

Warmup

What is $0101\ 0001 + 0000\ 1101$?

Respond at **pollev.com/stevenbell**

ES 4: Boolean equations and logic gates

Steven Bell

8 September 2022

By the end of class today, you should be able to:

- Write truth tables for AND, OR, XOR, NOT, NAND, NOR, XNOR
- Write a boolean equation from an English description and vice-versa
- Draw a logic diagram from a boolean equation and vice-versa
- Write a truth table from a boolean equation and vice-versa

Key representations

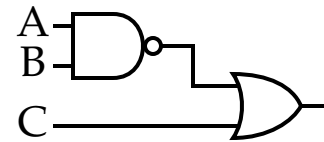
Boolean equation

$$\overline{AB} + C$$

Truth table

A	B	C	Y
0	0	0	1
0	0	1	1
0	1	0	1

Logic diagram



Key representations

English
description

Not both apples and
bananas...

Code

```
!(A&&B) || C
```

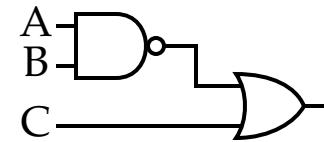
Boolean equation

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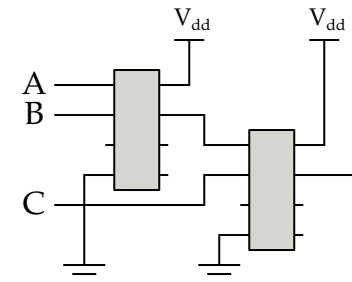
Truth table

A	B	C	Y
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Logic diagram



Circuit diagram



Name	Buffer	AND	OR	XOR	Inverter	NAND	NOR	XNOR																																																																																																						
Logic symbol																																																																																																														
Logic equation	A	AB	$A+B$	$A\oplus B$	\bar{A}	\overline{AB}	$\overline{A+B}$	$\overline{A\oplus B}$																																																																																																						
Truth table	<table border="1"> <thead> <tr><th>A</th><th>Y</th></tr> </thead> <tbody> <tr><td>0</td><td>0</td></tr> <tr><td>1</td><td>1</td></tr> </tbody> </table>	A	Y	0	0	1	1	<table border="1"> <thead> <tr><th>A</th><th>B</th><th>Y</th></tr> </thead> <tbody> <tr><td>0</td><td>0</td><td>0</td></tr> <tr><td>0</td><td>1</td><td>0</td></tr> <tr><td>1</td><td>0</td><td>0</td></tr> <tr><td>1</td><td>1</td><td>1</td></tr> </tbody> </table>	A	B	Y	0	0	0	0	1	0	1	0	0	1	1	1	<table border="1"> <thead> <tr><th>A</th><th>B</th><th>Y</th></tr> </thead> <tbody> <tr><td>0</td><td>0</td><td>0</td></tr> <tr><td>0</td><td>1</td><td>1</td></tr> <tr><td>1</td><td>0</td><td>1</td></tr> <tr><td>1</td><td>1</td><td>1</td></tr> </tbody> </table>	A	B	Y	0	0	0	0	1	1	1	0	1	1	1	1	<table border="1"> <thead> <tr><th>A</th><th>B</th><th>Y</th></tr> </thead> <tbody> <tr><td>0</td><td>0</td><td>0</td></tr> <tr><td>0</td><td>1</td><td>1</td></tr> <tr><td>1</td><td>0</td><td>1</td></tr> <tr><td>1</td><td>1</td><td>0</td></tr> </tbody> </table>	A	B	Y	0	0	0	0	1	1	1	0	1	1	1	0	<table border="1"> <thead> <tr><th>A</th><th>Y</th></tr> </thead> <tbody> <tr><td>0</td><td>1</td></tr> <tr><td>1</td><td>0</td></tr> </tbody> </table>	A	Y	0	1	1	0	<table border="1"> <thead> <tr><th>A</th><th>B</th><th>Y</th></tr> </thead> <tbody> <tr><td>0</td><td>0</td><td>1</td></tr> <tr><td>0</td><td>1</td><td>1</td></tr> <tr><td>1</td><td>0</td><td>1</td></tr> <tr><td>1</td><td>1</td><td>0</td></tr> </tbody> </table>	A	B	Y	0	0	1	0	1	1	1	0	1	1	1	0	<table border="1"> <thead> <tr><th>A</th><th>B</th><th>Y</th></tr> </thead> <tbody> <tr><td>0</td><td>0</td><td>1</td></tr> <tr><td>0</td><td>1</td><td>0</td></tr> <tr><td>1</td><td>0</td><td>0</td></tr> <tr><td>1</td><td>1</td><td>0</td></tr> </tbody> </table>	A	B	Y	0	0	1	0	1	0	1	0	0	1	1	0	<table border="1"> <thead> <tr><th>A</th><th>B</th><th>Y</th></tr> </thead> <tbody> <tr><td>0</td><td>0</td><td>1</td></tr> <tr><td>0</td><td>1</td><td>0</td></tr> <tr><td>1</td><td>0</td><td>0</td></tr> <tr><td>1</td><td>1</td><td>1</td></tr> </tbody> </table>	A	B	Y	0	0	1	0	1	0	1	0	0	1	1	1
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C++ equivalent	A	$A\&\&B$	$A B$	$A!=B$	$!A$	$!(A\&\&B)$	$!(A B)$	$A==B$																																																																																																						
74-series IC	7407	7408	7432	7486	7404	7400	7402	various																																																																																																						



