SOLUTIONS

PROBLEM 1:

\[ V_{id} \]

\[ 2V_T \]

\[ I_{bias} \]

\[ I_{c1} \]

\[ I_{c2} \]

\[ \frac{I_{bias}}{2} \]

\[ \frac{I_{bias}}{2} \]

\[ \frac{I_{bias}}{2} - g_m \cdot \frac{V_{id}}{2} \]

\[ \frac{I_{bias}}{2} + g_m \cdot \frac{V_{id}}{2} \]

* For \(-V_T < V_{id} < V_T\) or \(-25\text{mV} < V_{id} < 25\text{mV}\)
the collector current is linear w.r.t. \(V_{id}\) in differential input voltage, i.e.,

\[ I_{c1} = \frac{I_{bias}}{2} + g_m \cdot \frac{V_{id}}{2} \]

\[ I_{c2} = \frac{I_{bias}}{2} - g_m \cdot \frac{V_{id}}{2} \]

* For \(V_{id} > V_T\) or \(V_{id} < -V_T\) use the non-linear equation, i.e.,

\[ I_{c1} = I_{ce1} = \frac{I_{bias}}{1 + e^{-\frac{V_{id}}{V_T}}} \]

\[ I_{c2} = I_{ce2} = \frac{I_{bias}}{1 + e^{\frac{V_{id}}{V_T}}} \]
**Problem 2:**

Since \( V_{ip} = V_{im} = -2V \)

\[ I_{e1} = I_{e2} = 0.5mA. \]

\[ I_{c1} = I_{c2} = \frac{\beta}{(\beta+1)} \cdot 0.5mA = 0.49mA. \]

\[ V_{c1} = V_{c2} = 5 - (3k \cdot 0.49mA) = 3.53V \]

\[ V_{e} = -2 - 0.7 = -2.7V \]

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**Problem 3:**

\[ V_{ip} = +1 \quad V_{im} = -2V \]

\[ V_{id} = 3V \]

\[ 0^\circ \quad I_{c1} = 1mA \quad \text{and} \quad I_{c2} = 0 \]

\[ V_{c1} = 5V - 3k \cdot 1mA = 2V \]

\[ V_{c2} = 5V. \]

\[ V_{e} = 1 - 0.7 = 0.3V. \]
Problem 4:

We know the differential input impedance:

\[ R_{id} = 2 \cdot V_T = 2 \cdot \beta \cdot V_T \frac{I_C}{I_C} \]

where \[ I_C = I_{C1} = I_{C2} \]

\[ : \quad R_{id} = \frac{2 \times 150 \times 25 \text{mV}}{150 \mu\text{A}} = 50 \text{k}\Omega \]

Problem 5:

\[ V_{id} = V_{ip} - V_{im} = 20 \text{mV} \]

\[ \because V_{id} < V_T(25 \text{mV}) \quad \text{we can use the linear eqn} \]

\[ I_{C1} = \frac{I_{bias}}{2} + g_m \cdot \frac{V_{id}}{2} \]

\[ I_{C2} = \frac{I_{bias}}{2} - g_m \cdot \frac{V_{id}}{2} \]

\[ g_m = \frac{I_C}{V_T} = \frac{I_{C1}}{V_T} = \frac{0.5 \text{mA}}{25 \text{mV}} = 20 \times 10^{-3} \]

\[ : \quad I_{C1} = 500 \mu\text{A} + 20 \times 10^{-3} \times 10 \text{mV} = 700 \mu\text{A} \]

\[ I_{C2} = 500 \mu\text{A} - 200 \mu\text{A} = 300 \mu\text{A} \]
Problem 6

\[ V_{id} = 100 \text{mV} \quad (v_{ip-v_{im}}) \]

**.** \[ V_{id} > V_T \] we’ll use the non-linear large signal equation.

\[ I_{c1} = I_e1 = \frac{1 \text{mA}}{1 + e^{-\frac{100 \text{mV}}{25 \text{mV}}}} = \frac{1 \text{mA}}{1.135} \approx 880 \mu\text{A} \]

\[ I_{c2} = I_e2 = 1 \text{mA} - 880 \mu\text{A} = 120 \mu\text{A}. \]

Problem 7:

Since \[ V_{id} = 20 \text{mV} \] we can use the linear equation.

\[ I_{on} = I_{c1} - I_{c2} = 9 \mu\text{A} \cdot V_{id} \]

\[ = 20 \times 10^{-3} \times 20 \text{mV} = 400 \mu\text{A}. \]
Problem 8:

