PROBLEM #1

\[ Z(j\omega) \rightarrow \]

\[ Z(j\omega) = R + j \cdot L \omega + \frac{1}{jC \omega} = R + j \cdot (L \omega - \frac{1}{C \omega}) \]

Say the resonance frequency is \( \omega_0 \) where the reactances cancel each other, i.e., the imaginary part is zero.

\[ \therefore (L \omega_0 - \frac{1}{C \omega_0}) = 0 \Rightarrow \omega_0 = \frac{1}{\sqrt{LC}} \]

Impedance \( |Z(j\omega)| \)

As seen from network as well as \( Z(j\omega) \),

\[ |Z(j\omega)| = \infty \text{ at } \omega = 0 \text{ and } \omega = \infty \]

\[ |Z(j\omega)| = R \text{ at } \omega = \omega_0 = \frac{1}{\sqrt{LC}} \]

Phase \( \angle Z(j\omega) \)

\[ \angle Z(j\omega) = \tan^{-1} \left( \frac{L \omega - \frac{1}{C \omega}}{R} \right) \]

(Contd.)
\[
\angle Z(j0) = -90^\circ \quad (-\pi/2)
\]
\[
\angle Z(j\omega) = +90^\circ \quad (+\pi/2)
\]
\[
\angle Z(j\omega_0) = 0
\]

Now we can plot the magnitude & phase of the impedance.
Finding \( Q \)

Use the definition:

\[
Q = W_0 \cdot \left( \frac{\text{energy stored}}{\text{average power dissipated}} \right)
\]

In a LC tank, we know that at any point of time, the sum energy in the L & C is constant. It sloshes back and forth between the inductor and the capacitor.

Also, we know that the maximum current through the network is \( V_{pk}/R \) since \( R \) is the minimum magnitude of impedance at \( w_0 \).

\[V_{pk}/R\]

\[\text{Total energy stored} = \text{Peak inductor energy}\]

\[
= \frac{1}{2} L I_{pk}^2
\]

If the excitation is sinusoidal, the average power dissipated on the resistor \( R \) is

\[
P_{avg} = \frac{1}{2} I_{pk}^2 \cdot R
\]

\[
Q = w_0 \cdot \frac{1/2 L I_{pk}^2}{\frac{1}{2} I_{pk}^2 \cdot R} = \frac{1}{\sqrt{LC}} \times \frac{L}{R} = \frac{\sqrt{L/C}}{R}
\]

\[
Q = \frac{\sqrt{L/C}}{R}
\]
It intuitively makes sense since decreasing $R$ increases $Q$ for there is less loss in the LC tank. And increasing characteristic impedance $(\frac{2\pi}{\omega}) = \sqrt{L/C}$ increases the $Q$. This is true because the impedance increases with respect to the loss $R$ and this in turn increases the $Q$. 
we have already derived (in lecture 22)

\[ B(j\omega) = \frac{R}{3R + j(\omega R^2 C - \frac{1}{\omega C})} \]

\[ = \frac{1}{3 + j(\omega RC - \frac{1}{\omega RC})} \]

Phase angle \( \phi = -\tan^{-1}\left(\frac{\omega RC - \frac{1}{\omega RC}}{3}\right) \)

Since near \( \omega_0 = \frac{1}{RC} \) \( \phi \approx 0 \), we can write

\[ \phi \approx -\left(\frac{\omega RC - \frac{1}{\omega RC}}{3}\right) \]

Now let's find the phase sensitivity w.r.t. \( \omega \) at around \( \omega_0 \)

i.e. \( \frac{d\phi}{d\omega} \bigg|_{\omega=\omega_0} = -\frac{4}{3} \times \left(\frac{RC + \frac{1}{\omega^2 RC}}{3}\right) \bigg|_{\omega=\omega_0} \)
On substituting $w = w_o = \frac{1}{\sqrt{RC}}$

we get

$$\left. \frac{d\phi}{dw} \right|_{w=w_0} = -\frac{2}{3} \times RC = -\frac{2}{3} \times \frac{1}{w_o}$$

(1)

It's given that $\Delta \phi$ in the amplifier is $0.1$ rad.

In order to compensate this phase-shift in the amplifier, the RC network needs a phase-shift $\Delta \phi = 0.1$ rad such that the total phase-shift is zero in the loop for the oscillation frequency.

