On the Primary Exclusive Region of Cognitive Networks

Mai Vu, Natasha Devroye, and Vahid Tarokh

***Abstract*—We study a cognitive network consisting of a single primary transmitter and multiple secondary, or cognitive, users. The primary transmitter, located at the center of the network, communicates with primary receivers within a disc called the *primary exclusive region* (PER). Inside the PER, no cognitive users may transmit, in order to guarantee an outage probability for the primary receivers within. Outside the PER, uniformly distributed cognitive users may transmit, provided they are at a certain *protected radius* from a primary receiver. We analyze the aggregated interference from the cognitive transmitters to a primary receiver within the PER. Based on this interference and the outage guarantee, we derive bounds on the radius of the PER, showing its interdependence on the receiver protected distance and other system parameters. We also extend the analysis to allowing the cognitive users to scale their power according to the distance from the primary transmitter. These studies provide a closed-form, theoretical analysis of such a network geometry with PER, which may be relevant in the upcoming spectrum sharing actions.**

***Index Terms*—Cognitive networks, exclusive regions, outage probability, primary and secondary transmissions, spectrum sharing, secondary access, TV bands.**

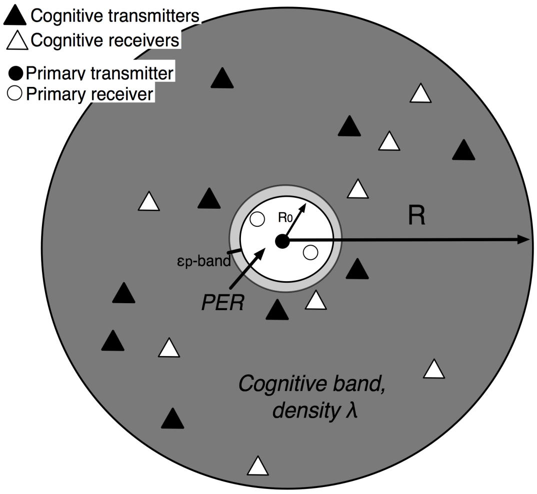

Fig. 1. A cognitive network consists of a single primary transmitter at the center of a *primary exclusive region* (PER) with radius R_0, which contains its intended receiver. Surrounding the PER is a protected band of width $\epsilon > 0$. Outside the PER and the protected bands, n cognitive transmitters are distributed randomly and uniformly with density λ.

I. Introduction

A COGNITIVE network usually consists of primary nodes, which have priority access to the spectrum, and cognitive (secondary) nodes, which access the spectrum according to some defined *secondary spectrum licensing* rules [3]. For example, consider a TV station which broadcasts in a currently licensed and exclusive band. Despite the high prices paid for these exclusive bands in spectral auctions [4], measurements show that *white space*, or temporarily unused time or frequency slots, are alarmingly common [5]. Notably, TV bands are wasted in geographic locations barely covered by the TV signal. This has prompted various regulatory and legislative bodies to put forth procedures [6] which would open up TV channels 2-51 (54 MHZ - 698 MHz) for use by secondary devices. These devices, often cognitive radios [7], [8], would be able to dynamically access the spectrum provided any degradation they cause to the primary license holders' transmissions is within an acceptable level. While the definition of what is acceptable is a still topic of much debate [9], this cognitive network model is of great interest. This re-licensing of exclusive bands is often termed *secondary spectrum licensing* [3] or *dynamic spectrum access* [10]–[12]. For practical feasibility studies of such TV-band networks, see [13] and references therein.

In this paper, we perform a theoretical study of network geometry in secondary-spectrum licensing problem. We consider a network with a single primary transmitter and multiple cognitive [1] users. The primary transmitter is located at the center of the network, and the primary receivers are within a circle of radius R_0, which we call the *primary exclusive region* (PER). This region is void of cognitive transmitters in order to guarantee a certain performance for the primary receivers within. Furthermore, any cognitive transmitter must be at least an ϵ radius away from a primary receiver. This practical assumption is to ensure that the primary receiver does not suffer infinite interference. Assuming the location of the primary receiver is unknown to the cognitive users, this ϵ-radius results in a guard band of width ϵ around the PER, in which no cognitive transmitters may operate (See Figure 1). Such a model can be applied to a TV network, in which the primary transmitter may be thought of as the TV broadcaster, and the primary receivers as TV subscribers. It can also apply to other scenarios, such as the downlink in a cellular network.

