LINEAR PRECODING FOR MIMO WIRELESS CORRELATED CHANNELS WITH NON-ZERO MEANS: K FACTOR ANALYSIS, EXTENSION TO NON-ORTHOGONAL STBC

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ABSTRACT

Linear precoder for MIMO channels exploiting both the channel mean and transmit correlation has been shown to improve performance of an orthogonal space-time coded system [7]. In this paper, we extend the precoder design to systems with non-orthogonal space-time code, and provide asymptotic analysis at high K factor. The precoder is designed by minimizing the Chernoff bound on the pair-wise error probability. While a linear precoder can be viewed as a multi-mode beamformer, it converges to a single beam as K factor increases. Design criterion based on the minimum codeword distance and a new criterion based on the average codeword distance are considered. Numerical simulations using quasi-orthogonal STBC give examples of the performance gain that can be achieved with these designs.

1. INTRODUCTION

In MIMO wireless systems, it is well known that channel side information (CSI) at the transmitter, including partial information, can help enhance the systems performance [1, 2]. Precoder is a processing technique at the transmitter which exploits the available CSI.

Due to the fluctuating wireless channel, channel information at the transmitter is usually not complete. Space-time block codes (STBC) are useful in such scenarios to provide a robust measure against the unknown fading. Linear precoder in concatenation with STBC architecture has been widely studied in the literature [3]-[7], with various forms of transmit CSI involving channel mean and/or transmit antenna correlation.

In this paper, we study a linear precoder exploiting both channel mean and transmit correlation. This is an extension of the previous result in [7] to cover the non-orthogonal STBC case. Apart from analyzing the pair-wise codeword error (PEP) probability based on the minimum codeword distance, we also consider a new average codeword distance, which represents the covariance of the codeword error statistics. This criterion can be convenient in designing a precoder with non-orthogonal STBC. Asymptotic analysis on the effect of high K factor, which corresponds to a channel with strong mean component or to having a good channel estimate [3], shows that the precoder converges to a single mode beamformer on the dominant right singular vector of the channel mean as K factor increases. Thus for channels with high K factor, the STBC used with this precoder should have a rate limit to one. Precoder design with quasi-orthogonal STBC (QSTBC) [8, 9] gives an example for precoding with non-orthogonal STBC.

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The rest of the paper is organized as follows: In the next Section we outline the channel and signal models. Section 3 discusses the problem setup based on uncoded pair-wise error probability and the design criteria. Section 4 then analyzes the asymptotic effect of K factor on the optimal precoder. Precoder design with non-orthogonal STBC is discussed in Section 5, for both general case and QSTBC specifically. Section 6 presents performance results for the precoder with QSTBC using various modulation constellations. We conclude in Section 7.

2. CHANNEL AND SIGNAL MODELS

We consider a MIMO wireless communication system with N transmit and M receive antennas. The channel is frequency flat quasi-static fading which is represented by matrix $\mathbf{H}$ of size $M \times N$. Assuming a non-zero mean channel with transmit antenna correlation, the channel matrix can be written in the form

$$\mathbf{H} = \sqrt{\frac{K}{K+1}}\mathbf{H}_0 + \mathbf{H}_w\sqrt{\frac{1}{K+1}}\mathbf{R}_0^{1/2} . \qquad (1)$$

Here K is the ratio of the power in the mean component to the average power in the random components of the channel. $\mathbf{H}_w$ is a complex Gaussian random matrix with independent zero-mean and unit variance entries, i.e. $\mathbf{H}_w \in \mathcal{N}(0, \mathbf{I})$. $\mathbf{H}_0$ is the normalized channel mean and $\mathbf{R}_0$ is the normalized transmit correlation matrix such that

$$\begin{aligned} \operatorname{tr}(\mathbf{H}_0^*\mathbf{H}_0) &= MN \\ \operatorname{tr}(\mathbf{R}_0) &= N . \end{aligned} \qquad (2)$$

We assume that the transmit correlation matrix $\mathbf{R}_0$ is full rank and therefore is invertible. This assumption is due to the fact that if the transmit correlation matrix is not full-rank, the channel itself is likely rank-deficient, and the null-space of the channel must be avoided.

