SMALL-SIGNAL OUTPUT IMPEDANCE

Let's define the output impedance as

\[ Y_o = \left. \frac{V_{ce}(t)}{I_{ce}(t)} \right|_{I_B(t)=0} \]

\[ i_c(t) = i_{ca} + i_c(t) \]

\[ V_{BEQ} \]

\[ + \]

\[ - \]

\[ V_{CEQ} \]

**Figure: 8.1**

We can write the instantaneous collector current (with early voltage effect)

\[ i_c(t) = I_s \cdot e^{\frac{V_{BEQ}}{V_T}} \cdot \left( 1 + \frac{V_{CEQ} + V_{ce}(t)}{V_A} \right) \]

\[ = I_s \cdot e^{\frac{V_{BEQ}}{V_T}} \cdot \left( 1 + \frac{V_{CEQ}}{V_A} + \frac{V_{ce}(t)}{V_A} \right) \]

\[ i_c(t) = I_{ca} + i_c(t) \]

\[ \text{or, } \]

\[ i_c(t) = I_{co} \cdot \frac{V_{ce}(t)}{V_A} \]

Where \( I_{co} = I_c @ V_{ce}=0 \)
\[ r_0 = \frac{V_A}{I_{co}} \quad - \text{Eq: 8.1} - \]

**Small-Signal Equivalent CKT.**

Figure: 8.2

\[ V_{be} - r_0 - \beta \cdot i_b \]

Note: The small signal model for pnp is exactly the same excluding reference directions.
If $\Delta V_{BE}$ is small enough, the slope can be approximated at $V_{BE}$.

The slope $\frac{\Delta I_B}{\Delta V_{BE}}$ can be mathematically written as:

$$\lim_{\Delta V_{BE} \to 0} \frac{\Delta I_B}{\Delta V_{BE}} = \frac{dI_B}{dV_{BE}} \bigg|_{V_{BE}}$$

And we'll define the term

$$\frac{1}{\eta_T} = \frac{\partial I_B}{\partial V_{BE}} \bigg|_{V_{BE}}$$
Since $\Delta V_{BE} \to 0$, we can write

$$I_B = \frac{I_s}{B} \cdot e^{V_{BE}/V_T}$$

or

$$\frac{dI_B}{dV_{BE}} = I_s \cdot e^{V_{BE}/V_T} \cdot \frac{1}{B \cdot V_T}$$

$$= \frac{I_c}{B \cdot V_T} = \frac{1}{R_n} \quad \text{or} \quad R_n = \frac{B \cdot V_T}{I_c}$$

Similarly, we can define,

$$g_m = \left. \frac{dI_C}{dV_{BE}} \right|_{V_{BE}}$$

$$I_C = I_s \cdot e^{V_{BE}/V_T}$$

$$\frac{dI_C}{dV_{BE}} = \frac{1}{V_T} \cdot I_s \cdot e^{V_{BE}/V_T} = \frac{I_c}{V_T}$$

$$\therefore \quad g_m = \frac{I_c}{V_T}$$

Using the above relation in $R_n$ we can write

$$R_n = \frac{B}{g_m}$$
Output Impedance.

\[ R_0 = \lim_{\Delta V_{CE} \to 0} \frac{1}{\Delta I_C/\Delta V_{CE}} = \frac{1}{\frac{dI_C}{dV_{CE}}} = \frac{V_A}{I_C} \]

where \( I_{CO} = I_C @ V_{CE} = 0 \).
SMALL-SIGNAL EQUIVALENT CIRCUIT

* The small-signal equivalent circuit represents the linear model for a circuit which only responds to A.C. signal or change in signal. Therefore, there is no D.C. information in the model.

* Based on that principle, you can follow a simple two step process to create a small-signal equivalent circuit from any linear or non-linear circuit.

* The steps are:

  1) Replace all non-linear elements (diode, npn, pnp, etc) with their equivalent small-signal (linear) model.

  2) Make all independent D.C. sources zero. i.e.

   i) All D.C. voltage sources are short.

   ii) All D.C. current sources are open.
Example:

Assume mid-band frequency where $C_C$ is short. $V_{CC}$ short.

$n$ $p$ $n$ model

$n$ $p$ $n$ model.
PnP version

\[ V_s \]

\[ R_B \]

\[ R_C \]

\[ V_{cc} \]

\[ V_{cc} \text{ short} \]

**pnp model**

\[ i_b \]

\[ r_h \]

\[ r_o \]

\[ R_C \]

\[ V_{0} \]

\[ pnp \text{ model.} \]