Such a network geometry is also considered in [14], of which we were unaware until a late stage of the current paper's research. In [14], the question of how cognitive radios must scale their power to meet a desired maximal interference constraint at a primary receiver is studied, first for a single cognitive transmitter, then for a large network of cognitive transmitters. By analyzing the aggregated secondary

[1] We use the terms cognitive and secondary interchangeably

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interference power, the authors of [14] provide bounds on the allowable cognitive transmit power.

The focus of this paper is on the radius of the primary exclusive region, subject to a primary outage constraint rather than a maximal interference constraint. Specifically, consider a network in which the cognitive users are mobile, or are static but joining and leaving the network at random. In the presence of the random interference from the cognitive users, the primary user must be guaranteed an outage capacity. This constraint ensures the primary user a minimum rate for a certain portion of time or network spatial realizations. The outage constraint must hold in the worst case scenario, which has the primary receiver at the edge of the PER in a network with an infinite number of cognitive users.

The impact of cognitive users on the primary user can be captured by the expected amount of interference from the cognitive users. Since many cognitive users may operate concurrently, the aggregated interference is relevant. We derive upper and lower bounds on this interference and show that, if the path loss exponent is greater than 2, the average interference remains bounded irrespective of the number of cognitive users. Based on these interference bounds, we provide an upper bound on the radius of the PER that satisfies the outage constraint on the primary user's rate. The bound also allows us to study the interdependence and trade-offs between the PER radius, the protected radius around each primary receiver and the primary transmit power.

The structure of this paper is as follows. In Section II, we introduce our network model and formulate the problem. In Section IV, we derive lower bounds and upper bounds for the expected interference seen at the primary receiver. Using these expressions, we then examine the outage constraint on the primary user and derive the relations among the radius of the *primary exclusive region*, R_0, the receiver protected radius ϵ, and all the other network parameters. In Section V, we provide our concluding remarks.

II. Problem Formulation

Consider a cognitive network with two types of users: primary and cognitive users. Of interest is the distance from the primary users at which the cognitive users can operate in order to ensure an outage probability for the primary users. We first discuss the network model and the channel and signal models, then define the primary exclusive region parameters. Our analysis first assumes constant transmit power for the cognitive users. Later we will briefly discuss the extension of the results to the case when the cognitive users scale their power according to the distance to the primary transmitter.

A. Network model

We consider an extended network with transmitters and receivers located on a planar circle of radius R, as shown in Figure 1. Assume that the single primary transmitter is located at the center of this network, a model suitable for a broadcast scenario. Surrounding the primary transmitter is a primary exclusive region (PER) of radius R_0. All primary receivers of interest are located in this region. Each primary receiver is protected by a disc of radius ϵ, within which no cognitive transmitters may operate. This receiver protected radius is to ensure that the aggregated interference at primary receiver does not reach infinite. Since these primary receivers may be passive devices, such as a TV receiver, their exact locations may be unknown to the cognitive users. (The location of the primary transmitter, on the other hand, can be easily detected.) Thus for the cognitive transmitters to meet the ϵ constraint, they must lie outside the circle of radius $R_0 + \epsilon$ centered at the primary transmitter. There are n cognitive users, each with a single transmitter and a single receiver, randomly and uniformly distributed with density λ in the cognitive band between radii $R_0 + \epsilon$ and R. All parameters R_0, ϵ and R are network design parameters and are known to the cognitive users.

B. Channel and signal models

We consider a path-loss only model for the wireless channel. Given a distance d between a transmitter and its receiver, the channel h is given by

$$h = \frac{A}{d^{\alpha/2}}, \tag{1}$$

where A is a frequency-dependent constant and α is the power path loss. In the subsequent analysis, we normalize A to 1 for simplicity. We consider $\alpha > 2$ which is typical in practical scenarios.

For the signal model, we assume no multiuser detection. Each user, either primary or cognitive, has no knowledge of other users' signals and treats their interference as noise. Furthermore, the signals of different users are statistically independent. With a large number of cognitive users, all independent and power-constrained, their interference to the primary receiver will be approximately Gaussian. Hence the transmit signals for both types of users are assumed to be zero-mean Gaussian.

