Warmup

Write a boolean equation in canonical form for this truth table:

A	B	C	Y
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

$\bar{A}BC + A\bar{B}C + ABC = Y$

ES 4: Boolean algebra

Steven Bell

10 September 2024

By the end of class today, you should be able to:

- Use boolean algebra to manipulate equations, using theorems 1-12
- Use "bubble pushing" to manipulate the logic of a circuit
- Use the above manipulation techniques to implement a circuit with constraints (e.g., only NAND gates)

Key representations

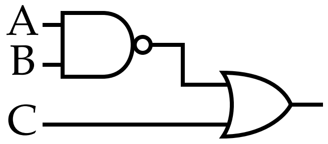
and how to simplify them

Boolean equation

$$\overline{AB} + C$$

Boolean algebra

Logic diagram



Bubble pushing

Truth table

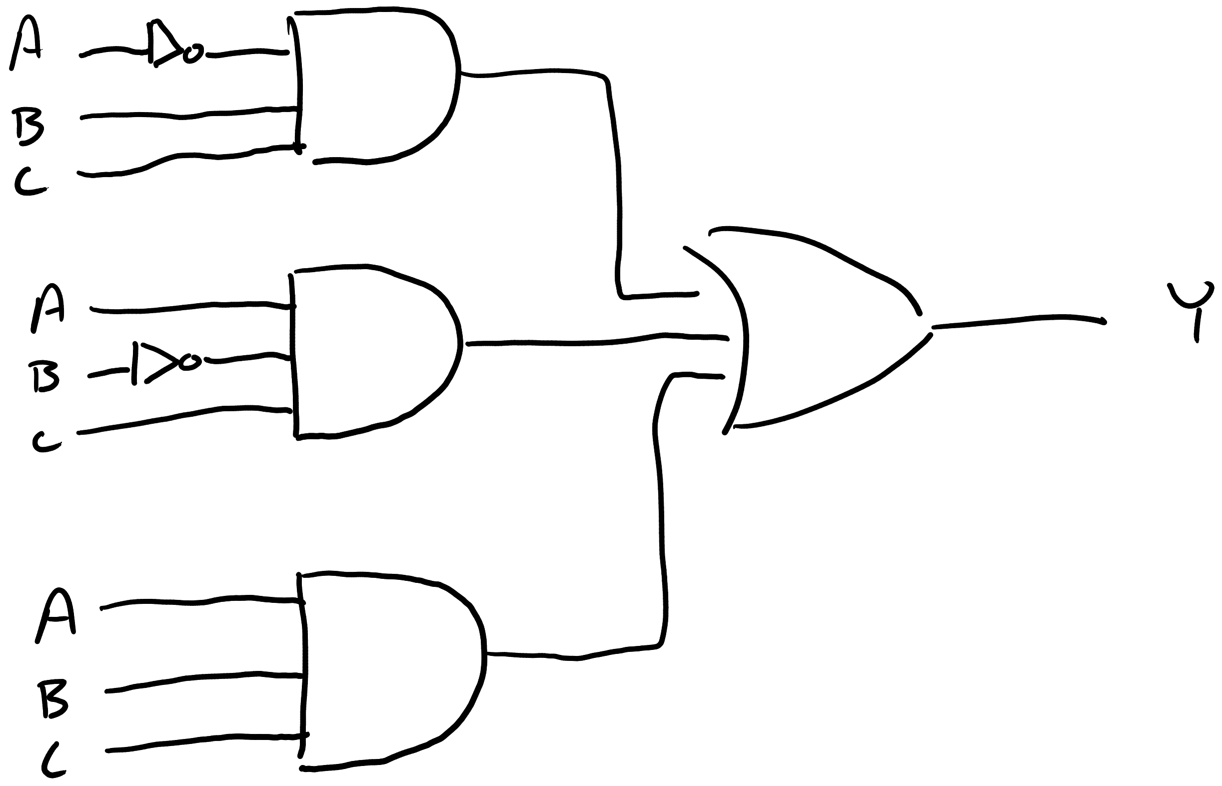
A	B	C	Y
0	0	0	1
0	0	1	1
0	1	0	1

Karnaugh maps

(next class)