The receiver is assumed to know the channel perfectly (i.e. it knows the channel realization $\mathbf{H}$), whereas the transmitter only knows the channel mean $\mathbf{H}_0$, transmit correlation $\mathbf{R}_0$ and the K factor. The channel mean and transmit antenna correlation are more stable quantities than the instantaneous channel and hence can be obtained reliably at the transmitter, either by measuring the reverse channel or using feedback from the receiver.

Let $\mathbf{X}$ be the transmit signal block over T symbols, then the receive signal block is

$$\mathbf{Y} = \mathbf{H}\mathbf{X} + \mathbf{V} ,$$

where $\mathbf{V} \in \mathcal{N}(0, \mathbf{I}\sigma^2)$ is the additive complex white Gaussian noise with σ^2 being the noise power.

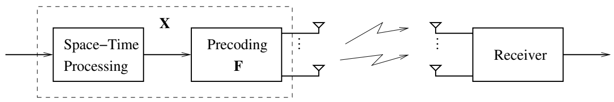

Fig. 1. MIMO channel precoding systems architecture.

3. PEP CRITERION AND LINEAR PRECODER DESIGN

3.1. PEP criterion

Follow the same setup in [7], we consider the uncoded PEP with ML detection. The probability of a signal block $\mathbf{X}$ being decoded incorrectly as $\hat{\mathbf{X}}$, averaged over the channel statistics, is

$$\text{PEP} \leq \frac{\exp\left[\text{tr}(K\mathbf{H}_0\mathbf{W}_0^{-1}\mathbf{H}_0^*)\right]}{\det(\mathbf{W}_0)^M}\det(\mathbf{R}_0)^M\exp\left[-\text{tr}(K\mathbf{H}_0\mathbf{R}_0^{-1}\mathbf{H}_0^*)\right] \quad (3)$$

where

$$\mathbf{W}_0 = \frac{1}{K+1}\frac{\mathbf{R}_0\Delta\Delta^*\mathbf{R}_0}{4\sigma^2} + \mathbf{R}_0 \ ,$$

and $\Delta = \mathbf{X} - \hat{\mathbf{X}}$. By ignoring the constant terms, minimizing this bound is equivalent to minimizing the following objective function

$$J(\mathbf{W}_0) = \text{tr}(K\mathbf{H}_0\mathbf{W}_0^{-1}\mathbf{H}_0^*) - M\log\det(\mathbf{W}_0). \quad (4)$$

3.2. Linear precoder with STBC setup

We study a linear precoding structure where a precoding matrix $\mathbf{F}$ is placed between a space-time processing block and the antennas as depicted in Figure 1. At each time instance, the linear precoder functions as a beamformer with either one or multiple orthogonal modes, which are the left singular vectors of $\mathbf{F}$. To provide a measure against unknown fading in the channel, space-time block code is used in this system. The combination of STBC and precoder makes the system robust against a changing environment while delivering both diversity and antenna array gains. It also gives the flexibility of adapting to various transmit channel knowledge parameters.

To maintain the total average transmit power, the precoding matrix needs to satisfy the power constraint

$$\text{tr}(\mathbf{F}\mathbf{F}^*) = 1. \quad (5)$$

Let $\mathbf{C}$ of size $N \times T$ be the codeblock in the STBC, the transmit signal block $\mathbf{X}$ then has the structure $\mathbf{X} = \mathbf{F}\mathbf{C}$. Denote

$$\mathbf{A} = (\mathbf{C} - \hat{\mathbf{C}})(\mathbf{C} - \hat{\mathbf{C}})^* \ ,$$

as the codeword distance product matrix then $\Delta\Delta^* = \mathbf{F}\mathbf{A}\mathbf{F}^*$.

3.3. Worst-case versus average design

The precoder design problem aims to find $\mathbf{F}$ to minimize the objective function (4) or (3), subject to the power constraint (5). Since $\mathbf{A}$ depends on the specific pair of codewords, a design criterion is needed for selecting $\mathbf{A}$ in order to find the optimal precoder.

Previous studies [3, 6, 7] have employed worst-case analysis where $\mathbf{A}$ corresponds to the minimum distance over all pairs of codewords. This is effectively the same as minimizing the bound on the maximum pair-wise error probability. This design guarantees a minimum performance gain for the precoder.