C. The primary outage constraint

The radius R_0 of the primary exclusive region is determined by the outage constraint on the primary user's transmission rate T_0 given as

$$\Pr\left[T_0 \leq C_0\right] \leq \beta \tag{2}$$

where C_0 and β ($\beta < 1$) are pre-chosen constants. This constraint guarantees the primary user a rate of at least C_0 for all but β fraction of the time. Alternatively, the outage can be guaranteed over different realizations of the network spatial locations. This outage model can apply to networks in which the cognitive nodes are mobile, or are static but joining and leaving the network at random.

The outage constraint (2) must hold for all primary receivers within the PER, including the worst case which is when the receiver is on the edge of the PER. Let I_0 be the *aggregated interference power* from all cognitive users to this worst-case primary receiver. Let the transmit power of the primary user be P_0, and of each cognitive user be P. With Gaussian signaling, assume the Shannon capacity for the rate T_0 of the primary user, then in the worst case, $T_0 = \log\left(1 + \frac{P_0}{R_0^{\alpha}(I_0+\sigma^2)}\right)$. The outage constraint (2) is then equivalent to a probability on

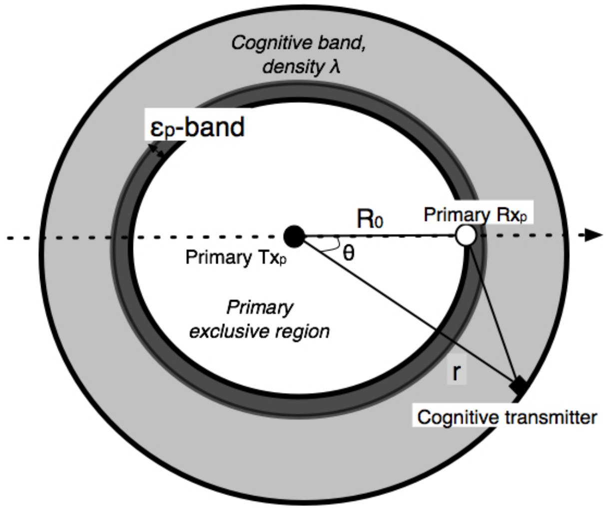

Fig. 2. Worst-case interference to a primary receiver: the receiver is on the boundary of the primary exclusive region of radius R_0. We seek to find R_0 to satisfy the outage constraint on the primary user.

the primary receiver's SINR (signal to interference and noise ratio) as

$$\Pr\left[I_0 \geq I_{\text{thres}}\right] \leq \beta \tag{3}$$

where

$$I_{\text{thres}} = \frac{P_0/R_0^\alpha}{(2^{C_0}-1)} - \sigma^2 \tag{4}$$

is the interference power threshold. As we consider channels with only path loss, the outages that occur here are not due to small-scale multipath fading as in traditional schemes, but are rather due to the path loss and random placement of the cognitive users.

III. Worst Interference to the Primary Receiver

In this section, we study the impact of the cognitive users on the primary users through the generated interference. We consider the worst case scenario in which the primary receiver is at the edge of the PER, on the circle of radius R_0, as shown in Figure 2. The outage constraint (2) must also hold in this case.

Consider interference at the worst-case primary receiver from a cognitive transmitter at radius r and angle θ. The distance $d(r,\theta)$ (the distance depends on r and θ) between this interfering transmitter and the primary receiver is given by

$$d(r,\theta) = \left(r^2 + R_0^2 - 2R_0 r\cos\theta\right)^{1/2}.$$

For uniformly distributed cognitive users, θ is uniform in $[0, 2\pi]$, and r has the density

$$f_r(r) = \frac{2r}{R^2 - (R_0+\epsilon)^2}. \tag{5}$$

The expected interference *power*, $E[I_0]$, experienced by the primary receiver from all $n = \lambda\pi(R^2 - (R_0+\epsilon)^2)$ cognitive users is then given by

$$E[I_0] = \int_{R_0+\epsilon}^{R}\int_0^{2\pi} \frac{\lambda r P\, dr\, d\theta}{(r^2+R_0^2-2R_0 r\cos\theta)^{\alpha/2}}. \tag{6}$$