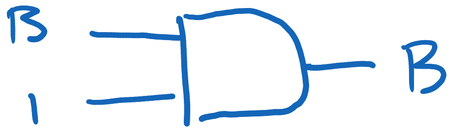
What if we implemented this with gates?

$$\overline{A}BC + A\overline{B}C + ABC$$



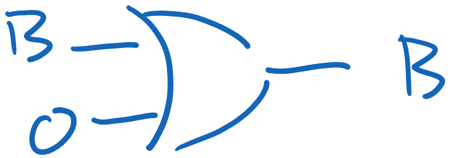
Boolean theorems to the rescue! (??)

$$B \cdot 1 = B$$

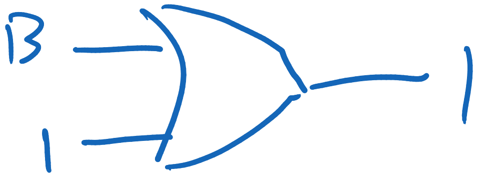


$$B \cdot 0 = 0$$

$$B + 0 = B$$



$$B + 1 = 1$$



More Boolean theorems to the rescue!

$$B \cdot B = B$$

$$B + B = B$$

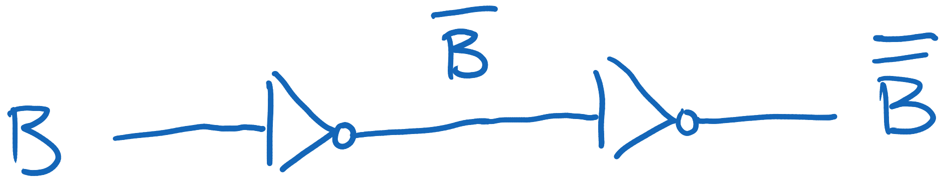
$$B \cdot \overline{B} = 0$$

$$B + \overline{B} = 1$$

Invterting twice

(aka "involution")

$$\overline{\overline{B}} = B$$



Associativity

Works just like "normal" algebra

$$ABC = (AB)C = A(BC)$$

$$A + B + C = (A+B) + C = A + (B+C)$$

Distributivity, what?

AND distributes over **OR**

$$A(B+C) = AB + AC$$

But also: **OR** distributes over **AND**

$$A + (BC) = (A+B)(A+C)$$

Simplifying from canonical form

$$\overline{A}BC + A\overline{B}C + ABC + ABC$$

$$\overline{A}(BC) + A(BC) + A\overline{B}C + ABC$$

$$(\overline{A} + A)(BC) + A\overline{B}C + ABC$$

$$BC + (\overline{B} + B)(AC)$$

$$BC + AC$$

$$(A + B)C$$

Practice

A	B	C	D	Y
0	**0**	**0**	**0**	**1**
0	**0**	**0**	**1**	**1**
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	0

A	B	C	D	Y
1	**0**	**0**	**0**	**1**
1	**0**	**0**	**1**	**1**
1	0	1	0	0
1	**0**	**1**	**1**	**1**
1	1	0	0	0
1	1	0	1	0
1	1	1	0	0
1	1	1	1	0

$$\bar{A}\bar{B}\bar{C}\bar{D} + \bar{A}\bar{B}\bar{C}D + A\bar{B}\bar{C}\bar{D} + A\bar{B}\bar{C}D + A\bar{B}CD$$

$$\bar{B}(\bar{A}\bar{C}\bar{D} + \bar{A}\bar{C}D + A\bar{C}\bar{D} + A\bar{C}D + ACD)$$

$$\bar{B}(\bar{A}\bar{C} + A\bar{C} + ACD + A\bar{C}D)$$

$$\bar{B}((\bar{A} + A)(\bar{C}) + (AD)(C + \bar{C}))$$

$$\bar{B}(\bar{C} + AD)$$

All the theorems

T1 $B \cdot 1 = B$ $\quad B + 0 = B$

T2 $B \cdot 0 = 0$ $\quad B + 1 = 1$

T3 $B \cdot B = B$ $\quad B + B = B$

T4 $\overline{\overline{B}} = B$

T5 $B \cdot \overline{B} = 0$ $\quad B + \overline{B} = 1$

T6 $B \cdot C = C \cdot B$ $\quad B + C = C + B$

T7 $(B \cdot C) \cdot D = B \cdot (C \cdot D)$ $\quad (B + C) + D = B + (C + D)$

T8 $(B \cdot C) + (B \cdot D) = B \cdot (C + D)$ $\quad (B + C) \cdot (B + D) = B + (C \cdot D)$