STAR TREK



Y



A



B

Bigger equations

We'll use a **bar** to indicate inversion: $\overline{A+B}$

You might also see "¬" for inversion: $\neg(A+B)$

AND takes precedence over OR:

$$AB + C = (AB) + C$$

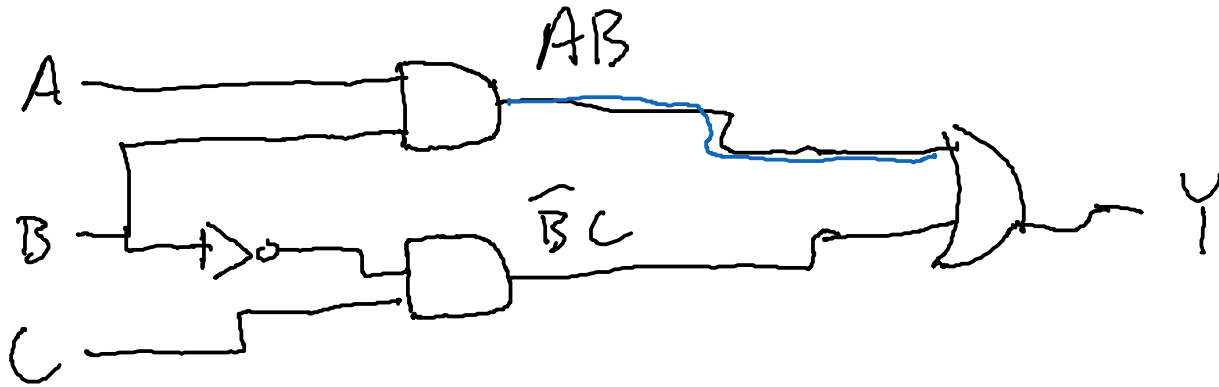
(Think multiplication over addition)

Drawing logic diagrams

Elements are objects

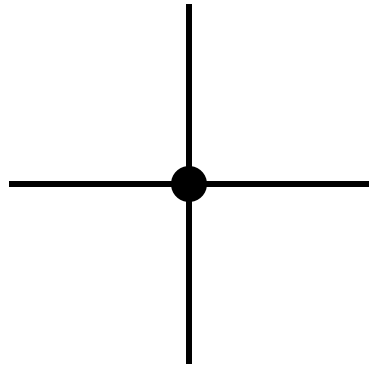
Nodes are the interconnections

$$Y = AB \oplus \overline{BC}$$

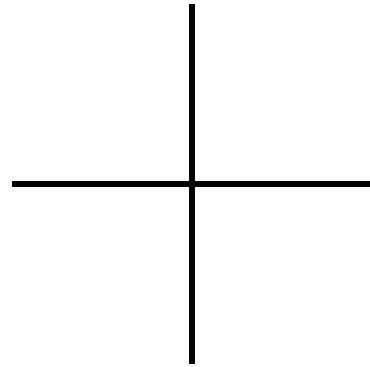


Logic diagram connections

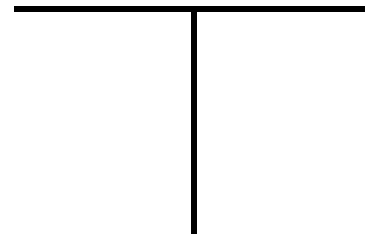
Wires are connected if there's a dot:



