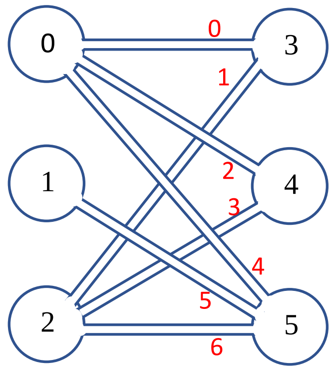
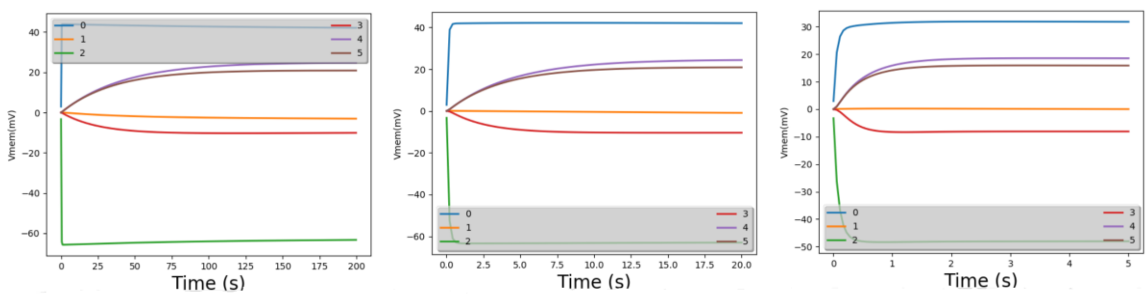
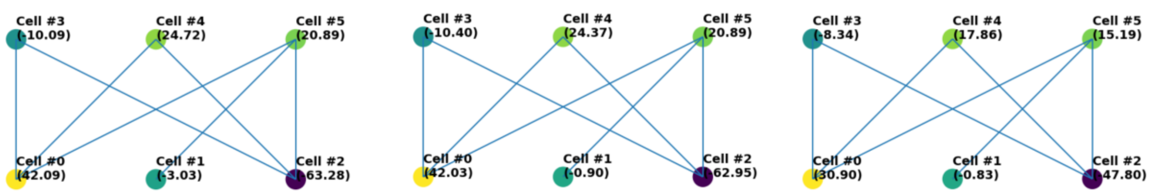
Lab #3 (Computing weighted sums)



Here are some typical graphs of V_{mem} at scale=.1, 1 and 10, and the corresponding pretty plots:

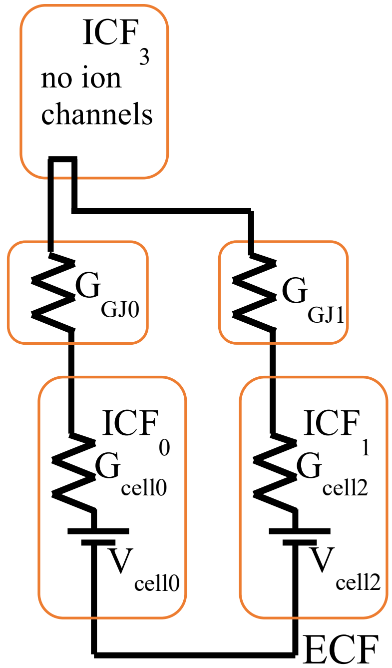




1. How close did your cells come to computing the correct answer (i.e., V_{mem})?
   As noted in the lab description, V_{mem} for the three input cells should be 44mV, 0mV and -66mV respectively. The desired V_{mem} for cell #3 is then (½ * 44) + (½ * -66) = -11mV. For cell #4 it's (⅚*44) + (⅙*-66) = 26mV, and for cell #5 it's (5/7 *44) + (1/7 * 0) + (1/7 * -66) = 22mV. For scale=.1, we got -19mV, 25mV and 21mV, which is quite close. For scale=1, they are still pretty close (but a bit worse). For scale=10, we have -8mV, 18mV and 15mV, which is not that close to the desired -11mV, 26mV and 22mV.
2. How long did it take to do the computation (i.e., for V_{mem} to settle to roughly its final QSS value)? Again, you can judge this roughly by eye.
   Based on the graphs above, it looks like we have settling times of roughly t=125, t=15 and t=2.5 seconds respectively.
3. Did the computation affect the input V_{mem} values? I.e., was the final V_{mem} for cells #0-2 affected by your computation? Note that if we had only built cells #0-2 without any GJs, their final V_{mem} values would have been 44mV, 0mV and -66mV respectively. At scale=.1, the input voltages are (42,-3,-63), which is quite close to (44,0,-66). By scale=10, they are substantially corrupted: (30,-1,-48), all moving towards a common middle ground.

To summarize, as ‘scale’ increases, the cell settles to a final value more quickly. However, the inputs tend to be corrupted and the outputs are less accurate. Finding a middle ground is a good thing!

Now for the explanations. Here is the equivalent-circuit model that we discussed in class:



When scale is large, then the GJ conductances are large and the GJ resistances are small. Essentially, the two GJ resistances form a short circuit between the two cells. This forces the two cells to collapse to some central value, as we observed.

When scale is small, and the GJ resistances are thus large, then the currents that flow are small. Small currents mean that voltages can change only slowly, which is why the circuit took longer to settle at scale=.1.

As for the accuracy, let’s do some algebra first. To simply the algebra a bit, first combine the two resistors G_{GJ0} and G_{cell0} into a single resistor G_0. Remember that resistances in series add, so if we were using resistance rather than conductance we would have $R_0 = R_{cell0} + G_{GJ0}$. Then, since conductance is the reciprocal of resistance, we have $\frac{1}{G_0} = \frac{1}{G_{cell0}} + \frac{1}{G_{GJ0}}$. Either way, we now just have two batteries (V_{cell0} and V_{cell1}) and two resistors (G_0 and G_1).

Next let’s do the circuit analysis. Define V_{out} as the output voltage in cell #3. The current flowing upwards in the left branch is $(V_{cell0}\text{-}V_{out})G_0$. The current flowing upwards in the right branch is $(V_{cell1}\text{-}V_{out})G_1$. The total current into any node must be zero, so we have $(V_{cell0}\text{-}V_{out})G_0 + (V_{cell1}\text{-}V_{out})G_1 = 0$. Rearranging gives us $V_{cell0}\, G_0 + V_{cell1}\, G_1 = V_{out}\,(G_0 + G_1)$, or $V_{out} = V_{cell0}\frac{G_0}{G_0+G_1} + V_{cell1}\frac{G_1}{G_0+G_1}$.

Clearly this is a weighted sum. If we were able to set, e.g., G_0=5 and G_1=1, then we would compute $V_{out} = \frac{5}{6}V_{cell0} + \frac{1}{6}V_{cell1}$ as desired for the second circuit. However, we are only directly setting G_{GJ0} and G_{GJ1}, not G_0 and G_1. Remember, though, that resistances in series add. If the GJ resistances are very large (i.e., their conductances are small), then the sum of $R_{GJ0}+R_{cell0} \approx R_{GJ0}$, which implies that our setting G_{GJ0}

will be a pretty good approximation to setting G_0. Thus, setting *scale* very low (which makes GJ resistances high) improves our accuracy.