T9 $B \cdot (B + C) = B$ $\quad B + (B \cdot C) = B$

T10 $(B \cdot C) + (B \cdot \overline{C}) = B$ $\quad (B + C) \cdot (B + \overline{C}) = B$

T11 $(B \cdot C) + (\overline{B} \cdot D) + \cancel{(C \cdot D)}$ $\quad (B + C) \cdot (\overline{B} + D) \cdot \cancel{(C + D)}$

T12 $\overline{B_0 \cdot B_1 \cdot B_2 \ldots} = \overline{B_0} + \overline{B_1} + \overline{B_2} \ldots$ $\quad \overline{B_0 + B_1 + B_2 \ldots} = \overline{B_0} \cdot \overline{B_1} \cdot \overline{B_2} \ldots$

What do I need to remember?

The axioms

Theorems 1-8, and DeMorgan's theorem (12)

What do I need to remember?

be able to use?

The axioms

Theorems 1-8, and DeMorgan's theorem (12)

Intermission: lab notes

With lab notes, we're trying to push you toward *professional practice*

DeMorgan's theorem

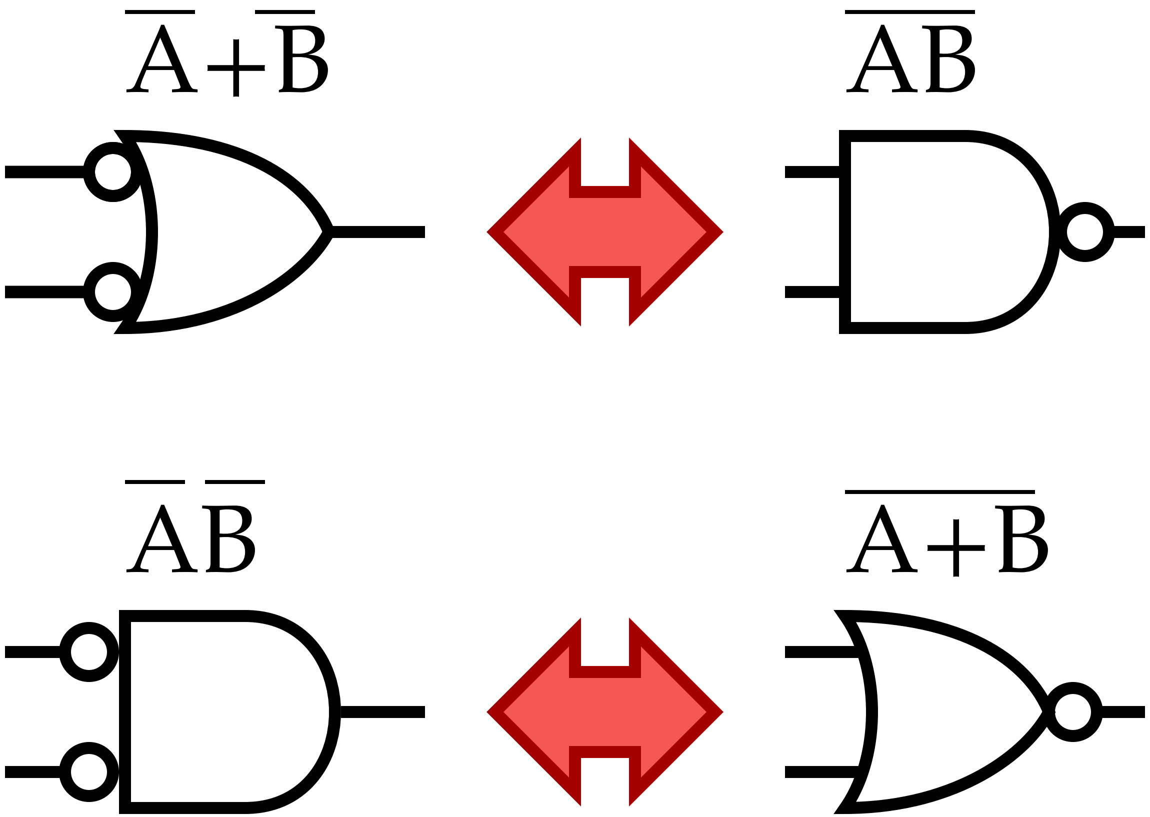
$$\overline{A} + \overline{B} = \overline{AB}$$