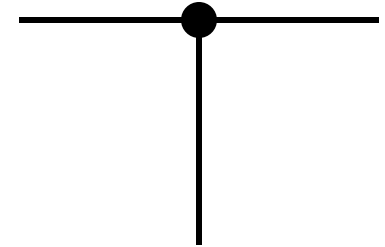
connected



not connected



connected



connected
(dot is only for clarity)

What is combinational?

A circuit is combinational if:

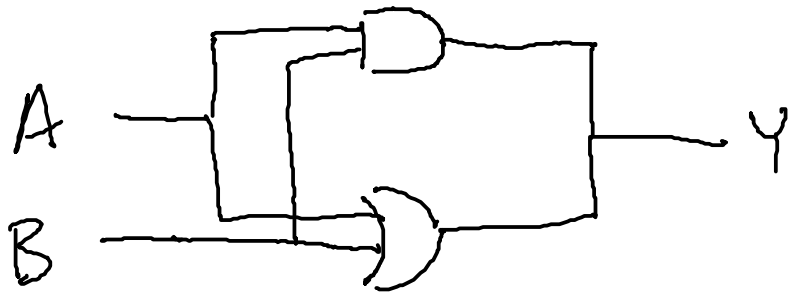
1. It is a discrete logic gate, or
2. It is composed of combinational elements such that:
 - There are no cycles, and
 - Every node (wire) is only driven by one gate

Who cares, anyway?

In a combinational circuit,
outputs are a function of only the current inputs

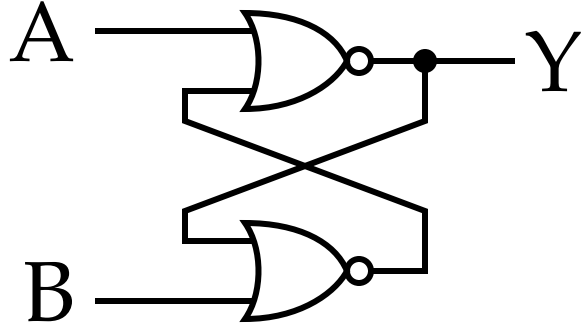
Feedback loops can break this!

Driving one wire with multiple gates breaks the digital abstraction!

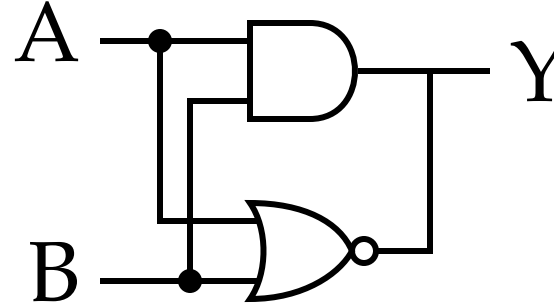


Which of the circuits below are combinational?

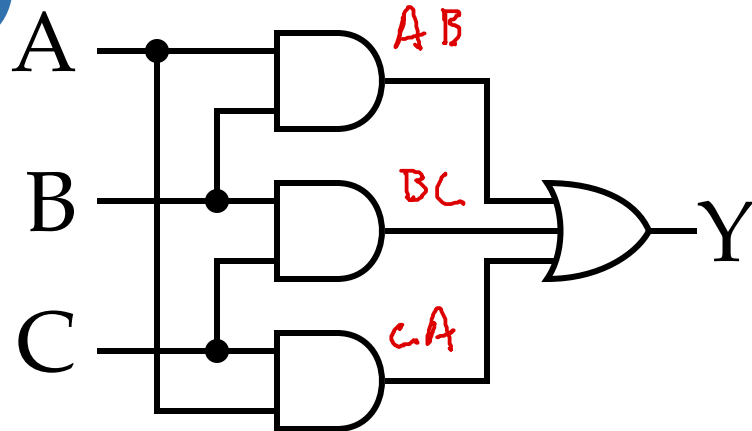
(1)