In this paper, we also examine another design criterion based on the average distance over all pairs of codewords. Since the precoder only acts on a column of $\mathbf{C}$ at a time, and the detection is done jointly over T symbol times, we propose the average distance measure for $\mathbf{A}$ as

$$\bar{\mathbf{A}} = \frac{1}{T}E\left[(\mathbf{C} - \hat{\mathbf{C}})(\mathbf{C} - \hat{\mathbf{C}})^*\right] = \frac{1}{T}\sum_{i\neq j} p_{ij}\Delta_{ij}\Delta_{ij}^*, \quad (6)$$

where $\Delta_{ij} = \mathbf{C}_i - \mathbf{C}_j$, and p_{ij} is the probability of the pair $(\mathbf{C}_i, \mathbf{C}_j)$ amongst all the distinct codeword pairs. $\bar{\mathbf{A}}$ therefore represents the covariance of the codeword error statistics.

Since the error bound is monotonic in $\mathbf{A}$, the average distance $\bar{\mathbf{A}}$ leads to a smaller value on the bound (3) compared to the minimum distance criterion. While this does not guarantee a minimum precoding gain, we find that it leads to a valid precoder. An attractive property of this distance measure is that often $\bar{\mathbf{A}}$ is a scaled identity matrix, where this may not be the case for the minimum distance $\mathbf{A}$ with non-orthogonal STBC. The implication of this will be made more obvious in Section 5. In the simulation section, we show comparisons between precoding performances based on these two design criteria.

4. EFFECT OF K FACTOR ON THE PRECODER

In this Section we investigate the effect on the optimal precoder as K factor increases to infinity. When K factor is infinite, this would correspond to a non-fading channel, or to the case where the instantaneous fading channel is known perfectly at the transmitter. In either case, it is useful to study this limit, so that applicable scenarios can be identified.

When K approaches infinity, the objective function (4) is invalid since it approaches infinity, and we need to use the full upper-bound (3) to analyze. Let

$$\mathbf{Q} = \frac{1}{K+1}\frac{\mathbf{R}_0^{\frac{1}{2}}\Delta\Delta^*\mathbf{R}_0^{\frac{1}{2}}}{4\sigma^2},$$

then $\mathbf{W}_0 = \mathbf{R}_0^{\frac{1}{2}}\left(\mathbf{Q} + \mathbf{I}\right)\mathbf{R}_0^{\frac{1}{2}}$. With sufficiently large K, the Hermitian matrix $\mathbf{Q}$ will have the largest eigenvalue with magnitude less than one, and the following expansion [12] can be applied

$$\mathbf{W}_0^{-1} = \mathbf{R}_0^{-\frac{1}{2}}\left(\mathbf{I} - \mathbf{Q} + \mathbf{Q}^2 - \mathbf{Q}^3 + \ldots\right)\mathbf{R}_0^{-\frac{1}{2}}.$$

Replace this into the upper bound (3), and by noticing that

$$K\mathbf{W}_0^{-1} - K\mathbf{R}_0^{-1} \rightarrow \frac{1}{4\sigma^2}\Delta\Delta^* \quad \text{as} \quad K \rightarrow \infty,$$

the limiting upper bound on the average PEP is

$$P_{\text{bound, limit}} = \exp\left[-\text{tr}\left(\frac{1}{4\sigma^2}\mathbf{H}_0\mathbf{F}\mathbf{A}\mathbf{F}^*\mathbf{H}_0^*\right)\right]. \quad (7)$$

Minimizing the above expression (7) is equivalent to maximizing the trace expression. Applying the inequality on trace of a matrix product $\text{tr}(ABC) \leq \sum\lambda_i(A)\lambda_i(B)\lambda_i(C)$, where $\lambda_i(A)$ are sorted eigenvalues of A [14], and taking into account the power constraint (5), the optimal $\mathbf{F}$ has the form

$$\mathbf{F} = \mathbf{u}\mathbf{v}^* \ ,$$

where $\mathbf{u}$ is the dominant eigenvector of $\mathbf{H}_0^*\mathbf{H}_0$ and $\mathbf{v}$ is the dominant eigenvector of $\mathbf{A}$. In other words, the optimal precoder in the limit of high K factor (or perfect channel knowledge) is a single mode beamformer that matches the dominant right singular vector of the channel mean. This applies regardless of the STBC used or the choice of worst-case or average distance criterion, as the left singular vectors of $\mathbf{F}$ does not depend on $\mathbf{A}$. Therefore at high K factors, the precoder based on the PEP criterion is suitable for MIMO/MISO systems employing a STBC with rate one or less. At low K factors, higher rate STBC may be used.