$$\overline{A}\,\overline{B} = \overline{A + B}$$

NAND! NOR

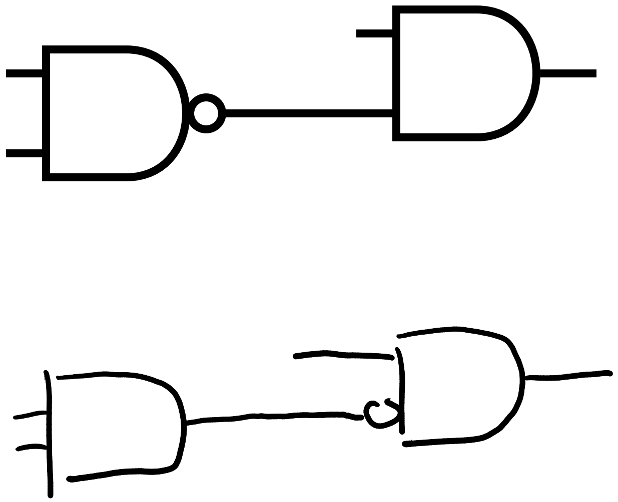
A	B	$\overline{A} + \overline{B}$	$\overline{A + B}$
0	0	1	1
0	1	1	0
1	0	1	0
1	1	0	0

DeMorgan's theorem on a schematic



Moving bubbles

A bubble just represents inversion, so it can be moved up or down a wire without changing the function.