Using (1) we can write

$$\Delta w = \frac{dw}{d\phi} \times \Delta \phi = \frac{1}{\frac{dw}{d\phi}} \times \Delta \phi$$

$$= \left( -\frac{3}{2} \times w_0 \right) \times \left( 0.1 \text{ rad} \right)$$

$$= -0.15w_o$$

$$w_o \text{ new} = w_o - \Delta w = 0.85w_o$$
PROBLEM #3

\[ KCL \ @ \ V_C \]

\[ g_m \cdot V_e = (V_C - V_e) \cdot C_1 + \frac{V_C}{8 \cdot L} \]

or \[ V_e (g_m + \frac{1}{8L}) = V_C \left(8C_1 + \frac{1}{8L}\right) \]  \( - (1) \)

\[ KCL \ @ \ V_e \]

\[(V_C - V_e) \cdot C_1 = g_m \cdot V_e + \frac{V_e}{R_{in}} + V_e \cdot \beta \cdot C_2 \]

or \[ V_C \cdot \beta \cdot C_1 = V_e \left\{8 \left(C_1 + C_2\right) + G_e\right\} \]  \( - (2) \)

where, \[ G_e = g_m + \frac{1}{R_{in}} \]

on substituting \[ V_e \] from (1) in (2) we get

\[ V_C \cdot \beta \cdot C_1 = \frac{V_C \left(8C_1 + \frac{1}{8L}\right)}{g_m + 8C_1} \cdot \left\{8 \left(C_1 + C_2\right) + G_e\right\} \]
\[ \Rightarrow \omega_c \left\{ \frac{\pi^2 LC_1 \omega_m + \omega^3 L C^2 - \omega^3 L (C_1 + C_2)}{\omega (C_1 + C_2) - \omega} \right\} = 0 \]

Since we want this circuit to oscillate \( \omega_c \neq 0 \), on substituting \( \omega = j \omega \) we get:

\[ \left\{ \omega^2 L C_1 (\omega_m - \omega) \right\} + j \left\{ \omega^3 L C_1 \omega_m - \omega (C_1 + C_2) \right\} = 0 \]

On making the Imaginary part = 0

\[ \omega \left\{ \omega^2 L C_1 \omega_m - (C_1 + C_2) \right\} = 0 \]

\[ \therefore \quad \omega_0 = \sqrt{\frac{(C_1 + C_2)}{\sqrt{L \cdot C_1 \cdot C_2}}} \]

On substituting the \( \omega_0 \) expression in the real part and solving it for zero we get:

\[ \frac{(C_1 + C_2)}{L \cdot C_1 \cdot C_2} \times \omega_m (\omega_m - \omega) - \omega = 0 \quad [\omega = \omega_m + \frac{1}{R_{in}}] \]

\[ = \frac{C_1 + C_2}{C_2} (\frac{\omega_m}{\omega} + \frac{1}{R_{in}} - \omega_m) = \omega_m + \frac{1}{R_{in}} \]

\[ \therefore \quad \omega_m = \omega_{min} = \frac{C_1/C_2}{R_{in}} \quad \text{Re}||R_{\pi} \]
On substituting $R_{in} = \frac{RE \cdot r_{\pi}}{RE + r_{\pi}}$ in the $g_m$ expression:

$$g_m = \frac{C_1}{C_2} \left( \frac{g_m}{\beta} + \frac{1}{RE} \right)$$  \[ \therefore r_{\pi} = \frac{\beta}{g_m} \]

Or

$$\frac{C_1}{C_2} \cdot \frac{g_m}{\beta} = g_m - \frac{C_1/C_2}{RE}$$

$$\therefore \beta = \beta_{\text{min}} = \frac{C_1/C_2 \cdot g_m}{g_m - \frac{C_1/C_2}{RE}}$$
(b) D.C. equivalent circuit for the large signal model.

\[ V_{CC} = +9V \]

Assuming \( V_{BE} = 0.7 \)

For \( I_c = 1mA \), (Assume \( I_c \approx I_E \))

\[ R_E = \frac{V_{EE} - V_{BE}}{I_c} = \frac{8.3}{1mA} = 8.3 \text{k}\Omega \]

\[ \therefore R_E = 8.3 \text{k}\Omega \text{ & } I_c = 1mA \]

(a) The expression for oscillation frequency is already derived to be

\[ 2\pi f_0 = \omega_0 = \frac{1}{\sqrt{C_1 \cdot (L_1 + L_2)}} \]

or \( L_1 + L_2 = \frac{1}{C_1 \cdot (2\pi f_0)^2} \)
For 1 MHz oscillation frequency: \( C_1 = 270 \, \text{pF} \)

\[
L_1 + L_2 = \frac{1}{\frac{270 \times 10^{-12}}{(2\pi \times 10^6)^2}} = 93.8 \, \text{mH}
\]

\[ L_1 + L_2 \approx 94 \, \text{mH} \]

The expression for \( g_{m \text{min}} \) has been derived to be

\[
g_{m \text{min}} = \frac{1}{R_E} \left( \frac{1}{L_2} - \frac{1}{L_1} \right)
\]

Now, let's say the \( g_m \) is given because of a bias constraint (e.g. required \( I_C = 1 mA \)). Then, \( \frac{L_2}{L_1} \) & \( \frac{1}{B} \) has to be bounded to still satisfy the oscillation condition.