5. PRECODER FOR NON-ORTHOGONAL STBC

5.1. General case design

The case of precoding with orthogonal STBC is solved in [7] using the minimum distance criterion. In this Section, we extend the analysis to the non-orthogonal STBC case. In both cases, an optimization problem can be formulated as follows

$$\begin{aligned}\min_{\mathbf{W}_0} \quad & J(\mathbf{W}_0) \;=\; \mathrm{tr}(K\mathbf{H}_0\mathbf{W}_0^{-1}\mathbf{H}_0^*) - M\log\det(\mathbf{W}_0) \\ \text{s.t.} \quad & \mathbf{W}_0 \;=\; \frac{1}{4(K+1)\sigma^2}\mathbf{R}_0\mathbf{F}\mathbf{A}\mathbf{F}^*\mathbf{R}_0 + \mathbf{R}_0 \\ & \mathrm{tr}(\mathbf{F}\mathbf{F}^*) \;=\; 1\end{aligned} \tag{8}$$

The difference of non-orthogonal STBC from orthogonal STBC is that the codeword distance product matrix $\mathbf{A}$ may not be a scaled identity matrix. This makes solving the above optimization problem difficult in the general case.

To overcome the non-identity $\mathbf{A}$ problem, we use the following matrix inequality [13]

$$\lambda_{\min}(\mathbf{A})\mathrm{tr}(\mathbf{F}\mathbf{F}^*) \leq \mathrm{tr}(\mathbf{F}\mathbf{A}\mathbf{F}^*) \leq \lambda_{\max}(\mathbf{A})\mathrm{tr}(\mathbf{F}\mathbf{F}^*).$$

Noting that the precoder $\mathbf{F}$ should not allocate power in null-directions of the signal space, we only need to consider the eigenvalues of $\mathbf{A}$ that are non-zero. Based on this, we define the following value for the worst case distance criterion

$$\lambda_0 = \frac{1}{P}\min_{\Delta_{ij}} \lambda_{\min}(\Delta_{ij}\Delta_{ij}^*) \tag{9}$$

where $\lambda_{\min}(.)$ is the minimum non-zero eigenvalue of the matrix and P is the average transmit symbol power. For the average distance criterion, similarly we take $\lambda_0 = \lambda_{\min}(\bar{\mathbf{A}})/P$. Let $\mathbf{A} = \mathbf{U}_A\mathbf{\Lambda}_A\mathbf{U}_A^*$ be the eigenvalue decomposition of $\mathbf{A}$, then the unitary matrices $\mathbf{U}_A$ can be absorbed into the precoder $\mathbf{F}$. Now we relax the problem by replacing $\mathbf{\Lambda}_A$ with $\lambda_0 P\mathbf{I}$. This effectively makes $\mathbf{W}_0$ in (8) smaller (in the positive semi-definite sense), hence loosen the upper bound (3) on the error probability.

The relaxed problem formulation becomes

$$\begin{aligned}\min_{\mathbf{W}_0} \quad & J(\mathbf{W}_0) \;=\; \mathrm{tr}(K\mathbf{H}_0\mathbf{W}_0^{-1}\mathbf{H}_0^*) - M\log\det(\mathbf{W}_0) \\ \text{s.t.} \quad & \mathrm{tr}\left(\mathbf{R}_0^{-1}\mathbf{W}_0\mathbf{R}_0^{-1} - {\mathbf{R}_0}^{-1}\right) = \frac{\lambda_0}{4(K+1)}\rho \\ & \mathbf{R}_0^{-1}\mathbf{W}_0\mathbf{R}_0^{-1} - {\mathbf{R}_0}^{-1} \geq 0\end{aligned} \tag{10}$$

with ρ as the SNR. This problem is convex in the matrix variable $\mathbf{W}_0$ and can be solved analytically. The solution as given in [7] is restated here. Let

$$\mathbf{\Psi} = M^2\mathbf{I}_N + 4\nu K(K+1)\mathbf{R}_0^{-1}\mathbf{H}_0^*\mathbf{H}_0\mathbf{R}_0^{-1} ,$$

then $\mathbf{W}_0$ is given as

$$\mathbf{W}_0 = \frac{1}{2\nu(K+1)}\mathbf{R}_0\Big(M\mathbf{I}_N + \mathbf{\Psi}^{\frac{1}{2}}\Big)\mathbf{R}_0 .$$

Here ν is the Lagrange multiplier. The algorithms for solving ν can be found in [7]. Depending on the K factor and the SNR, the solution may require mode-dropping. In that case, the precoder does not allocate power in some directions.