Involution with bubbles

$$\overline{\overline{A}} = A$$

Bubble pushing

Modify this circuit to use NAND/NOR/NOT instead of AND/OR

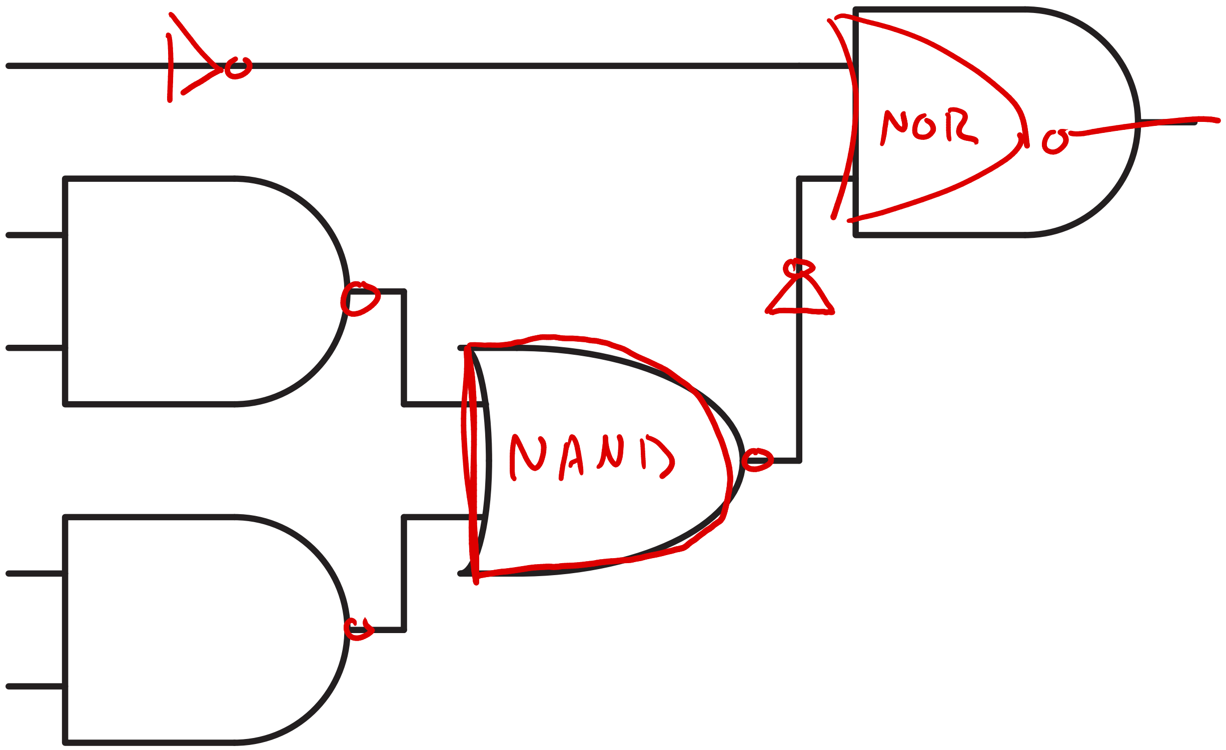