For $\alpha = 2k$ with integer k, we can calculate $E[I_0]$ analytically. As an example, for $\alpha = 4$, we obtain the values of $E[I_0]$ as

$$E[I_0]_{\alpha=4} = \lambda\pi P\left[-\frac{R^2}{(R^2-R_0^2)^2} + \frac{(R_0+\epsilon)^2}{\epsilon^2(2R_0+\epsilon)^2}\right]. \tag{7}$$

The derivation is in the Appendix. Letting $R \to \infty$, this average interference becomes

$$E[I_0]_{\alpha=4}^{\infty} = \lambda\pi P\left[\frac{(R_0+\epsilon)^2}{\epsilon^2(2R_0+\epsilon)^2}\right] \tag{8}$$

For other values of α, closed-form evaluation of (6) is not available. We derive bounds on this expected interference power $E[I_0]$ at the primary receiver for a general α. These bounds are then used to analyze the interference versus the radius R_0 and the path loss α and establish an explicit dependence of R_0 on ϵ and other design parameters.

A. Upper and lower bounds on the average interference

In this subsection we obtain two lower bounds and an upper bound on $E[I_0]$. These bounds are established by slightly altering the geometry of the network.

1) A first lower bound on $E[I_0]$: A first lower bound on $E[I_0]$ can be established by re-centering the network at the *primary receiver* Rx_p. We then make a new exclusive region of radius $2R_0$, and a new outer radius of $R-R_0$, both centered at Rx_p. The set of cognitive users included in the new ring will be a subset of the original, making the interference a lower bound as

$$\begin{aligned} E[I_0]_{\text{LB1}} &= \int_{2R_0+\epsilon}^{R-R_0} \frac{2\pi\lambda P r}{r^\alpha} dr \\ &= \frac{2\pi\lambda P}{\alpha-2}\left(\frac{1}{(2R_0+\epsilon)^{\alpha-2}} - \frac{1}{(R-R_0)^{\alpha-2}}\right). \end{aligned} \tag{9}$$

As $R \to \infty$, this bounds approach the limit

$$E[I_0]_{LB1}^{\infty} = \frac{2\pi P\lambda}{\alpha-2}\frac{1}{(2R_0+\epsilon)^{\alpha-2}}. \tag{10}$$

This lower bound is tight when R_0 is small, but becomes loose as R_0 increases. The next lower bound has the opposite property.

2) A second lower bound on $E[I_0]$: Another lower bound on the interference can be derived by approximating the interference region by two half-planes, similar to [14]. As illustrated in Figure 3, consider only interference from the cognitive users in the two half-planes P_A and P_B which touch the circle of radius $R_0+\epsilon$. Consider a line in P_A that makes an angle ϕ at Rx_p, the distance d from any point on this line to Rx_p satisfies $\frac{\epsilon}{\cos(\phi)} \leq d < \infty$. Since the cognitive users are distributed uniformly, as $R \to \infty$, the distribution of d becomes similar to the distribution of r given in (5), and ϕ will be uniform in $[-\frac{\pi}{2}, \frac{\pi}{2}]$. Similar analyses hold for P_B. Hence the average total interference from the cognitive users in P_A and P_B to Rx_p is

$$\begin{aligned} E[I_0]_{\text{LB2}} &= P\lambda\left(\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\int_{\frac{\epsilon}{\cos(\phi)}}^{R}\frac{rdr}{r^\alpha}\,d\phi + \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\int_{\frac{2R_0+\epsilon}{\cos(\phi)}}^{R}\frac{rdr}{r^\alpha}\,d\phi\right) \\ &= \frac{P\lambda}{\alpha-2}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\left(\frac{\cos^{\alpha-2}(\phi)}{\epsilon^{\alpha-2}} + \frac{\cos^{\alpha-2}(\phi)}{(2R_0+\epsilon)^{\alpha-2}} - \frac{1}{R^{\alpha-2}}\right)d\phi. \end{aligned} \tag{11}$$