If we re-write the \( g_m \) expression in terms of the bounds,

\[
g_{m \text{min}} = \frac{1}{R_E} \left( \frac{1}{(\frac{L_2}{L_1})_{\text{min}}} - \frac{1}{B_{\text{min}}} \right)
\]

On rearranging we get,

\[
(\frac{L_2}{L_1})_{\text{min}} = \frac{1}{g_m \cdot R_E} + \frac{1}{B_{\text{min}}}
\]
\[ B_{\text{min}} = 50 \text{ from data sheet.} \]
\[ R_E = 8.3 \, k\Omega \]
\[ g_m = \frac{I_c}{V_t} = \frac{4mA}{25mV} \]

\[ 0.06 \left( \frac{L_2}{L_1} \right)_{\text{min}} = \frac{25mV}{1mA \times 8.8k} + \frac{1}{50} \]

\[ = 0.023 \]

Since this is the minimum requirement, let's choose a convenient ratio,

\[ \frac{L_2}{L_1} \approx 0.05 \]

Using the above ratio & \( L_1 + L_2 = 94 \, \mu H \) we get \( L_1 = 89.52 \) \, \mu H \), let's choose \( L_1 = 90 \, \mu H \) \( \)

\[ 0.06 \, L_2 = 4 \, \mu H \]

So the resonator elements are:

\[ L_2 = 4 \, \mu H \]
\[ L_1 = 90 \, \mu H \]
\[ C_1 = 270 \, \mu F \]
MODEL for NPN 2N2222A

* General Purpose
.MODEL Q2N2222A NPN(Is=15f Vaf=75 Bf=250 Ne=1.3 Ise=15f
+ Ikf=0.3 Xtb=1.5 Br=6 Nc=2 Isc=0 Ikr=0 Rc=1 Cjc=18p
+ Mjc=0.33 Vjc=0.75 Fc=0.5 Cje=22p Mje=0.33 Vje=0.75
+ Tr= 17.9n Tf=430p Itf=0.6 Vtf=1.7 Xtf=3 Rb=19)

Parameters used in the circuit

.PARAM  C1=270p VBAT=9  L1=90u L2=4u Re=8.3k Cc=1n

Main circuit netlist

V_VCC VCC 0 {VBAT} ; +Supply Voltage
V_VEE 0 VEE {VBAT} ; -Supply Voltage
L_L1 l1fb {L1} IC=1mA ; First inductor of the tank
L_L2 l1fb col {L2} IC=1mA ; second inductor of the tank
C_1 col 0 {C1} IC=9 ; tank capacitor
C_cc l1fb emit {Cc} IC=9.7 ; AC coupling capacitor
Q_Q1 col 0 emit Q2N2222A ; NPN transistor
R_re emit VEE {Re} ; Emitter degeneration resistor

Analysis directives:

.OP ; DC Operating point
.TRAN 0 10m 0 50n ; Transient Analysis
  * Syntax: .TRAN TSTEP TSTOP TSTART TMAX
  * TSTEP= printing step size
  * TSTOP= simulation stop time
  * TSTART= Simulation start time
  * TMAX= maximum step size

.FOUR 939.4K 5 V([col]) ; Fourier Analysis for distortion measurement
  *Syntax: .FOUR <fund> <harm> <out var>
  * fund : fundamental frequency of oscillation.
  * harm : no. harmonics
  * out var : Output variable to be analyzed

.PROBE ; Probes all nodes in the circuit

.END
Transient Simulation Result

FOURIER COMPONENTS OF TRANSIENT RESPONSE V(col)

<table>
<thead>
<tr>
<th>HARMONIC NO</th>
<th>FREQUENCY (HZ)</th>
<th>FOURIER COMPONENT</th>
<th>NORMALIZED COMPONENT</th>
<th>PHASE (DEG)</th>
<th>NORMALIZED PHASE (DEG)</th>
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<tr>
<td>1</td>
<td>9.394E+05</td>
<td>1.096E+00</td>
<td>1.000E+00</td>
<td>-3.375E+01</td>
<td>0.000E+00</td>
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<tr>
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<td>1.879E+06</td>
<td>3.911E-02</td>
<td>3.567E-02</td>
<td>-2.854E+01</td>
<td>3.896E+01</td>
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<tr>
<td>3</td>
<td>2.818E+06</td>
<td>2.050E-02</td>
<td>1.870E-02</td>
<td>-1.155E+01</td>
<td>8.971E+01</td>
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<tr>
<td>4</td>
<td>3.758E+06</td>
<td>1.531E-03</td>
<td>1.396E-03</td>
<td>2.352E+01</td>
<td>1.585E+02</td>
</tr>
<tr>
<td>5</td>
<td>4.697E+06</td>
<td>1.725E-03</td>
<td>1.573E-03</td>
<td>5.282E+01</td>
<td>2.216E+02</td>
</tr>
</tbody>
</table>

TOTAL HARMONIC DISTORTION = 4.033282E+00 PERCENT