Figure 2.36 from the textbook

Bubble pushing

Modify this circuit to use NAND/NOR/NOT instead of AND/OR

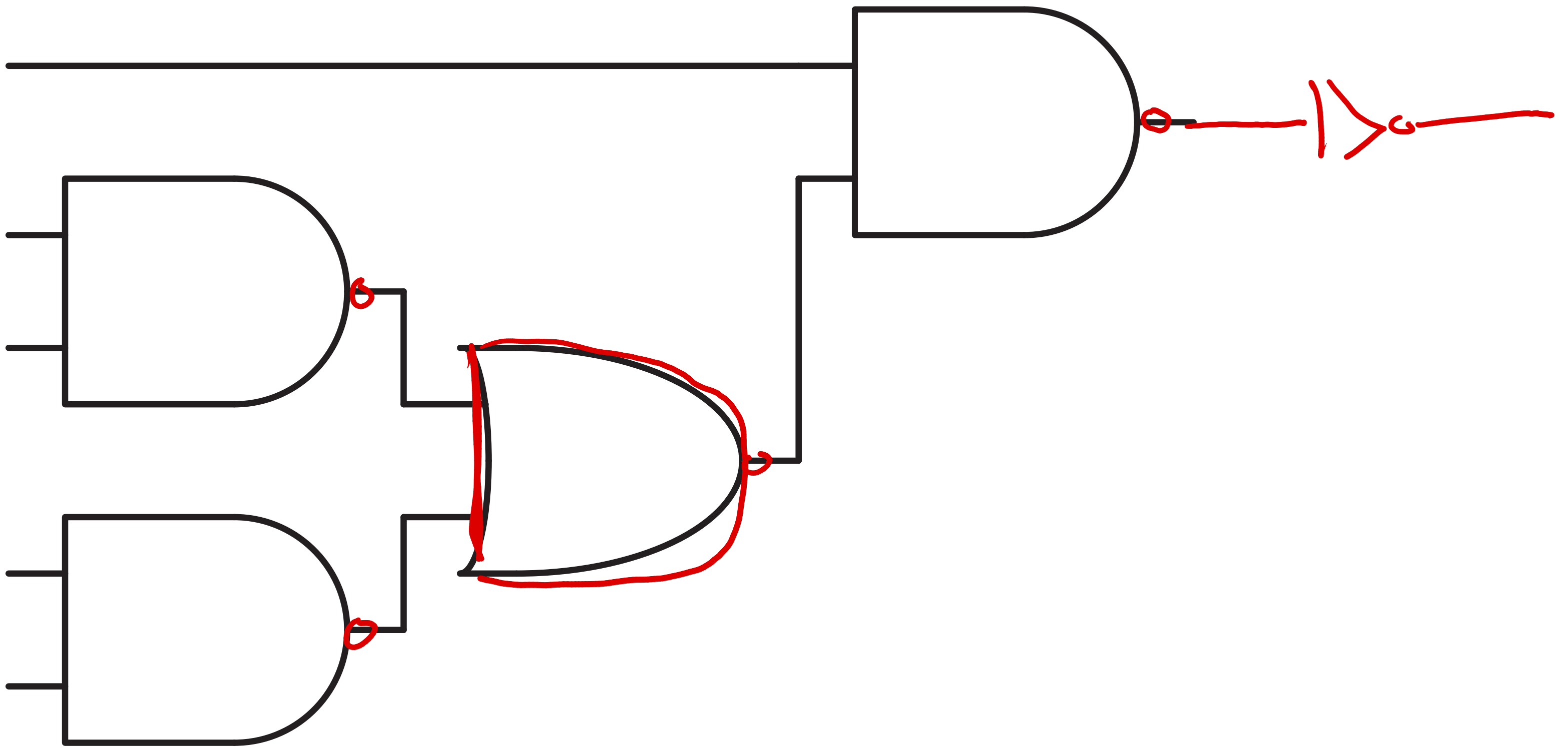
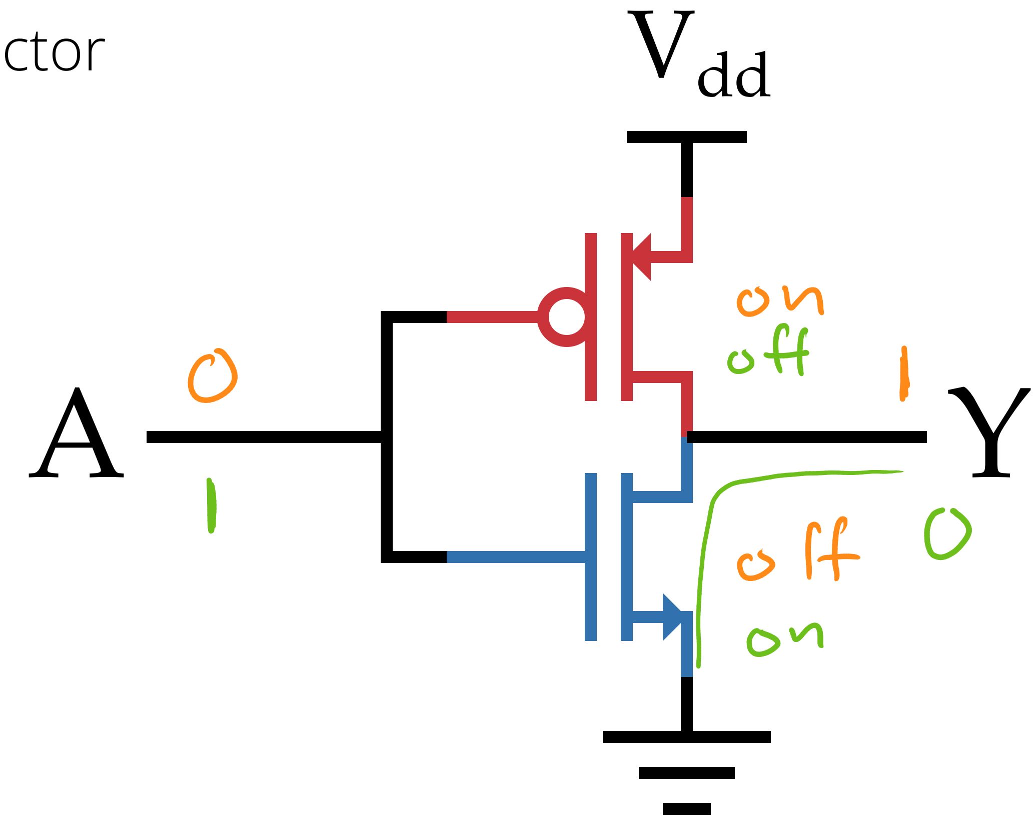