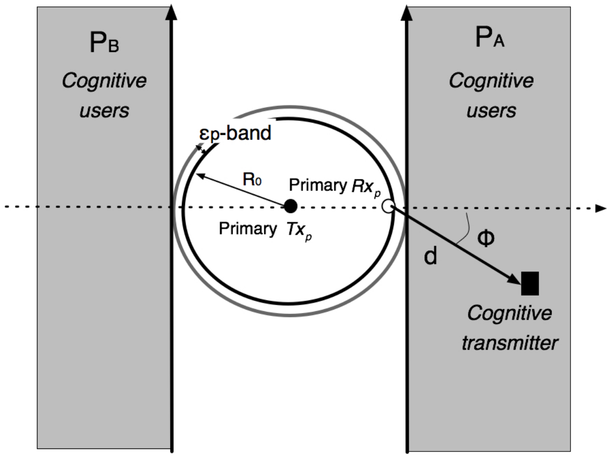

Fig. 3. Another lower bound on the expected interference at the primary Rx is obtained by approximating the interference region by two half-planes P_A and P_B. The region between these planes is free from cognitive transmitters.

Denote

$$A(\alpha) = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^{\alpha-2}(\phi)\, d\phi. \quad (12)$$

For an integer α, we can compute $A(\alpha)$ in closed form. We demonstrate a table for some values of $A(\alpha)$ in the Appendix, which we use in simulations. For other α, numerical evaluation of $A(\alpha)$ is possible. Now we can write the second lower bound on the average interference as

$$E[I_0]_{\rm LB2} = \frac{P\lambda}{\alpha-2}\left(\frac{A(\alpha)}{\epsilon^{\alpha-2}} + \frac{A(\alpha)}{(2R_0+\epsilon)^{\alpha-2}} - \frac{\pi}{R^{\alpha-2}}\right). \quad (13)$$

When $R \to \infty$, this lower bound approaches

$$E[I_0]_{LB2}^{\infty} = \frac{P\lambda A(\alpha)}{\alpha-2}\left(\frac{1}{\epsilon^{\alpha-2}} + \frac{1}{(2R_0+\epsilon)^{\alpha-2}}\right). \quad (14)$$

Since this bound takes into account the interfering transmitters close to the primary receiver, for a small ϵ or large R_0, this lower bound is tighter than the previous one in (10).

3) An upper bound on $E[I_0]$: For the upper bound, similar to the first lower bound, we re-center the network at the primary receiver. We now reduce the exclusive region radius, centered at Rx_p, to ϵ and extend the outer network radius, also centered at Rx_p, to $R_0 + R$. The set of cognitive transmitters contained within these two new circles is a superset of the original, creating an upper bound on the interference as

$$E[I_0]_{\rm UB} = \int_{\epsilon}^{R_0+R} \frac{2\pi P\lambda r}{r^\alpha} dr = \frac{2\pi P\lambda}{\alpha-2}\left(\frac{1}{\epsilon^{\alpha-2}} - \frac{1}{(R+R_0)^{\alpha-2}}\right). \quad (15)$$

As $R \to \infty$, this upper bound becomes

$$E[I_0]_U^{\infty} = \frac{2\pi P\lambda}{\alpha-2}\frac{1}{\epsilon^{\alpha-2}}. \quad (16)$$

B. Comparisons of the bounds on $E[I_0]$

As an example, we compare the upper bound in (16) and the lower bounds in (10) and (14) for various values of R_0, while fixing $\alpha = 4$, $\lambda = 1, P = 1$, and $\epsilon = 2$ and assuming an infinite network ($R \to \infty$). Figure 4 shows the upper and lower bounds compared to the exact expression of (8). The first lower bound is tight for small values of R_0, while the second lower bound is asymptotically tight as $R_0 \to \infty$. The upper bound is quite loose. However, these bounds provide a good indication for the range of interference power for different path loss α when the exact interference value is not analytically available.

