**Problem 1**

(a) \( V_0 = -Gm \cdot Z_0 \cdot V_i \)

\[ = -Gm \cdot Z_0 \cdot \frac{V_s \cdot Z_i}{R_s + Z_i} = -Gm \cdot Z_0 \cdot V_s \quad [R_s = 0] \]

\[ \Rightarrow \frac{V_0}{V_s} = -Gm \cdot Z_0 = -1 \text{MA/V} \cdot 1 \text{M} = -1000 \]

(b) with \( R_s = 1 \text{K} \) & \( R_L = 1 \text{M} \)

\[ V_0 = -Gm \cdot V_i \cdot \left( Z_0 || R_L \right) \]

\[ = -Gm \cdot \frac{V_s \cdot Z_i}{R_s + Z_i} \cdot \left( Z_0 || R_L \right) \]

\[ \therefore \frac{V_0}{V_s} = -Gm \cdot \left( Z_0 || R_L \right) \cdot \frac{Z_i}{R_s + Z_i} = -1 \text{MA/V} \times 500 \text{K} \times 0.5 \]

\[ \therefore \frac{V_0}{V_s} = -250 \]
**Problem #2**

**(a)**

\[ \Delta = 1 - (L_1 + L_2) = 1 + G_1H_2 + G_2H_1 \]

\[ P_1 = G_1 \cdot G_2 \quad \Delta_1 = 1 \]

\[ \frac{O}{I} = \frac{P_1 \Delta_1}{\Delta} = \frac{G_1 \cdot G_2}{1 + G_1H_2 + G_2H_1} \]

**(b)**

\[ \Delta = 1 - (L_1 + L_2) \]

\[ = 1 - (-H_1 \cdot G_1 \cdot G_2 \cdot G_3 - H_2 \cdot G_2 \cdot G_3) = 1 + G_2 \cdot G_3 \left( G_1H_1 + H_2 \right) \]

\[ P_1 = G_1 \cdot G_2 \cdot G_3 \quad \Delta_1 = 1 \]

\[ P_2 = G_1 \cdot G_2 \cdot G_4 \quad \Delta_2 = 1 \]

\[ \frac{O}{I} = \frac{P_1 \Delta_1 + P_2 \Delta_2}{\Delta} \]

\[ \frac{O}{I} = \frac{G_1 \cdot G_2 \cdot (G_3 + G_4)}{1 + G_2 \cdot G_3 \left( G_1H_1 + H_2 \right)} \]
**PROBLEM #3**

Barkhausen Criteria:

\[ A_v \cdot \beta(s) = 1 \quad \text{where} \quad A_v = 1 + \frac{R_2}{R_1} \]

\[ \beta(s) : \]

\[ \frac{V_i - V_o}{R} = \beta C V_o \Rightarrow V_1 = V_o (1 + \beta CR) - (1) \]

KCL at \( V_o \):

\[ \frac{V_1 - V_o}{R} + \beta C (V_1 - V_i) + \beta C V_o = 0 \quad -(2) \]

On substituting \( V_1 \) in \( (2) \):

\[ V_o (1 + \beta CR) + \beta CR (V_o + V_o \beta CR) - \beta CR V_i + \beta CR V_o = 0 \]
\[ \beta(s) = \frac{V_0}{V_i} = \frac{8CR}{s^2C^2R^2 + 3.8CR + 1} \]

In order to satisfy Barkhausen Criterion:

\[ A_0 \cdot \beta(j\omega) = 1 \]

\[ \Rightarrow A_0 \cdot j\omega \cdot 8CR = -w^2C^2R^2 + 1 + j \cdot 3w \cdot CR \]

\[ \Rightarrow \sqrt{w^2C^2R^2 - 1} + j \sqrt{A_0 \cdot w \cdot 8CR} = 0 \]

In order for the complex number to be zero, both the real & imaginary part has to be zero.

\[ w^2C^2R^2 - 1 = 0 \quad \Rightarrow \quad \omega_0 = \frac{1}{RC} \]

\[ 2 \cdot wCR \{ 4A_0 - 3 \} = 0 \]

\[ \omega_0 \cdot CR \neq 0 \quad \Rightarrow \quad A_0 = 3 \]

\[ \frac{R_2}{R_1} = 2 \]
PROBLEM # 4 (a)

Small-Signal Model:

\[ i_b = (V_i - V_o) \times \frac{1}{R_\pi} \]

\[ V_o = \left( \frac{1}{2} (B+1) \right) i_b + i_0 \times R_E' \]

where \( R_E' = R_E || R_0 \)

From the above equations we get the following graph:

There's only one forward path: for \( Av = \frac{V_o}{V_i} \)

\[ P_1 = \frac{1}{R_\pi} \times (B+1) R_E' \]

Since the only loop \( L_1 \) is touching \( P_1 \)

\[ \Delta_1 = 1 \]
\[ \Delta = 1 - L_1 = 1 - \left( -\frac{(B+1)R_E'}{R_t} \right) \]

\[ \alpha \Delta = 1 + \frac{(B+1)R_E'}{R_t} \]

\[ \therefore \frac{U_0}{U_i} = \frac{P_1 \cdot \Delta}{\Delta} = \left( \frac{(B+1)R_E' / R_t}{1 + \frac{(B+1)R_E'}{R_t}} \right) \]

or it can be written as:

\[ \frac{U_0}{U_i} = \frac{R_E'}{R_t + \frac{R_E'}{B+1}} \]