Once the solution for ν is found, we can form the following matrix

$$\mathbf{\Phi} = \frac{2}{\nu\rho\lambda_0}(M\mathbf{I}_N + \mathbf{\Psi}^{\frac{1}{2}}) - \frac{4(K+1)}{\rho\lambda_0}\mathbf{R}_0^{-1} .$$

Let the eigenvalue decomposition of this matrix be $\mathbf{\Phi} = \mathbf{U}_\Phi\mathbf{\Lambda}\mathbf{U}_\Phi^*$ then the precoder is given as

$$\mathbf{F} = \mathbf{U}_\Phi\mathbf{\Lambda}^{\frac{1}{2}}\mathbf{U}_A^* . \tag{11}$$

Here $\mathbf{U}_A$ is the eigenvector of $\mathbf{A}$. For orthogonal STBC, since $\mathbf{A}$ is always a scaled identity matrix, $\mathbf{U}_A$ is an arbitrary unitary matrix and hence, it can be omitted. For non-orthogonal STBC, $\mathbf{U}_A$ depends on the STBC structure.

5.2. Precoder with quasi-orthogonal STBC

In this Section, we give an example of designing the linear precoder specifically for quasi-orthogonal STBC (QSTBC) [8, 9]. Consider the following form of QSTBC

$$C = \begin{pmatrix} c_1 & c_2 & c_3 & c_4 \\ -c_2^* & c_1^* & -c_4^* & c_3^* \\ c_3 & c_4 & c_1 & c_2 \\ -c_4^* & -c_3^* & -c_2^* & c_1^* \end{pmatrix} \tag{12}$$

In [6], precoder with this STBC is derived for channel mean feedback based on asymptotic analysis.

For this code, $\mathbf{A}$ has the form

$$A = \begin{pmatrix} a & 0 & b & 0 \\ 0 & a & 0 & b \\ b & 0 & a & 0 \\ 0 & b & 0 & a \end{pmatrix} \tag{13}$$

where $a = \sum_{i=1}^4 |\Delta c_i|^2$ and $b = \Delta c_1\Delta c_3^* + \Delta c_1^*\Delta c_3 + \Delta c_2\Delta c_4^* + \Delta c_2^*\Delta c_4$.

Note that although the minimum rank of $\mathbf{A}$ is two in this case, the precoder is not limited to rank two. The reason is that, at each time instance, the precoder acts on a separate column of the space-time code as a beamformer. Since there are 4 different symbols in each column, the precoder can form maximumly four orthogonal beams, one per symbol, that match the statistically preferred directions in the channel. Due to this beamforming effect, the rank of the precoder matrix is not a function of the diversity order of the STBC. Rather, it depends on the number of different symbols in each column of the STBC. Only when the SNR is not high enough that the precoder drops modes.

Assume that the symbols c_i come from a constellation $\mathcal{C}$. For the minimum distance criterion, $\mathbf{A}$ is given by the case where there is only one symbol difference, so that $\mathbf{A}_{\min} = \min_{\mathcal{C}}(|\Delta c|^2)\mathbf{I}$. For the average distance criterion, assuming all symbols c_i are independent, equally likely and that $E[c_i] = 0$, $\bar{\mathbf{A}}$ is also a scaled identity matrix $\bar{\mathbf{A}} = \alpha\mathbf{I}$ where

$$\alpha = \frac{2M_c^8}{M_c^8 - M_c^4}E\left[|c|^2\right] ,$$

with M_c as the number of signal constellation points in $\mathcal{C}$. In this QSTBC example, the matrix $\mathbf{A}$ is a scaled identity matrix in both criteria, hence the matrix $\mathbf{U}_A$ can be omitted in the precoder solution (11).

6. SIMULATION RESULTS

In this Section we present the numerical simulation results for a 4×1 system using QSTBC. The channel used in the simulations has a K factor of 0.1. The mean and the transmit correlation matrices are generated arbitrarily and normalized according to (2).