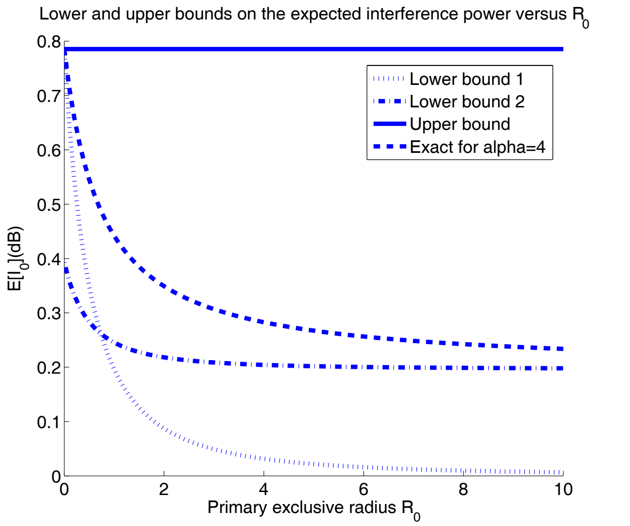

Fig. 4. Upper bound (16), first lower bound (10), second lower bound (14) for $\alpha = 4$, $\lambda = 1$, $P = 1$, $\epsilon = 2$. In this case we have the exact expression for $\alpha = 4$, which we compare to the other bounds to give an indication of their tightness.

C. Distance-dependent cognitive power scaling

As a special case, consider the cognitive transmitters which are allowed to scale their power according to the distance from the primary transmitter, similar to [14]. The transmit power P of a cognitive user may now be a function of the radius r, at which this cognitive user is located, as

$$P(r) = P_c r^\gamma \quad (17)$$

for some constant power P_c and a feasible power exponent γ.

Following the same development as in the constant power case, the two lower bounds and single upper bound can be derived to reflect this power scaling as follows.

$$E[I_0]_{LB1}^{\infty}(\gamma) = \frac{2\pi P_c\lambda}{\alpha-2-\gamma}\frac{1}{(2R_0+\epsilon)^{\alpha-2-\gamma}} \quad (18)$$

$$E[I_0]_{LB2}^{\infty}(\gamma) = \frac{P_c\lambda A(\alpha-\gamma)}{\alpha-2-\gamma}\left(\frac{1}{\epsilon^{\alpha-2-\gamma}} + \frac{1}{(2R_0+\epsilon)^{\alpha-2-\gamma}}\right) \quad (19)$$

$$E[I_0]_U^{\infty}(\gamma) = \frac{2\pi P_c\lambda}{\alpha-2-\gamma}\frac{1}{\epsilon^{\alpha-2-\gamma}}. \quad (20)$$

Based on these bounds, for the interference to stay bounded, the power scaling exponent must satisfy $\gamma < \alpha-2$. For a given path loss α and acceptable power scaling of γ, these bounds are equivalent to those of a channel with no power scaling and path loss $\alpha^* = \alpha - \gamma$. Thus a network with power scaling may be thought of as an equivalent network without power scaling but with a slower decay of the power with distance (a smaller path loss parameter).

IV. The Primary Exclusive Region

A. Bounding the radius R_0

The bounds on the expected interference in Section III can be used to provide bounds on the radius R_0 of the primary exclusive region. First assume that the primary network operates in the region that there is no outage due to noise, then the interference threshold (4) must be non-negative, which leads to

$$R_0 \le \left(\frac{P_0}{\sigma^2(2^{C_0}-1)}\right)^{1/\alpha} \triangleq R_0^u. \quad (21)$$

If R_0 is larger than R_0^u, the receivers at the edge of the PER will be in outage. This is because attenuation over the distance R_0^u causes an insufficient (in terms of the outage capacity) received signal in presence of noise alone. Thus, R_0^u is the maximum radius to ensure that the outage constraint holds even without any cognitive users.

The outage probability has also been applied to analyze a network of spectrum-sharing radios in [15]. There, several numerically based characteristic functions and a Gaussian approximation were used for the interference. It was stated that the Gaussian approximation is only valid when the interfering agile radios are not too close to the static receiver, which may be applied to our model for ϵ not too small. In [2], we used the Markov inequality to obtain a bound on the PER radius. In this paper, we use this Gaussian interference assumption to obtain a tighter bound.

