The application of Mason rule. Example from workbook.

Figure:

Choosing the capacitor as tree branches, the branch equations that describe the network of Figure are:

\[ I_1 = \frac{(V_{in} - V_1)}{R_1} \quad V_1 = \frac{(I_1 - I_2)}{8C_1} \]
\[ I_2 = \frac{(V_1 - V_2)}{R_2} \quad V_2 = \frac{(I_2 - I_3)}{8C_2} \]
\[ I_3 = \frac{(V_2 - V_3)}{R_3} \quad V_3 = \frac{I_3}{8C_3} \]

Taking all capacitors to be equal to say 'C' and all resistors equal to say 'R', the equations can be represented by the following graph:

Let's find the transfer function:

\[ \frac{V_3}{V_{in}} = ? \]
Now, we can apply Mason's Gain Formula to the graph:

\[
\frac{V_3}{V_{in}} = P = \frac{1}{\Delta} \sum_{k} P_k \Delta_k
\]

\[
P_1 = \frac{1}{R} \times \frac{1}{8C} \times \frac{1}{R} \times \frac{1}{8C} \times \frac{1}{R} \times \frac{1}{8C} = \left(\frac{1}{R8C}\right)^3
\]

\[
\Delta_1 = 1 \quad [^0^0^0 \text{ no non-touching loops for } P_1]
\]

\[
\Delta = 1 - \frac{\sum L_a}{a} + \sum L_b \cdot L_c - \ldots \ldots
\]

\[
= 1 - \left\{ \frac{1}{R8C} \frac{1}{L_1} - \frac{1}{R8C} \frac{1}{L_2} - \frac{1}{R8C} \frac{1}{L_3} - \frac{1}{R8C} \frac{1}{L_4} - \frac{1}{R8C} \frac{1}{L_5} \right\}
\]

\[
+ \left\{ L_1 \cdot L_3 + L_1 \cdot L_4 + L_1 \cdot L_5 + L_2 \cdot L_4 + L_2 \cdot L_5 + L_2 \cdot L_5 \right\} \Rightarrow \text{two non-touching loops}
\]

\[
- \left\{ L_1 \cdot L_3 \cdot L_5 \right\} \rightarrow \text{all combination of 3 non-touching loops.}
\]

\[
0^0 \frac{V_o}{V_{in}} = \frac{\left(\frac{1}{R8C}\right)^3}{1 + 5\left(\frac{1}{R8C}\right) + 6 \cdot \left(\frac{1}{R8C}\right)^2 + \left(\frac{1}{R8C}\right)^3}
\]

\[
0^0 \frac{V_o}{V_{in}} = \frac{1}{(SCR)^3 + 5 \cdot (SCR)^2 + 6 \cdot (SCR) + 1}
\]
Now let's find the input Impedance \( \left( \frac{I_1}{V_{\text{in}}} \right) \) using the Mason's Gain formula.

\[
\frac{I_1}{V_{\text{in}}} = \frac{1}{\Delta} \sum_k P_k \cdot \Delta_k.
\]

**IMPORTANT NOTE:**

Since the network structure hasn't changed from the previous example, the denominator \( \Delta \) will exactly be the same as in the previous example and hence we do not have to re-calculate. This denominator is also known as the characteristic polynomial of the network. And when we solve \( \Delta = 0 \), we get the "poles" of the network.

This is a very important observation since it tells us that if the network structure doesn't change, then finding transfer function for different nodes will only change the "numerator" of the transfer function and not the denominator. Or in other words, the characteristic polynomial or the "poles" remain the same.
Now coming back to the problem, we only need to evaluate \( \sum_{k} p_k \Delta_k \).

There is only one forward path, say \( P_1 \), from \( V_{\text{in}} \) to \( I_1 \).

\[ P_1 = \frac{1}{R} \]

\[ \Delta_1 = \Delta - \overline{l_1} - \overline{l_3} - \overline{l_4} - \overline{l_5} \]

\[ \overline{l_1} \text{ touching } P_1 \]

\[ \overline{l_5} \text{ touching } P_1 \]

\[ \Delta_1 = 1 + 4/2CR + 3(1/2CR)^2 \]

\[ \frac{I_1}{V_{\text{in}}} = \frac{P_1 \cdot \Delta_1}{\Delta} = \frac{(1/R)[1 + 4/2CR + 3(1/2CR)^2]}{1 + 5(1/2CR) + 6(1/2CR)^2 + (1/2CR)^3} \]

\[ \frac{I_1}{V_{\text{in}}} = \frac{(SC)[3 + 4BCR + SCR^2]}{(SCR)^3 + 5(SCR)^2 + 6(SCR) + 1} \]
IMPORTANT NOTE ABOUT MASON’S GAIN FORMULA

A transfer function \( \frac{0}{1} \) using Mason’s gain:

\[ O \rightarrow \text{has to be dependent variable} \]

\[ I \rightarrow \text{has to be independent variable} \]

\[ \text{OR} \]

\[ O \rightarrow \text{has to be a output or mixed node} \]

\[ I \rightarrow \text{has to be a input node}. \]