On neglecting $r_o$, 

$$I_o = g_m \cdot V_{id}$$

$$V_o = i_o \times \left( \frac{1}{R_L \| s \cdot C_L} \right)$$

$$= g_m \cdot V_{id} \cdot \left( \frac{\frac{1}{C_L}}{s + \frac{1}{R_L C_L}} \right)$$

or 

$$\frac{V_o(s)}{V_{id}} = \frac{g_m / C_L}{s + \frac{1}{R_L C_L}}$$

$$g_m = \frac{I_{ci}}{V_T} = \frac{0.5mA}{25mV} = 20 \times 10^{-3}$$

$$R_L = 10k\Omega, \quad C_L = 10pF$$

Pole frequency, $\omega = \frac{1}{R_L C_L}$

$$\omega = \frac{1}{2\pi \cdot R_L \cdot C_L} = \frac{1}{2\pi \cdot 10k\Omega \cdot 10pF} \approx 1.6 \text{MHz}$$

D.C. gain (i.e. $s=0$)

$$A_v = g_m \cdot R_L = 20 \times 10^{-3} \times 10k\Omega = 200$$
**Problem #9.**

(a) Large-signal transfer characteristic of $V_0/V_i$

1. $Q_1$ is cut-off and $Q_2$ is in deep saturation.
2. $Q_1$ is entering active & $Q_2$ is in saturation.
3. $Q_1$ & $Q_2$ are in forward active region.
4. $Q_1$ is in deep saturation and $Q_2$ in active.

(b) Small-signal model
It has been shown (lecture #32)

\[ A_v = \frac{V_o}{V_i} = -gm_1(R_{01} \parallel R_{02}) = -\frac{1}{\frac{V_T}{V_{A1}} + \frac{V_T}{V_{A2}}} \]

Since \( V_{A1} = V_{A2} = 75V \)

\[ A_v = -\frac{75V}{2 \times 25mV} = 1500 \]
Problem #10:
Small-signal model of the given Wilson's current mirror.

The output impedance \( R_{ot} = \frac{V_t}{I_t} \)

Before we find the expression for \( \frac{V_t}{I_t} \), we'll do a few simplification.

Since Q2 is diode connected, it can be replaced with an equivalent resistance of \( \frac{1}{g_{m2}} \)

\[ v_{x2} \quad \overrightarrow{g_{m2}} \quad \theta_{x2} \implies \theta_{x2} = \frac{1}{g_{m2}} \]

\[ \implies \frac{1}{g_{m2}} = \frac{1}{g_{m2}} \]

\[ \frac{1}{g_{m}} = \frac{1}{\theta_{x1}} + \frac{1}{\theta_{x2}} + \frac{1}{\theta_{o2}} + g_{m2} \]

where \( \frac{1}{g_{m}} = \frac{1}{\theta_{x1}} + \frac{1}{\theta_{x2}} + \frac{1}{\theta_{o2}} + g_{m2} \).
Assuming $\beta$ of each transistor is sufficiently large, the dependent source $g_{m1}v_{x1}$ can be replaced with current source $i_{x3}$.

So, the simplified small-signal model is:

We can write,

$$ V_t = i_1 \cdot R_{gm} + (i_t - g_{m3}v_{x3})R_{o3} \quad \text{(1)} $$

Using KCL at node (2)

$$ \frac{v_{x3}}{R_{x3}} + i_1 + \frac{i_1 \cdot R_{gm} + v_{x3}}{R_{o1}} = 0 \quad \text{(2)} $$

Rearranging (2) we get,

$$ v_{x3} = -i_1 \cdot R_{x3} \left( \frac{1 + \frac{R_{gm}}{R_{o1}}}{1 + \frac{R_{x3}}{R_{o3}}} \right) \quad \text{(3)} $$
KCL at 1

\[ i_t = i_4 - \frac{V_{x3}}{R_{x3}} \]  - (4)

Substituting (3) in (4) we get

\[ i_4 = \frac{i_t}{1 + \left( \frac{1 + \frac{R_{gm}}{R_{o1}}}{1 + \frac{R_{x3}}{R_{o1}}} \right)} \]  - (5)

Substituting (5) in (3)

\[ V_{x3} = -i_t R_{x3} \left( \frac{1 + \frac{R_{gm}}{R_{o1}}}{2 \frac{R_{x3}}{R_{o1}} + \frac{R_{gm}}{R_{o1}}} \right) \]  - (6)

Substituting (5) & (6) in (1)

\[ R_{ot} = \frac{V_t}{i_t} \]

\[ \Rightarrow R_{ot} = \frac{1}{\frac{1}{R_{gm}} \left[ 1 + \left( \frac{1 + \frac{R_{gm}}{R_{o1}}}{1 + \frac{R_{x3}}{R_{o1}}} \right) \right]} + R_{o3} + \frac{g_{ms} R_{x3} R_{o3} (1 + \frac{R_{gm}}{R_{o1}})}{2 \frac{R_{x3}}{R_{o1}} + \frac{R_{gm}}{R_{o1}}} \]
If we assume $r_0 \to \infty$

$$R_{\text{ot}} = \frac{r_{\text{qm}}}{2} + r_0 + \frac{g m_3 r_{\text{r}3} r_0}{2}$$

$$\frac{\nu^2}{\beta \cdot r_0^2}$$