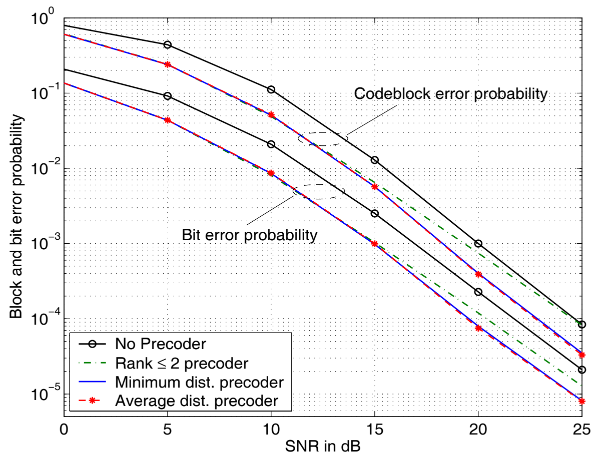

Fig. 2. 4×1 system using QSTBC and QPSK modulation.

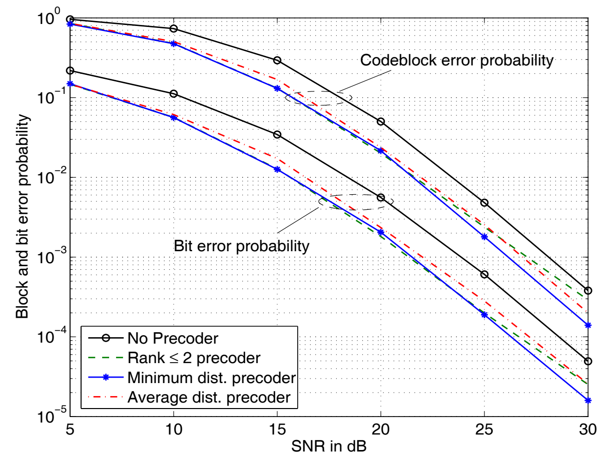

Fig. 3. 4×1 system using QSTBC and 16QAM modulation.

Figure 2 shows performance curves for QPSK modulation. Precoders based on the minimum distance and average distance criteria are included. Results show that both perform exactly the same for QPSK constellation, with a gain of around 1.7dB - 2dB compared to systems without precoder. Also shown is performance of the precoder that has rank limited to 2 only. This precoder performance gets worse as the SNR increases, and eventually it performs even worse than without precoding at very high SNRs. This serves as an illustrative example that the rank of the precoder should not depend on the diversity order of the STBC.

Figure 3 shows similar performance curves for 16QAM constellation, assuming Gray bit mapping. In this case, the precoder based on the minimum distance performs slight better than the one based on the average distance. The reason is that with a larger constellation, the number of minimum distance codeword pairs become larger, therefore worst-case design precoder gives more gain. The difference, however, is small in this case, and it shows that average distance is also a valid precoding design criterion. An advantage of using the average distance criterion is that, the matrix $\mathbf{A}$ is more likely to be a scaled unitary matrix and therefore helps simplifying the optimization problem (8). With a larger constellation size, the precoding gain is higher; the gains are at about 1.8-2.5dB in this case.

Notice that the precoder based on long-term channel statistics only picks up antenna array gain but not diversity gain. This is an attribute to statistical channel knowledge where the precise directions of each channel realization are unknown to the transmitter. Hence, this shows the role of the STBC in a precoded system in achieving a robust performance against unknown fading in the channel.

7. CONCLUSION

In this paper, we have studied a linear precoding structure that exploits long-term channel statistics in terms of both the channel mean and transmit antenna correlation. The precoder works in conjunction with a STBC. We use a framework similar to [7] and apply a relaxation to solve for the precoder with non-orthogonal STBC. We then provide precoder design and performance examples using QSTBC [8, 9]. Two different design criteria are examined: minimum distance and average distance. Simulation results show that precoders based on these two criteria perform quite closely to each other. Limiting analysis of high K factor effect on the precoder shows that it converges to a single mode beamformer which matches the dominant right singular vector of the channel mean matrix. Thus, for channels with high K factor, this precoder design is suitable for systems employing STBC with rate one or less. At low K factors, higher rate STBC may be used.

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