For large B i.e \( \frac{1}{B+1} = 0 \)

\[ \frac{U_0}{U_i} = \frac{R_E'}{R_E'} = 1 \]

Input Impedance \( Z_{in} = \frac{U_i}{I_i} \)

\[ Z_{in} = R_B || Z_{it} \]

where \( Z_{it} \) is the impedance looking into the base of the transistor i.e \( Z_{it} = \frac{U_i}{I_i} \)

Note: Since \( U_i \) is an independent variable & \( I_b \) is dependent variable, we cannot apply the Mason's gain formula. Instead we find \( Y_{it} = \frac{I_b}{U_i} \) & then take the inverse.
\[ Y_{it} = \frac{ib}{V_i} \]

**One forward path** \( P_1 = \frac{1}{r_{\pi}} \) \& \( \Delta_1 = \Delta \)

\( \Delta = \) same as previous, since network hasn't changed.

\[ Y_{it} = \frac{P_1 \cdot \Delta_1}{\Delta} = \frac{1/r_{\pi}}{1 + (B+1)RE'} \]

\[ Z_{it} = V_{it} = r_{\pi} + (B+1)RE' \]

\[ Z_{in} = \frac{RB \times Z_{it}}{RB + Z_{it}}. \]

**Output Impedance**

\[ Z_0 = \frac{V_o}{I_o} \]

\( Z_0 = \frac{P_1 \cdot \Delta_1}{\Delta} = \frac{RE'}{1 + (B+1)RE'/r_{\pi}} \]

\[ Z_0 = \frac{RE'}{r_{\pi} + (B+1)RE'} \]
Problem # 4 (b):

Small-signal model:

![Circuit Diagram](image)

Considering $R_C$ as the only branch, we can write:

$$i_c = -g_m V_i + (V_o - V_i) \times \frac{1}{r_0}$$

$$V_o = (I_o - I_c) R_C$$

From these equations we can draw the following graph:

$$\Delta_1 = R_C \times (g_m + \frac{1}{r_0})$$

$$\Delta = 1 - \left( -\frac{R_C}{r_0} \right) = 1 + \frac{R_C}{r_0}$$
\[
\frac{V_0}{V_i} = \frac{P_1 \cdot \Delta I}{\Delta} = \frac{(g_m + \frac{1}{r_o}) \times R_c}{1 + \frac{R_c}{r_o}}
\]

For large \( r_0 \) i.e. \( \frac{1}{r_0} = 0 \)

\[
A_v = \frac{V_0}{V_i} = g_m \times R_c
\]

**Input Impedance**

Since \( V_\pi \) is in parallel with the independent source \( (V_i) \), it cannot be considered as an branch voltage.

\[
0^0_zin = -\frac{1}{Z_{\pi}} \parallel Z_{it} \quad \left[ -\frac{1}{Z_{\pi}} \right.
\]

\[
\lim_{(g_m + 1/r_0) \times 1}^{(g_m + 1/r_0) \times 1}
\]

\[
\left. \frac{1 + \frac{R_c}{r_0}}{g_m + \frac{1}{r_0}} \right]
\]

Since we cannot apply Mason's gain formula to \( Z_{it} \) (reason's explained in previous problem). We find \( Y_t \) first instead.

\[
\frac{Y_{it}}{V_i} = \frac{V_{it}}{V_i} = \frac{P_1 \cdot \Delta I}{\Delta} = \frac{-(g_m + 1/r_0) \times 1}{1 + \frac{R_c}{r_0}}
\]

or

\[
Z_{it} = -\left[ \frac{\frac{1 + \frac{R_c}{r_0}}{g_m + \frac{1}{r_0}}} \right]
\]
Output Impedance:

\[ Z_0 = \frac{V_0}{I_0} = \frac{P_i \cdot A_i}{\Delta} \]

Or,

\[ Z_0 = \frac{V_0}{I_0} = \frac{R_c \times 1}{1 + \frac{R_c}{V_0}} \]

where \( V_0 = \text{large} \quad \frac{I_0}{V_0} = 0 \quad Z_0 = R_c \]
Problem #1(a) (Alternative solution to finding Input Impedance)

\[ R_{E'} = R_E / R_0 \]

\[ Z_{in} = \frac{\frac{V_i}{I_i}}{Vi} \quad \text{dependent variable} \]
\[ \text{independent variable} \]

Let's write the network equations.

\[ V_i = (i_i - i_b) \times R_B \]
\[ V_o = i_b \times (B+1) \times R_{E'} \]
\[ i_b = (V_i - V_o) \times \frac{1}{Y_T} \]

Now we can draw the graph:

\[ P_1 = R_B \quad \Delta_1 = 1 - L_2 \quad \text{(not touching \( P_1 \))} \]
\[ \Delta = 1 - (L_1 + L_2) = 1 - \frac{-R_B Y_T - (B+1) R_{E'}}{Y_T} \]

\[ \frac{V_i}{I_i} = \frac{P_1 \Delta_1}{\Delta} = \frac{R_B \frac{Y_T + (B+1) R_{E'}}{Y_T}}{Y_T + R_B + (B+1) R_{E'}} \]

Or

\[ Z_{in} = \frac{R_B \frac{Y_T + (B+1) R_{E'}}{Y_T}}{R_B + (Y_T + (B+1) R_{E'})} \]