Figure 2.36 from the textbook

A simple gate

CMOS: *complimentary* metal-oxide semiconductor

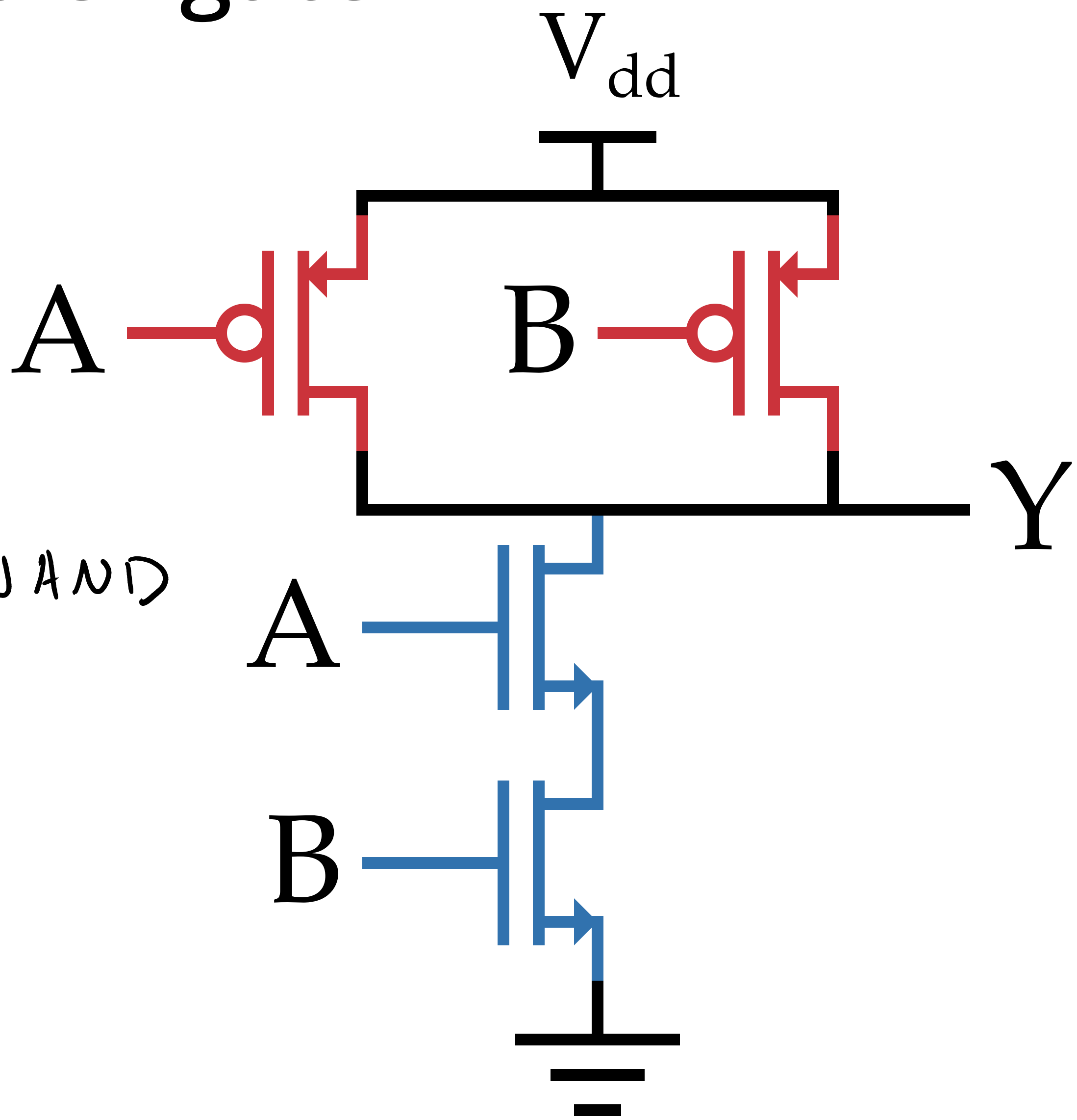


PMOS: "closed" when gate is **low**.

NMOS: "closed" when gate is **high**.

Bonus content

A more complex gate



NAND

A	B	Y
0	0	1
0	1	1
1	0	1
1	1	0

Bonus content

What about the consensus theorem?

It's pretty clever, and you can illustrate it with a Venn diagram
but it's not super helpful...

Bonus content

For Thursday

1. Read the book (2.6-2.7) and complete the reading check on Canvas

   Reading check is due at **11AM** the day of class, so I can review it

2. Lab sections are posted, plan to come to lab this week

3. Gradescope invites sent; submit your lab notes