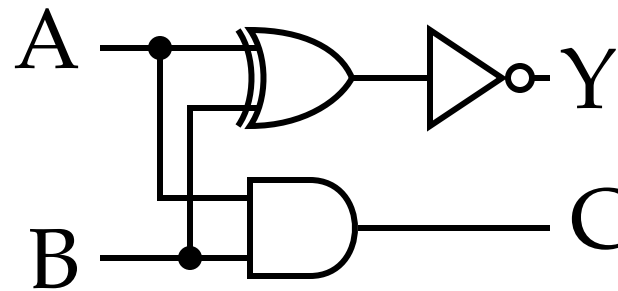
(2)



(3)



(4)

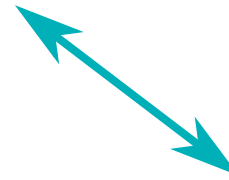


Respond at pollev.com/stevenbell

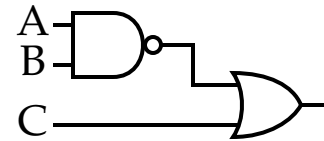
Translating representations

Boolean equation

$$\overline{AB} + C$$



Logic diagram



Truth table

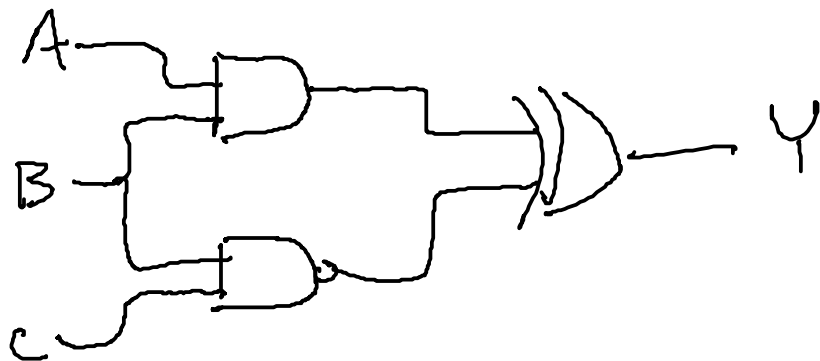
A	B	C	Y
0	0	0	1
0	0	1	1
0	1	0	1

Boolean equation \leftrightarrow logic diagram

Replace operations with gates, following precedence

Replace gates with operations, nesting to create precedence

