$

$\Omega = \frac{\overset{1.37}{\cancel{1.04}}\,mm^2}{s_o^2}$

So flux per unit area of the scene is

$2\times10^4 \frac{lm}{m^2} \cdot \frac{\overset{1.37}{\cancel{1.04}}\,mm^2}{s_o^2}$

To find total flux through the lens, we need the area of the scene. Remembering that rays through the center of the lens are not deflected, we can use pinhole geometry:



Assuming $s_i \approx f$ as in problem (1), and solving for both width and height,

$h = s_o \cdot \frac{3.6\,mm}{3.7\,mm}$, $w = s_o \cdot \frac{4.8\,mm}{3.7\,mm}$

$A_{scene} = s_o^2 \cdot 1.26$

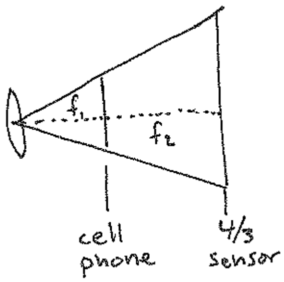
So total flux through the lens is

$2\times10^4 \frac{lm}{\cancel{m^2}} \cdot \frac{\overset{1.37}{\cancel{1.04\,mm^2}}}{\cancel{s_o^2}} \cdot \cancel{s_o^2}\cdot 1.26 = 0.0346$ lumens

Dividing by the area of the sensor gives illuminance:

$\frac{0.0346\,lm}{} = 2004 \frac{lm}{m^2}$

5) Assume focus at infinity, so $s_i = f$:



By similar triangles, $f_2 = 17.3\,mm \cdot \frac{3.7mm}{4.8mm} = 13.3\,mm$

6) Redoing the calculation with $w = 17.3\,mm$, $h = 13mm$
we get the same thing: $2004 \frac{lm}{m^2}$

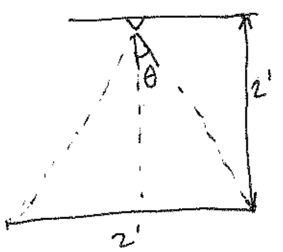
But our sensor is much larger, so we're capturing more light total.

7) Assume the desk is 2' x 4'. Bookshelf is ~ 2' above desk

The BXIB-L1092A-35E3000-A-13

1092 mm long (almost as long as the desk)

3000 lumens



$\theta = atan(\frac{1}{2}) \approx 27°$

so only light between ~ -27 and +27 degrees ≈ 60 deg
will hit the desk.

From the datasheet, radiation pattern is almost 180 deg

So let's assume ⅓ of the light hits the desk = 1000 lm.

Desk area = 61 x 122 cm = $0.74\,m^2$

Illuminance = $\frac{1000\,lm}{0.74m^2} = 1350 \frac{lm}{m^2}$