The outage constraint on the interference power (3) transfers to a constraint on the variance of the Gaussian interference. Let z be the zero-mean Gaussian interference, note that the interference power $I_0 = z^2$, and apply the bound on the Gaussian tail, $Q(t) \le \frac{1}{2}\exp(-t^2/2)$, then

$$\Pr[I_0 \ge I_{\text{thres}}] = 2\Pr[z \ge \sqrt{I_{\text{thres}}}] \le \exp\left(-\frac{I_{\text{thres}}}{2E[I_0]}\right). \quad (22)$$

This analysis assumes real signals and interference, but it also applies to complex signals with independent and equal-variance real and imaginary parts. Using the upper bound on $E[I_0]$ in (16) for an infinite network, together with the expression for I_{thres} in (4) and applying the bound on the outage probability in (3) leads to the following upper bound on R_0:

$$R_0^\alpha \le \frac{P_0}{(2^{C_0}-1)}\left(-\frac{4\pi P\lambda \ln\beta}{(\alpha-2)\epsilon^{\alpha-2}} + \sigma^2\right)^{-1}. \quad (23)$$

This bound is always smaller than the bound in (21). As expected, the maximum distance that we can guarantee an outage probability for a primary receiver will be reduced in the presence of cognitive users.

When α is an even integer, the exact value of $E[I_0]$ may be used to obtain a tighter bound on R_0. Using the example for $\alpha = 4$ in (8), we obtain an implicit equation for the exclusive region radius R_0 as

$$\frac{(R_0+\epsilon)^2}{\epsilon^2(2R_0+\epsilon)^2} \le -\frac{1}{2\lambda\pi P\ln\beta}\left(\frac{P_0/R_0^4}{2^{C_0}-1} - \sigma^2\right). \quad (24)$$

Equations (23) and (24) provide a relation among the system parameters: P_0 (the primary transmit power), P (the cognitive users' power), C_0 (the outage capacity), β (the outage probability), λ (the cognitive user density), σ^2 (the noise power), and R_0 (the exclusive region radius). These equations may be of particular interest when designing the primary system to guarantee the primary outage constraint Pr[primary user's rate $\le C_0] \le \beta$. By fixing several of the parameters, we can obtain relations among the others. Specifically, we can relate the primary outage target rate C_0 to the capacity without interference $C = \log_2(1 + P_0/\sigma^2)$ as $C_0 = \eta C$, where $0 \le \eta \le 1$ represents the fraction of the interference-free capacity that we wish to guarantee with probability β in the presence of the cognitive users.

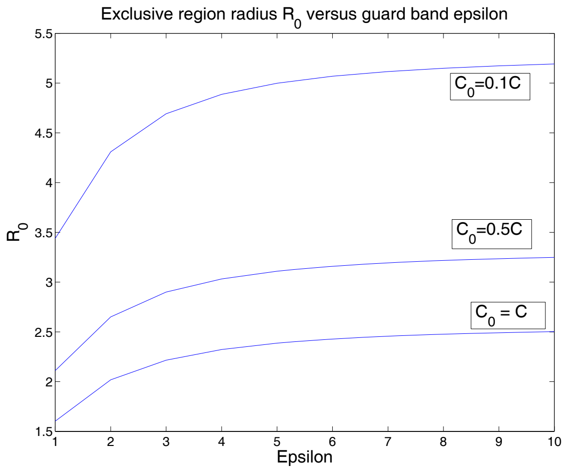

Fig. 5. The relation between the exclusive region radius R_0 and the guard band ϵ according to (24) for $\lambda = 1, P = 1, P_0 = 100, \sigma^2 = 1, \beta = 0.1$ and $\alpha = 4$.

B. Numerical examples

As an example, we plot in Figure 5 the relation between the exclusive region radius R_0 and the guard-band width ϵ for various values of the outage capacity C_0, while fixing all other parameters according to (24) for $\alpha = 4$. The plots show that, as expected, R_0 increases with ϵ. This is because of the trade-off between the interference seen from the secondary users, which is of a minimum distance ϵ away, and the desired signal strength from the primary transmitter, which is of a maximum distance R_0 away. The larger the ϵ, the less interference, and thus the further away the primary receiver may lie from the transmitter. Somewhat surprising, however, is that the plot shows an early diminishing effect of ϵ on R_0. Increasing ϵ beyond a certain value has little impact on R_0. As $\epsilon \to \infty$, R_0 approaches the limit of the interference-free bound in (21) for $\alpha = 4$. On the other hand, the PER radius R_0 decreases with increasing C_0. This is again intuitively appealing since to guarantee a higher capacity, the received signal strength at the primary receiver must increase, requiring the receiver to be closer to the transmitter.