$$Y = AB \oplus \overline{BC} \quad (\text{not } \overline{BC})$$



B	C	\overline{BC}	\overline{BC}
0	0	1	1
0	1	1	0
1	0	1	0
1	1	0	0

Translating representations

Boolean equation

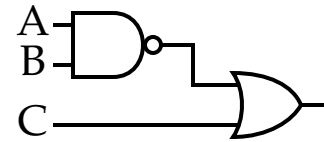
$$\overline{AB} + C$$

Evaluate

Truth table

A	B	C	Y
0	0	0	1
0	0	1	1
0	1	0	1

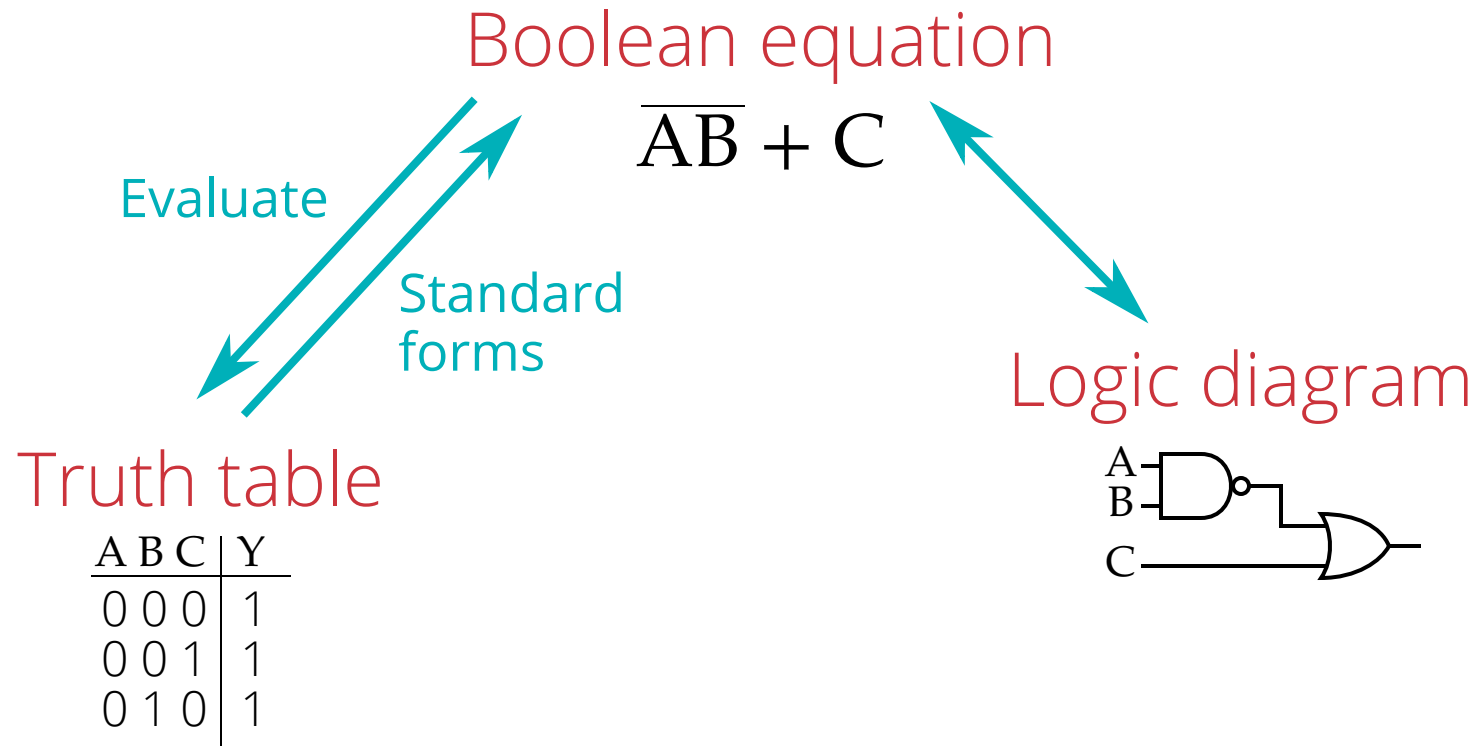
Logic diagram



$$Y = AB \oplus \overline{BC}$$

A	B	C	AB	\overline{BC}	Y
0	0	0	0	1	1
0	0	1	0	1	1
0	1	0	0	1	1
0	1	1	0	0	0
1	0	0	0	1	1
1	0	1	0	1	1
1	1	0	1	1	0
1	1	1	1	0	1

