One simple way to realize a differential amplifier is to take two separate identical single-ended amplifiers and subtract their output to create the single-ended output.

Example:

\[
\begin{align*}
\text{If } Q_1 \text{ and } Q_2 \text{ are identical, then their individual gains (small-signal)} \\
\text{ie. } \frac{v_o}{v_i} &= -g_m R_L = -A \text{ (say)}
\end{align*}
\]

Then, the differential gain \( A_d \) can be written as

\[
A_d = \frac{v_o}{v_{id}} = \frac{v_{op} - v_{om}}{v_{ip} - v_{im}} = \frac{-A v_{im} + A v_{ip}}{v_{ip} - v_{im}} = A
\]

\[
\therefore A_d = A = g_m R_L
\]
Similarly, the common-mode gain can be written as

$$A_{cm} = \frac{V_o}{V_{cm}} = \frac{A_v (V_{ip} - V_{im})}{V_{ip} + V_{im}} = 2A_v \left(\frac{V_{ip} - V_{im}}{V_{ip} + V_{im}}\right)$$

Since the above expression is unreducible to a closed form, we will feed an ideal common-mode signal and find the gain.

An ideal common-mode signal is when
$$V_{ip} = V_{im}$$

$$\Rightarrow A_{cm} = 0.$$  

* Although this appears to be an ideal differential amplifier small-signal wise, any large common-mode signal will result in a non-linear differential gain.

* The reason being, any large common-mode signal will change the bias current of the amplifier.

i.e.  $$A_d = g_m R_L = \frac{I_C(V_{cm})}{\frac{V_T}{R_L}}$$  
non-linear.
BJT Emitter-Coupled Differential Pair:

Assume: $\beta = \infty$
$A_v = \infty$

Figure: The basic BJT differential-pair configuration.

Let's first start analyzing the differential pair by applying a common-mode signal i.e. $V_{ip} = V_{im} = V_{cm}$.
Since \( V_{be1} = V_{be2} = V_{om} - V_c \)

\[
I_{E1} = I_{E2} = \frac{2I}{2} = I.
\]

Now let's apply extreme differential signals:

Let \( V_{im} = -V_{EE} \)  

\( Q_1 \) is off.

\[
I_{E2} = 0
\]

\[
I_{E1} = 2I
\]

For \( V_{ip} = -V_{EE} \)  

\( Q_1 \) is off.

\[
I_{E1} = 0
\]

\[
I_{E2} = 2I.
\]
Now let's plot the asymptotes of $I_{c1}$ & $I_{c2}$ for the differential input i.e. $V_{ip} - V_{im}$

Need to find the exact current in this region.
Large-signal analysis for small change in input

\[ \begin{align*}
Q1 & : i_{e1} = i_e \\
Q2 & : i_{e2} = i_e \\
\text{We can write:} & \\
i_{e1} &= \frac{I_s}{\alpha} (V_{ip} - V_e) / V_t \\
i_{e2} &= \frac{I_s}{\alpha} (V_{im} - V_e) / V_t \\
\text{On dividing the above two equations we get,} & \\
\frac{i_{e1}}{i_{e2}} &= \frac{(V_{ip} - V_e)}{(V_{im} - V_e)} / V_t = e^{V_{id}/V_t} \quad \therefore \quad V_{id} = V_{ip} - V_{im} \\
\text{we can write the following two relations:} & \\
i_{e1} &= i_{e2} \cdot e^{V_{id}/V_t} \\
i_{e2} &= i_{e1} \cdot e^{-V_{id}/V_t} \\
\end{align*} \]
we also know \((KCl \text{ at } V_e)\),

\[ i_{e1} + i_{e2} = 2I \quad - (2) \]

On substituting \( i_{e2} \) from \((1) \) in \((2) \) we get

\[
i_{e1} = \frac{2I}{1 + e^{-\frac{(V_{id}/V_t)}}}
\]

and, on substituting \( i_{e1} \) from \((1) \) in \((2) \) we get

\[
i_{e2} = \frac{2I}{1 + e^{-\frac{(V_{id}/V_t)}}}
\]

When \( V_{id} = 0 \) \(, i_{e1} = I \) \& \( i_{e2} = I \)

\( V_{id} > 4V_t \) \( i_{e1} \approx 2I \) \& \( i_{e2} \approx 0 \)

\( V_{id} < -4V_t \) \( i_{e1} \approx 0 \) \& \( i_{e2} \approx 2I \)

* Agrees with conclusion we made by inspection.

\[ \Delta V_{id} = 8V_t \approx 200 \text{mV.} \]
As shown from the graph, the currents in each branch reach its full saturation within a span of 200 mV.

But doesn't mean we can have a linear operation of 200 mV.

Now let's analyze the range of linear operation.

Instead of looking at the individual current, we'll look at the single-ended output, which is the difference of the two collector current. i.e.,

$$i_o = i_{c1} - i_{c2}$$

or $$i_o = \frac{2I}{1 + e^x} - \frac{2I}{1 + e^x}$$ where $$x = \frac{V_{id}}{V_T}

On assuming $$|x| < 1$$, we can neglect the higher terms of $$e^x$$

$$i_o = \frac{2I}{1 + (1-x+x^2/2! + ...)} - \frac{2I}{1 + (1+x+x^2/2! + ...)}$$

Series for $$e^x = 1 + x + x^2/2! + x^3/3! + ...$$ valid for all $$x$$. 
on neglecting the higher-order terms,

\[ i_0 = \frac{2I}{2-x} - \frac{2I}{2+x} \]

\[ = I \times \left( \frac{1}{1-x_2} - \frac{1}{1+x_2} \right) \]

\[ \therefore |x| < 1 \quad , \quad |\frac{x}{3}| < 1 \quad \therefore \quad \text{on using the following Taylor expansions, and neglecting higher order terms.} \]

\[ \frac{1}{1+x} = 1-x+x^2-x^3+\ldots \]

\[ \frac{1}{1-x} = 1+x+x^2+x^3+\ldots \quad \quad |x| < 1 \]

we get,

\[ i_0 = I \times \left\{ \left( 1+x_2 \right) - \left( 1-x_2 \right) \right\} \]

\[ i_0 = I \times x = \frac{I \times \text{vid}}{V_T} \quad \therefore \quad \text{vid} = \frac{V_T}{V_T} \]

since this equation is valid for \( |x| < 1 \)

\[ \Rightarrow \quad \left| \frac{\text{vid}}{V_T} \right| < 1 \quad \text{or} \quad (\text{vid}) < V_T \]

On substituting \( g_m = \frac{I \times a}{V_T} = \frac{I}{V_T} \) we get

\[ i_0 = g_m \cdot \text{vid} \]
When \( V_T < |\text{Vid}| < 4V_T \) then

\[
I_o = \frac{2I}{1 + e^{\text{Vid}/V_T}} - \frac{2I}{1 + e^{-\text{Vid}/V_T}}
\]

which can be shown to be approximately:

\[
I_o = 2I \cdot \tanh\left(\frac{\text{Vid}}{V_T}\right)
\]

Now, the complete regions of operation can be shown to be approximately:

![Graph showing the regions of operation](image)