Alternatively, we can fix the guard band ϵ and the secondary user power P and seek the relation between the primary power P_0 and the exclusive radius R_0. In Figure 6, we plot this relation according to (24) for $\alpha = 4$. The fourth-order increase in power here is in line with the path loss $\alpha = 4$. Interestingly, at small values of ϵ, a little increase in ϵ can lead to a large

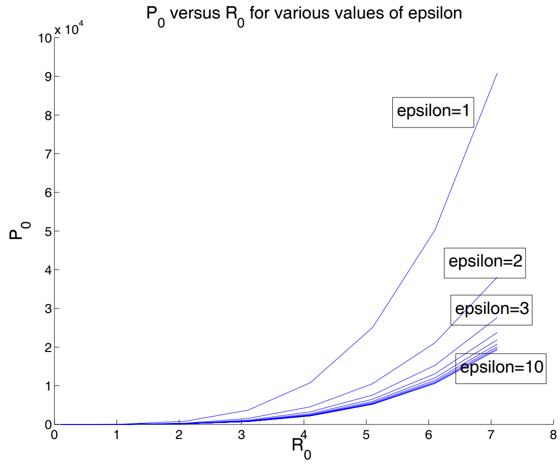

Fig. 6. The relation between the BS power P_0 and the exclusive region radius R_0 according to (24) for $\lambda = 1, P = 1, \sigma^2 = 1, \beta = 0.1, C_0 = 3$ and $\alpha = 4$.

reduction in the required primary transmit power P_0 to reach a receiver at a given radius R_0 while satisfying the given outage constraint. Here we also observe the diminishing impact of ϵ on the power P_0, similar to its impact on the PER radius R_0. These results suggest that there exists optimal operating values for ϵ, such that the receiver protected area is small enough while allowing sufficiently large R_0 and small transmit power P_0. This optimal ϵ value depends on other parameters, including the cognitive user transmit power P and density λ.

V. Conclusion

As cognitive networks are rapidly becoming a reality, it is of crucial importance to properly design the the network parameters to guarantee primary users a certain level of performance. In this paper, we guarantee an outage probability for the primary users: for any primary receiver within the PER of radius R_0, the probability that its rate falls below C_0 is less than β fraction of time or spatial realizations. By analyzing the average aggregated interference power at the worst-case primary receiver, we obtained bounds relating the design parameters: PER radius R_0, receiver protected radius ϵ, and transmit powers, to the desired parameters C_0 and β. These bounds can help in the design of cognitive networks with PERs.

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Appendix

Calculation of the Exact $E[I_0]$ when $\alpha = 4$

Here we derive the result in (7). For $a > |b|$, from pg. 383 [16], we obtain

$$\int_0^{2\pi} \frac{dx}{(a + b\cos(x))^2} = \frac{2\pi a}{(a^2 - b^2)^{3/2}}$$

In the integral of interest (6) we have $a = R_0^2 + r^2$ and $b = -2R_0 r$, and so $R_0^2 + r^2 > 2R_0 r$ as needed. Thus, the expected interference from all cognitive users is given by (25).

$$\begin{aligned} E[I_0] &= \lambda\pi P \int_{R_0+\epsilon}^{R} \int_0^{2\pi} \frac{2r\,dr\,d\theta}{2\pi(R_0^2 + r^2 - 2R_0 r\cos\theta)^2} \\ &= \lambda\pi P \int_{R_0+\epsilon}^{R} \frac{2r(r^2 + R_0^2)}{(r^2 - R_0^2)^3}\,dr \\ &= \lambda\pi P \left[-\frac{R^2}{(R^2 - R_0^2)^2} + \frac{(R_0+\epsilon)^2}{\epsilon^2(2R_0+\epsilon)^2} \right]. \end{aligned} \tag{25}$$

Evaluation of $A(\alpha)$

The function $A(\alpha)$ in (12) may be easily calculated (see for example pg. 161 of [16]) for integral values of α. For completeness, and reference for our simulations, we provide here a table of $A(\alpha)$.

α	2	3	4	5	6
$A(\alpha)$	π	2	$\frac{\pi}{2}$	$\frac{4}{3}$	$\frac{3\pi}{8}$

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