Translating representations



A	B	C	Y
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

A	B	C	Y
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

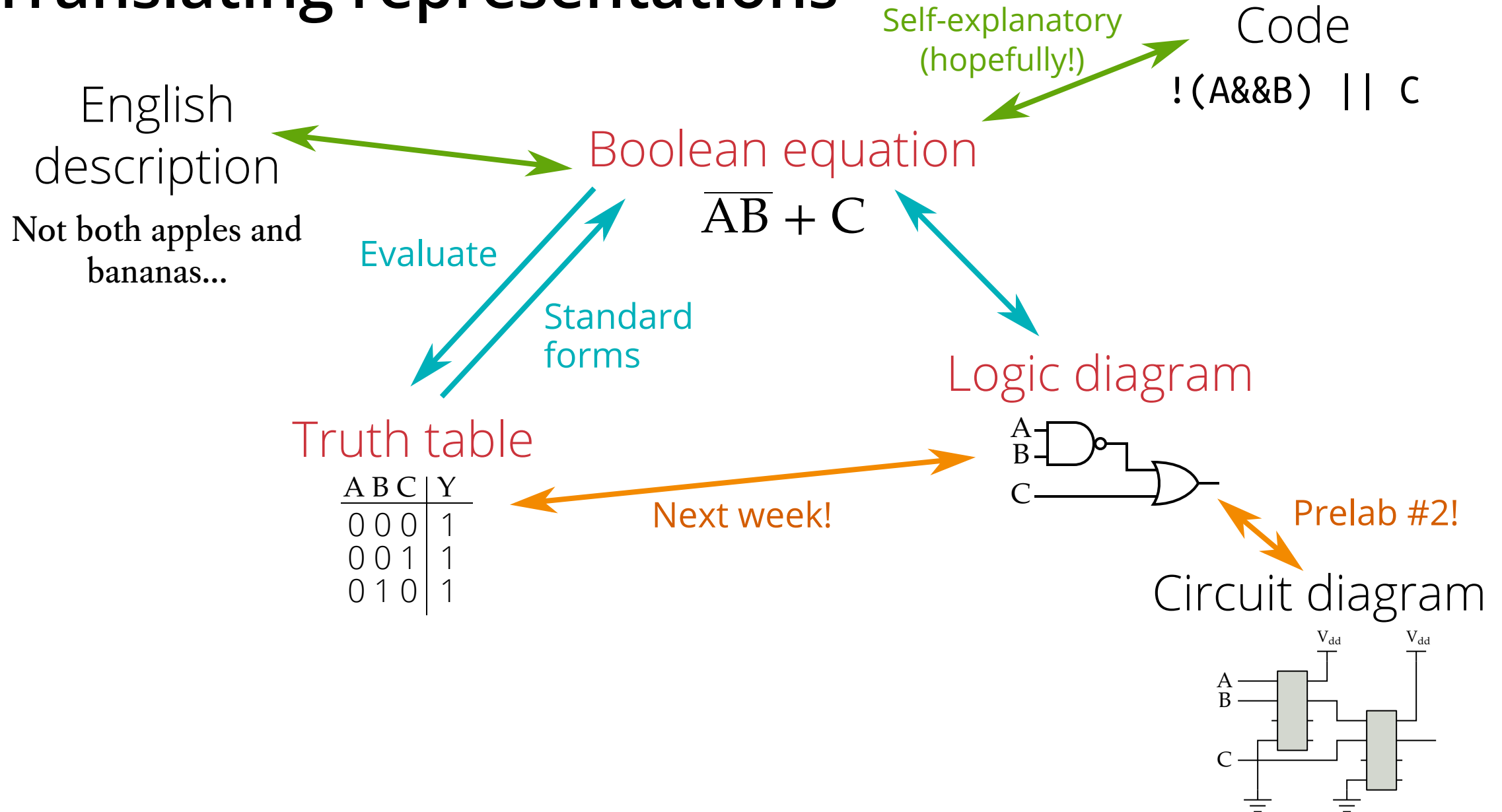
$$Y = ABC + \bar{A}\bar{B}C + \bar{A}B\bar{C} + \bar{A}BC + A\bar{B}\bar{C}$$

$$Y = (A+B+C) (\bar{A}+B+\bar{C}) (\bar{A}+\bar{B}+C)$$

$$Y = \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + \bar{A}BC + ABC$$

$$\overline{ABC} = \bar{A} + \bar{B} + \bar{C}$$

Translating representations



For next week

1. Read the book (2.3-2.7) and complete the pre-class reading check
Quiz is due at **11 AM** the day of class, so I can review it
2. Welcome survey posted as soon as I finalize lab times
3. Plan to come to lab next week!

Bonus material

Backup slides, extra information, or just stuff I had to cut out of the lecture for the sake of time.

A useful circuit

We have two 2-bit numbers. Design a circuit to calculate $A > B$.

Boolean or boolean?



George Boole
(1815-1864)

(Photo from Wikimedia)

Why is NAND a big deal?

