Calculating Gain for Common-Emitter Amplifier

\[ V_s = V_m \sin(\omega t) \]

**Figure 7.1: Common-Emitter Amplifier**

* Previously we analyzed the above problem to calculate D.C. operating point or the so-called quiescent point (Q-point).

* Now we add a A.C. coupled sinusoidal source \( V_s \) to the BE junction of the NPN transistor.

* The source is a A.C. coupled so not to disturb the biasing of the amplifier.

* Because of the sinusoidal source, all the voltages and currents in the circuit will have two components (a) D.C. component & (b) A.C. component or the sinusoidal component.
Graphical Representation of the new currents and voltages:

Gain = \( \frac{V_{om}}{V_M} \)
* New voltage and currents:

\[ i_B = I_B + i_b \]
\[ V_{BE} = V_{BE} + \delta_{BE} = V_{BE} + \delta_s \]
\[ i_C = I_C + i_c \]
\[ V_{CE} = V_0 = V_{CE} + \delta_{CE} \text{ or } V_0 + \delta_0 \]

where,

(a) \( I_B, I_C, V_{BE}, V_{CE}, V_0 \) are the D.C. components
(b) \( i_b, \delta_{BE}, i_c, \delta_{CE}, \delta_0 \) are the A.C. components
(c) \( i_B, i_C, V_{BE}, V_{CE}, V_0 \) are D.C. + A.C components.

* Using the transistor equation we can write

\[ i_C = I_s \cdot e^{(V_{BE} + \delta_s)/V_T} \]

\[ = I_s \cdot e^{V_{BE}/V_T} \cdot e^{\delta_s/V_T} \]

\[ = I_s \cdot \frac{e^{V_{BE}/V_T}}{I_C} \cdot e^{\delta_s/V_T} \]

\[ \Rightarrow i_C = I_C \cdot e^{\delta_s/V_T} \]

* Using KVL at the outer loop we can write

\[-V_{CC} + i_C \cdot R_C + V_0 = 0 \]

or \( V_0 = V_{CC} - I_C \cdot R_C \cdot e^{\delta_s/V_T} \)
In order to find the A.C gain or the time varying gain, we can find the maximum of the sinusoidal component at the output and divide by the maximum of the sinusoidal source.

Since $V_s = V_m \sin(\omega t)$, the maximum is simply $V_m$.

But for the output, $V_o = V_{cc} - I_c R_c \frac{V_m}{V_T}$, it is not obvious what is the maximum of the sinusoidal component.

**Approximation:**

The taylor series for an exponential is given as:

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots \quad -\infty < x < \infty$$

If $|x| \ll 1$ then we can eliminate higher-order terms:

$$e^x \approx 1 + x \quad |x| \ll 1$$

For very small signals, i.e., $\frac{V_m}{V_T} \ll 1$

$$e^{\frac{V_m}{V_T}} \approx 1 + \frac{V_m}{V_T}$$
Now we can write the output voltage as

\[ V_0 = V_{cc} - I_c \cdot R_e \left( 1 + \frac{V_m}{V_T} \right) \]

\[ = V_{cc} - I_c \cdot R_e - \frac{I_c \cdot R_e \cdot V_m}{V_T} \]

D.C. component \hspace{2cm} A.C. component

Now, we can write the gain as ratio of the A.C. component, i.e.

\[ \text{Gain} = \frac{V_{om}}{V_{sm}} = - \frac{I_c \cdot R_e}{V_T} \cdot V_m \times \frac{1}{V_{at}} \]

\[ \text{Gain} = - \frac{I_c \cdot R_e}{V_T} \]

**ERROR DUE TO APPROXIMATION**

* In order to get a feel for the validity of the approximation, let's compare the exact equation and that with the approximation for different values of \( V_m \) (the peak value of the sinusoidal wave).
* For the signal, it's sufficient to use the max. value of $V_m$ in the comparison.

<table>
<thead>
<tr>
<th>$V_m$</th>
<th>$4V_m/\sqrt{3}$</th>
<th>$1 + 4V_m/\sqrt{3}$</th>
<th>Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 mV</td>
<td>1.039</td>
<td>1.0385</td>
<td>0.07</td>
</tr>
<tr>
<td>5 mV</td>
<td>1.212</td>
<td>1.192</td>
<td>1.6</td>
</tr>
<tr>
<td>10 mV</td>
<td>1.469</td>
<td>1.384</td>
<td>5.7</td>
</tr>
<tr>
<td>15 mV</td>
<td>1.780</td>
<td>1.576</td>
<td>11.43</td>
</tr>
<tr>
<td>20 mV</td>
<td>2.158</td>
<td>1.769</td>
<td>18.04</td>
</tr>
</tbody>
</table>
For non-linear devices you need to have two kinds of models, large-signal and small-signal.

Since the I-V characteristics change with bias point, we cannot have one model for all regions of operation.

In order to find small-signal parameters, the device is first biased to its desired bias point or the Q-point, then the parameters are derived assuming the signal magnitude is small enough not to disturb the Q-point.

**Small-Signal I-V Characteristics for BJT**

\[ i_c(t) = I_cq + i_c(t) \]

\[ i_b(t) = I_bq + i_b(t) \]

**Figure:**
Let's find \( \frac{V_{be}(t)}{I_b(t)} \) (see pg-249 in textbook).

\[
I_B(t) = I_{BQ} + i_b(t) = \frac{I_S}{\beta} \left( \frac{V_{BEQ} + V_{be}(t)}{V_T} \right)
\]
\[
= \frac{I_S}{\beta} \cdot \frac{V_{BEQ}}{V_T} \cdot \frac{V_{be}(t)}{V_T}
\]
\[
= I_{BQ} \cdot \frac{V_{be}(t)}{V_T}
\]

or \( I_{BQ} + i_b(t) = I_{BQ} \cdot (1 + \frac{V_{be}(t)}{V_T}) \)

\[ e^x \approx 1 + x \]
\[ \text{for } |x| \ll 1 \]

or \( i_b(t) = V_{be}(t) \cdot \frac{I_{BQ}}{V_T} \)

Let's call the small-signal output resistance \( \left( R_T \right) \)

\[
R_T = \frac{V_{be}(t)}{i_b(t)} = \frac{V_T}{I_{BQ}} = \frac{\beta \cdot V_T}{I_{CQ}}
\]
Let's assume $\beta = \beta_c = \beta$ which is usually true.

Then
\[ i_d(t) = \beta \cdot i_b(t) \]

or,
\[ I_c(t) + i_c(t) = \beta \cdot I_{eq} + \beta \cdot i_b(t) \]

or,
\[ i_c(t) = \beta \cdot i_b(t) \]

- Eq 6.2

We know
\[ \frac{v_{be}(t)}{I_b(t)} = r_n = \frac{\beta \cdot V_T}{I_{eq}} \]

\[ i_c(t) = \frac{\beta \cdot v_{be}(t) \cdot I_{eq}}{\beta \cdot V_T} \]

- Eq 6.3

Let's define the transconductance of BJT
\[ g_m = \frac{i_c(t)}{v_{be}(t)} \]

Using Eq: 6.3
\[ g_m = \frac{I_{eq}}{V_T} = \frac{\beta}{r_n} \]

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