BANDWIDTH ESTIMATION USING OPEN-CIRCUIT TIME CONSTANT

As seen from the previous derivation for the high-frequency response, the second-order equation, although exact, hardly gives any insight into circuit elements dominating the -3dB frequency \( (\omega_h, \delta_h) \).

If the following two assumptions be made for a transfer function:

1. The zeros of the transfer function do not influence the \( \omega_h, \delta_h \) very much &
2. \( \omega_h, \delta_h \) is usually dominated by only one of the multiple poles.

then, open-circuit time-constant is a very powerful technique to estimate the high frequency -3dB point.

Even if the two approximations are not met strictly, the estimation is still very close to the real \( \omega_h \).
THE METHOD: (Ref: Gray & Searle Sec: 15.2)

The expression for $W_+$ is given by

$$W_+ = \sum_{i} \frac{1}{C_i \cdot R_{io}}$$

where,

- $C_i$ is the $i^{th}$ capacitor in the network
- $R_{io}$ is the impedance seen across $C_i$ with every other capacitor open.

Note: All independent sources are made zero.

Example:

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\[ \begin{align*}
\text{R}_{1} & \quad \text{R}_{2} \\
\text{C}_{1} & \quad \text{C}_{2} \\
\text{V}_{i} & \quad \text{V}_{0}
\end{align*} \]
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* Since there are two capacitors, we'll have two open circuit time constants associated with it.
Impedance associated with $C_1$ ($C_2$ open & $v_i$ short)

\[ R_{10} = R_1 \quad \Rightarrow \quad Z_1 = R_1 C_1 \]

Impedance associated with $C_2$ ($C_1$ open & $v_i$ short)

\[ R_{20} = R_1 + R_2 \quad \Rightarrow \quad Z_2 = (R_1 + R_2) C_2 \]

\[ \omega_+ \approx \frac{1}{\frac{1}{Z_1} + \frac{1}{Z_2}} = \frac{1}{\frac{1}{R_1 C_1} + \frac{1}{(R_1 + R_2) C_2}} \]
Redrawing the high-frequency small-signal model with $V_S$ shorted.

1) $R_{\pi 0}$: impedance across $G_t$ with $C_u$ open

$$R_{\pi 0} = R_s'$$

$C_t = G_t \cdot R_s'$

2) $R_{\mu 0}$: impedance across $C_u$ with $C_t$ open
\[ R_{uo} = \frac{V_t}{i_t} = \frac{V_o - V_x}{i_t} \]

\[ V_x = -i_t \cdot R_s' \]

\[ V_o = (i_t - q_m V_x) R_L = i_t (1 + g_m R_s') R_L \]

\[ R_{uo} = \frac{V_t}{i_t} = i_t \underbrace{\left\{ (1 + g_m R_s') R_L + R_s' \right\}}_{i_t} \]

\[ C_m = R_{uo} \cdot C_u = C_u \left\{ (1 + g_m R_s') R_L + R_s' \right\} \]

\[ W_H \equiv \frac{1}{Z_x + Z_m} = \frac{1}{C_x R_s' + C_u R_s' + C_u R_L (1 + g_m R_s')} \]

Note: This expression is the same as the co-efficient \( b' \) in the exact transfer \( w_1 \).

Also, \( W_H \) can be re-written as

\[ W_H \equiv \frac{1}{R_s' \left\{ C_x + C_u (1 + g_m R_L) \right\} + C_u R_L} \]

\( \downarrow \) gain of the amplifier

For the CE amplifier, usually, \( R_s' \left\{ C_x + C_u (1 + g_m R_L) \right\} \gg C_u R_L \)

\[ W_H \equiv \frac{1}{R_s' \left\{ C_x + C_u (1 + g_m R_L) \right\}} \]