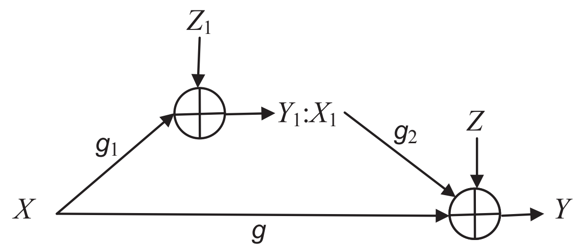
Homework 6

1. *AWGN Relay Channel*

   Figure 1 shows the AWGN-RC where $g, g_1, g_2 > 0$ are the channel gains, Z, Z_1 are iid $\sim N(0,1)$, and X and X_1 have the same power constraint P.



   (a) From the rate expressions for the discrete memoryless RC, derive the achievable rate of this Gaussian channel for decode-forward and compress-forward coding schemes. For decode-forward, show that joint Gaussian input is optimal.

   (b) Using Matlab, plot the rates derived in part (a) versus P for the following cases:

      i. $g = g_1 = 1$, $g_2 = 3$
      ii. $g = g_2 = 1$, $g_1 = 3$
      iii. $g_1 = g_2 = 3$, $g = 1$
      iv. $g_1 = g_2 = 1$, $g = 3$

2. *Compress-Forward Lower Bound*

   For the relay channel, the compress-forward lower bound is given as

$$C \geq \max_{p(x)p(x_1)p(\hat{y}_1|y_1,x_1)} \min\Big(I(X, X_1; Y) - I(Y_1; \hat{Y}_1|X, X_1, Y), I(X; Y, \hat{Y}_1|X_1)\Big)$$

   Starting from the error events given in El Gamal-Kim lecture notes page $(17-43)$, derive this lower bound using joint typicality lemma. Show all steps in your derivation.

3. *Relay with correlated noise.*

   Consider the Gaussian relay channel described as

$$Y_1 = g_1 X + Z_1$$
$$Y = gX + g_2 X_1 + Z$$

   where X and X_1 are the signals transmitted from the source and the relay respectively, Y_1 and Y are the signals received at the relay and the destination, respectively, and Z_1 and Z are Gaussian noises with zero mean and unit variance. Assume that X and X_1 are both independent of Z and Z_1 and both have power constraints P. The channel gains g, g_1, g_2 are real.

   Now assume that the noises Z and Z_1 are jointly Gaussian and are correlated with correlation factor of ρ. Evaluate an upper bound and achievable rates for this channel using

(a) Cut-set bound
(b) Direct transmission
(c) Decode-forward

Under what condition does each coding technique achieve the capacity? Interpret the results.

4. *Compress-forward relaying*

Consider a discrete-memoryless relay channel $p(y, y_1|x, x_1)$. In the original compress-forward scheme as discussed in class, the relay uses random binning technique (Wyner-Ziv encoding) to encode the compression index and forward to the destination.

Now consider an alternative technique for relay encoding which does not involve random binning. Here the relay just compresses its received signal, then encodes the compression index directly and forwards. Design a decoding technique and derive the achievable rate for this modified compress-forward scheme.

Evaluate this new achievable rate assuming Gaussian codes and compare with the original compress-forward rate.

5. *Composite decode-forward and compress-forward relaying*

Design a composite decode-forward and compress-forward relaying scheme for the discrete memoryless relay channel. Derive the achievable rate and show the error analysis.

Evaluate this rate for the Gaussian relay channel. Compare the performance with individual decode-forward scheme and compress-forward scheme. Show numerical plot of rates for these different schemes when the relay is moving on a straight line between the source and the destination (assuming a pathloss exponent of 3).

6. *Fading relay channels*

Consider the relay channel described as

$$\begin{aligned} Y_1 &= g_1 X + Z_1 \\ Y &= g_0 X + g_2 X_1 + Z \end{aligned}$$

where Z_1 and Z are *independent* zero-mean unit-variance Gaussian noises. The channel gains g_0, g_1, g_2 are now *complex* values.

(a) For fixed channel gains, evaluate the decode-forward achievable rate and the cut-set upper bound. Can decode-forward achieve the capacity of this channel for any case of channel gains?
(b) Now assume that the channel has phase fading with uniform phases in $[0, 2\pi]$. Specifically, each channel gain can be written in the form

$$g_i = \frac{e^{\phi_i}}{d_i^{\alpha/2}}, \quad , i = 0, 1, 2$$

where ϕ_i are independent random phases uniform in $[0, 2\pi]$, d_i are fixed distances and α is the path loss exponent.

Assume that the channel gains are known at the decoders but not at the encoders. The channel *ergodic capacity* is then the supremum of the achievable rates *averaged over the channel distribution*. Evaluate the average rate achievable by decode-forward and the average cutset bound (closed forms not required) and provide conditions when these two bounds meet and hence establish the capacity for this simple fading case.

(c) Now assume that the channel has Rayleigh fading with both phase and amplitude random. Formulate the end-to-end message outage probability for decode-forward relaying, taking into account outage events at both the relay and the destination. Perform a simulation to compare this decode-forward outage probability with direct transmission outage. Comment on your results.

7. *Network coding for the two-way relay channel*

Consider the Gaussian two-way relay channel

$$\begin{aligned} Y_1 &= h_{12}X_2 + h_{1r}X_r + Z_1, \\ Y_2 &= h_{21}X_1 + h_{2r}X_r + Z_2, \\ Y_r &= h_{r1}X_1 + h_{r2}X_2 + Z_r, \end{aligned}$$

where Z_1, Z_2 and Z_r are *independent* zero-mean unit-variance Gaussian noises. The parameters h_* are the channel gain coefficients. Assume the channel gains depend on the distance only with a pathloss exponent γ.

(a) Devise an algebraic network coding scheme for this channel. Show the rate region and plot an example for a specific distance setting.

(b) Compare the rate region of the above network coding scheme with the rate region achieved by applying decode-and-forward relaying. Provide several numerical comparison for different link configurations and comment on your results.

(c) How does the result change if the channel gains now include small scale fading and are not necessarily reciprocal? Provide a couple of numerical comparison on the rate regions and comment.