

ES 4 Problem Set 1 solutions

Problem Set 1 covers material in Sections 1.5, 2.1, and 2.2 in the textbook.

After solving the problems, look at the solutions posted on the course website and categorize your work for each problem on the following scale:

- ● Completely correct
- ◐ Nearly correct, but made a small mathematical or copying error
- ◑ Solved part of the problem correctly
- ◒ Started some work in the right direction
- ○ Incorrect, or didn't even know where to start on the problem
- Include a question mark (?) in addition to one of the above symbols if you don't feel like you understand the question or the solution well enough to make a definite judgement.

Problems with an asterisk (*) are optional.

1.a	1.b*	1.c	1.d	2.a	2.b*	2.c	2.d*	3.a	3.b	3.c*

What questions do you have about these concepts and skills?

What things are you uncertain about (even if you don't have a specific question)?

Approximately how long did it take you to complete the homework?

How long did you take going over the solutions and writing this reflection?

Turn in this self-assessment sheet on Gradescope. You do not need to turn in anything else, although we're happy to look at your work if you have questions!

Problem 1: Numbers

- (a) Write 35 and 22 in binary, and add them.

Solution:

$$35 = 32 + 2 + 1 = 100011 \text{ in binary}$$

$$22 = 16 + 4 + 2 = 10110 \text{ in binary}$$

$$\begin{array}{r} 100011 \\ + 10110 \\ \hline 111001 \end{array}$$

This is $32 + 16 + 8 + 1 = 57$, which checks.

- (b) Write 63 and 4 in binary, and add them.

Solution:

$$63 = 32 + 16 + 8 + 4 + 2 + 1 = 111111 \text{ in binary}$$

$$4 = 100 \text{ in binary}$$

$$\begin{array}{r} 111111 \\ + 100 \\ \hline 1000011 \end{array}$$

This is $64 + 2 + 1 = 67$, which checks.

- (c) What is the result of $01100101 + 01110110$? Write the answer in both binary and decimal.

Solution:

$$\begin{array}{r} 01100101 \\ + 01110110 \\ \hline 11011011 \end{array}$$

$$11011011 = 128 + 64 + 16 + 8 + 2 + 1 \text{ in decimal}$$

$$= 219$$

- (d) Suppose you have a circuit that can store only 6 bits, which currently has the value 20. If you add 46 to this and store the result back, what will the resulting 6-bit value be (in decimal)?

Solution:

$$20 = 16 + 4 = 010100 \text{ in binary}$$

$$46 = 32 + 8 + 4 + 2 = 101110 \text{ in binary}$$

$$\begin{array}{r} 010100 \\ + 101110 \\ \hline 1000010 \end{array}$$

Keeping only the lower 6 digits, we have $000010 = 2$ in decimal. This is called “overflow”, because the number has overflowed the storage available and the result is incorrect.

Problem 2: Truth tables

- (a) Write a truth table for the logic equation $Y = A \oplus \overline{B} \oplus C$.

Solution:

A	B	C	Y
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0

- (b) Write a truth table for the logic equation $Y = \overline{A \oplus BC}$.

Solution:

A	B	C	Y
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

- (c) Write a logic equation in canonical sum-of-products and product-of-sums form for the following truth table:

A	B	C	Y
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

Solution:

For sum-of-products form, we take the four terms with '1's in the truth table, and get $\overline{A}\overline{B}\overline{C} + \overline{A}BC + A\overline{B}\overline{C} + ABC$

For product-of-sums form, we take the four terms with '0's in the truth table: $(A + B + \overline{C})(A + \overline{B} + C)(\overline{A} + B + \overline{C})(\overline{A} + \overline{B} + C)$

- (d) Write a logic equation in canonical sum-of-products and product-of-sums form for the following truth table:

A	B	C	Y
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

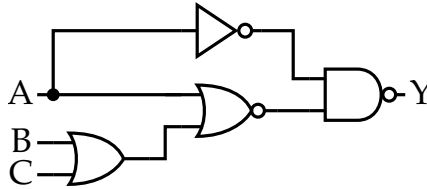
Solution:

For sum-of-products form, we take the six terms with '1's in the truth table, and get $\overline{A}\overline{B}C + \overline{A}B\overline{C} + A\overline{B}\overline{C} + A\overline{B}C + AB\overline{C} + ABC$

For product-of-sums form, we take the two terms with '0's in the truth table: $(A + B + C)(A + \overline{B} + \overline{C})$

Problem 3: Logic diagrams

- (a) Write a logic equation representing the circuit below.

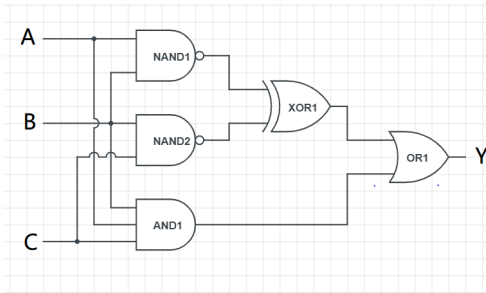


$$\overline{\overline{A}(A + B + C)}$$

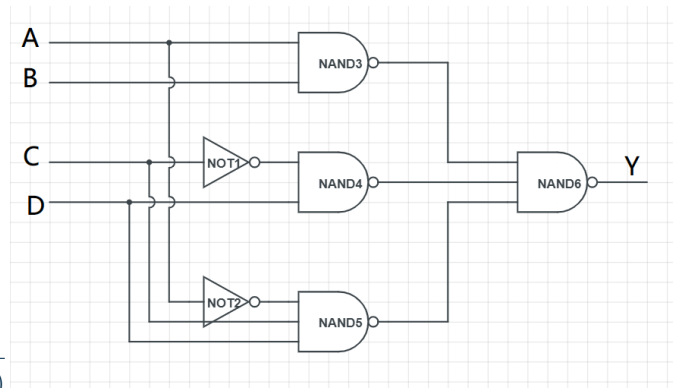
Using DeMorgan's theorem (either algebraically or with bubble pushing), this can be simplified to:

$$\begin{aligned} &= \overline{\overline{A}} + \overline{A + B + C} \\ &= A + \overline{A + B + C} \\ &= A + B + C \end{aligned}$$

- (b) Draw a logic diagram for the boolean equation $Y = (\overline{A}B \oplus \overline{B}C) + ABC$



- (c) Draw a logic diagram of the equation $AB + \overline{C}D + \overline{A}CD$ using only inverting logic (NAND/NOR/NOT). You can use whatever techniques you'd like to manipulate the equation/circuit.



$$AB + \overline{C}D + \overline{A}CD = \overline{\overline{AB + \overline{C}D + \overline{A}CD}} = \overline{(\overline{AB})(\overline{\overline{C}D})(\overline{\overline{A}CD})}$$

Additional practice

These are selected problems from the textbook (at the end of each chapter) which may be helpful for practice and review. The answers to these problems are online at <https://booksite.elsevier.com/9780128000564/solutions.php>.

- 1.13 (binary to decimal)
- 1.25 (decimal to binary)
- 1.53 (adding binary numbers)
- 2.1 (sum-of-products form)
- 2.3 (product-of-sums form)
- 2.5 (minimizing equations)
- 2.13 (minimizing equations)
- 2.7 (equations \rightarrow circuit)
- 2.27 (bubble pushing)