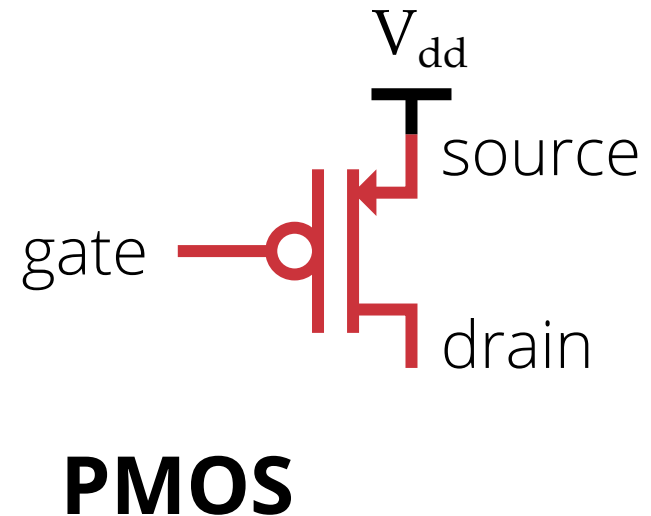
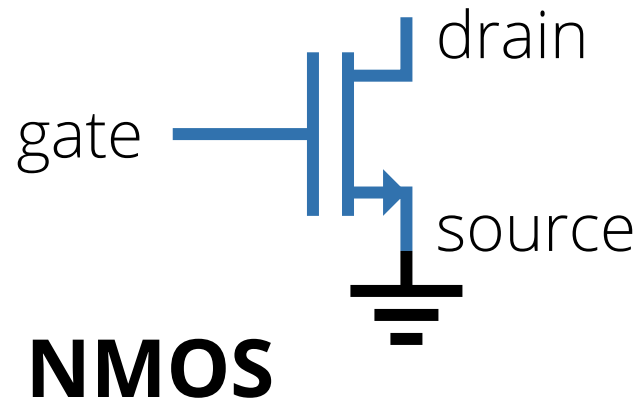
For the purposes of this course,

MOS transistors are electrically-controlled **switches**.

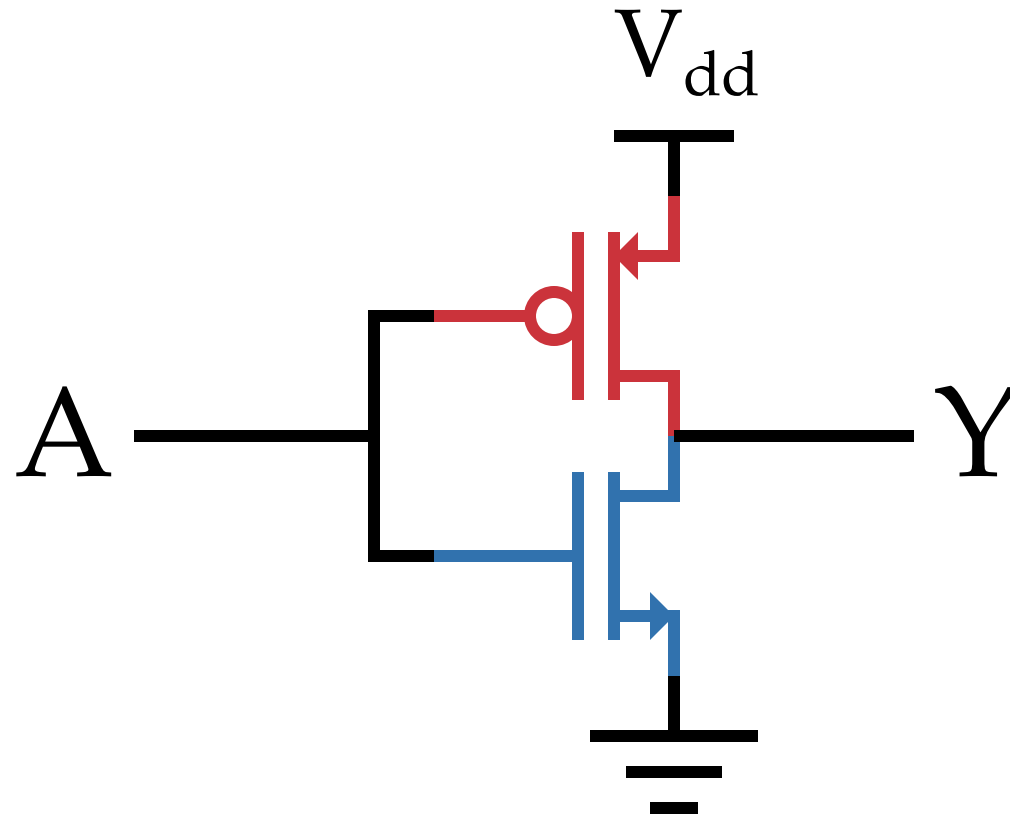
They have one terminal that controls the "switch", called the **gate**.

When the transistor is on, current can flow between the other terminals, called the **source** and **drain**.

They come in two types:



A simple gate



PMOS: "closed" when gate is **low**.

NMOS: "closed" when gate is **high**.

A more complex gate

