Wien-Bridge Oscillator:

Now, let's use the theory to analyze a popular linear oscillator called the Wien-bridge oscillator shown in the following figure.

![Wien-Bridge Oscillator Diagram]

Figure 20.1

* The amplifier is assumed to have its dominant pole at a much higher frequency than the frequency of oscillation. Therefore, the gain is approximated by its DC gain ($A_v$).

* The transfer function $B(s)$ can be written as

$$B(s) = \frac{V_{out}(s)}{V_{in}} = \frac{(R_1 || 1/C_{1s})}{R_2 + 1/C_{2s} + (R_1 || 1/C_{1s})}$$

Assuming $R_1 = R_2 = R$ and $C_1 = C_2 = C$ & $s = j\omega$

we get

$$B(j\omega) = \frac{R}{3R + j(\omega R^2 C - 1/wC)}$$
Now we can apply Barkhausen criterion

\[ \text{Av} \cdot B(j\omega) = 1 \]

or

\[ \frac{\text{Av} \cdot R}{2R + j \cdot (\omega R^2C - \frac{1}{\omega C})} = 1 \]

or

\[ \frac{R(3 - \text{Av})}{\text{Real}} + \frac{j(\omega R^2C - \frac{1}{\omega C})}{\text{Imaginary}} = 0 \]

For complex number to be zero, both the real and imaginary part should be equal to zero.

\[ \Rightarrow R(3 - \text{Av}) = 0 \quad \Rightarrow \quad \text{Av} = 3 \]

\[ \omega R^2C - \frac{1}{\omega C} = 0 \quad \Rightarrow \quad \omega = \frac{1}{RC} \]

\[ \Rightarrow f = \frac{1}{2\pi RC} \]

* The gain of the amplifier needs to be 3 (slightly greater) to sustain oscillation.

* The oscillation frequency is given by \( f = \frac{1}{2\pi RC} \)
AMPLIFIER REALIZATION:

1) Non-inverting Op-Amp realization as shown in the following figure:

\[ Au = 1 + \frac{R_2}{R_1} \]

* By choosing \( R_2 = 2R_1 + \Delta \) we can get 3x gain.

2) The amplifier can also be realized with discrete components as shown below in one particular example.
Since the amplifier has finite input and output impedance, we need to find an equivalent two-port model to analyze the oscillator circuit.

Two-port model:

\[
\begin{align*}
\text{Input} & \quad \text{Vin} \quad \text{Rin} \quad \text{Av} \cdot \text{Vin} \quad \text{Output} \\
\downarrow & \quad \uparrow \\
- & \quad + \\
& \quad \text{Rout}
\end{align*}
\]

Where,
- Rin \( \rightarrow \) Input impedance of the amplifier
- Rout \( \rightarrow \) Output impedance of the amplifier.
- \( \text{Av} \) \( \rightarrow \) No load voltage gain.

Before we analyze the amplifier, we'll make the following assumptions:

1. The current (I_E1) is much smaller than the current flowing in R_C & hence R_E (I_{R_C} = I_{RE})

2. The current flowing in R_B is much smaller compared to the base current I_E2.

\[ I_{E2} = I_{C0} \]
The exact derivations for $R_{in}$, $R_{out}$ & $A_v$ is left as an exercise for the students. We'll derive only the first-order or approximate expressions for the above quantities.

**NO LOAD VOLTAGE GAIN:**

We can find the approximate voltage gain without drawing the small-signal model.

* If the $B$'s are considerably large $Q_1$ along with $R_E$ can be approximated as an emitter-follower. And we know from previous experience that the gain of an emitter follower is close to unity.

\[ V_e \approx V_{in} \]

\[ I_E = \frac{V_e}{R_E} = \frac{V_{in}}{R_E} \]
Since we have assumed $I_{E1} \ll I_{RE}$

\[ \therefore I_{RC} = I_{RE} \]

\[ \therefore V_0 = (R_C + R_E) \times I_{RE} = \frac{V_{in}}{R_E} (R_C + R_E) \]

or
\[ \frac{V_0}{V_{in}} = A_v = 1 + \frac{R_C}{R_E} \]

Another way of analyzing the voltage gain is through a macroscopic method.

We know the gain of the following Op-Amp circuit to be:

We can model the two-stage amplifier in the above fashion as shown on the next page.
If we assume the input impedance to be high & the output impedance to be low, this approximation is a fair one.

We'll see in the next section that the above two assumptions are fairly true for the two-stage amplifier.